PHYSICS

COURSE - 3
MODERN PHYSICS

"We may forgo material benefits of civilization, but we cannot forgo our right and opportunity to reap the benefits of the highest education to the fullest extent...."

- Dr. B.R. Ambedkar

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PREFACE

This book deals with the topics in Modern Physics in the syllabus for the Third Year of the B.Sc course offered by the Dr. B. R. Ambedkar Open University. These topics cover the core area of the subject to be studied in the third year of the Degree courses in Sciences. The syllabus is for the sake of covenience divided into Blocks, each of which comprises a number of units. Each unit generally covers specific area of the subject. The units are prepared by specialists in accordance with a format so designed as to enable the student read and understand them without much difficulty. Each unit begins with a statement of its Aims followed by the objectives to be achieved after going through the Units. Generally technical terms with which the student may not be familiar are given at the end of each block under the head glossary.

Block 1 to 5 deals with the topics in Modern Physics. Block - 6 deals with the topics in Electronics. The University hopes that this course material will help the students to get acquainted with the concept and priciples of Modern Physics and Electronics.

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CONTENTS

MODERN PHYSICS

BLOCK - 1 : ELEMENTS OF QUANTITATIVE THEORY

UNIT–1: BLACK BODY RADIATION

Contents

1.1 AIMS AND OBJECTIVES

In this unit the various laws that explain the distribution of energy in the spectrum of black body radiation are discussed.

After going through this unit.

- you will be able to describe a perfect black body.
- you will also be able to explain the distribution of energy among the wave lengths of radiation emitted by a black body.

1.2 INTRODUCTION

Heat energy can be transmitted from one place to the other not only by conduction and convection but also by radiation. Heat, like light, is transmitted through vaccum. The radiation coming from a hot body has all the properties similar to that of light like rectilinar propagation, transmission through vaccum, same velocity of propogation, inverse square law of varation of intensity with distance, reflection, refraction and transmission, polarization interference and diffraction. More generally we can say that radiation or radiant energy is a more expressive term, light being only a kind of radiant energy which has the distinctive power of affecting the retina of the human eye. Radiation can be of any wavelength from 0 to ∞ and the spectrum is known as electromagnetic spectrum.

When radiation falls on matter, depending on the nature of the surface, any or all of the processes namely reflection and transmission may take place. The reflecting power of a surface is defined as the ratio of the amount of radiation reflected by the surface to the total amount of radiation normally incident on the surface. The absorptive power of a surface is defined as the ratio of the amount of radiant energy absorbed by the surface in a given time to the total radiation incident on the surface in that same time. The transmitting power of a surface is the ratio of the amount of radiation passing through it to the total radiation incident on it. Thus if r, a and t represent the reflecting power, absorptive power and transmitting power repectively of a surface of the matter, then when radiation falls on

it, we have

$$\mathbf{r} + \mathbf{a} + \mathbf{t} = 1 \qquad 1.1$$

The values of **r**, **a** and **t** of a given surface of matter for the incident radiation depend on the wavelength of the radiation. Depending on the values of **r**, **a** and **t** we can classify the substances.

A *perfect black body* is one for which r = 0, t = 0 and a = 1. That is, all the radiation incident on the surface will be completely absorbed and no radiation is reflected or transmitted. The body appears black. Perfect black body is an ideal realisation. Lamp black and platinum black are the nearest approaches to the ideal black body.

A perfect white body is one for which r = 1, t = 0 and a = 0. As far as the visible radiation is concerned, a piece of white chalk may be considered to a best approximation as perfect white body.

Before Prevost put forward his theory of exchanges in 1792 it was thought a hot body emits hot radiation and a cold body emits, cold radiation. According to Prevost substances at all finite temperatures emit radiant energy which increases with increase in temperature and is not affected by the presence of surroundings. The rise or fall in temperature of the body is due to its exchange of radiant energy with surrounding bodies, when the radiant energy received by a body equals the radiant energy emitted by the body then the temperature of the body remains constant. The body ceases radiating when it is at 0°K.

At any given temperature a body radiates heat in all directions and the radiation contains waves of several wavelengths. The emissivity of a surface is the amount of heat radiated in unit time by unit area of the surface per unit difference of temperature between the surface and the surroundings. The emissive power of a surface is defined as the ratio of quantity of heat emitted by a given surface in a certain time to the heat emitted in the same time by a perfectly black body of the same area at the same temperature. Kirchhoff proposed a law which states that at any temperature for any wavelength (λ), the ratio of the emisive power of a body (e_λ) to its absorptive power at that temperature for that wavelength (a_λ) is constant and is equal to the emissive power of a perfect black body (E_λ). Thus

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \qquad 1.2$$

In this Unit we shall study in detail how we can realize a black body and the various theories to explain the experimental observations regarding the radiation emitted by a black body.

1.3 THE BLACK BODY

When an enclosure is maintained at a constant temperature it becomes filled with radiation characteristic of a perfectly black body. When a small hole is made on the wall of the enclosure, the radiation coming out of it will be nearly identical with the radiation coming from a perfect black emissive surface. The smaller the hole the more completely

the radiation resembles black body radiation. The black body devised by Fery is shown in Fig. 1.1. Here the radiation

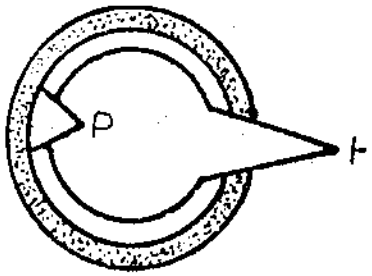


Fig. 1.1 Fery Black Body

comes out from the hole H. The conical projection P opposite to the hole is intended to avoid direct radiation from the surface opposite the hole which would otherwise make the body not perfectly black. The innerside of the enclosure is painted black. The enclosure behaves as a perfect black body towards incident radiation also. Any radiation passing through the hole into the enclosure will be reflected internally and gets absorbed.

The black body formulated by wien can be seen Fig. 1.2. Generally this model is in use nowadays. This consists of hollow cylinder C made of bronze or platinum. The inner walls of the cylinder are coated black. Porcelain Concentric tubes are wound round this cylinder. This tube is heated by passing current through the platinum foil wound round the cylinder. The radiation emmitted from the inside of the cylindrical metallic cyclinder passes through number of limited diaphragm and emmitted through H. The temperature of this black body is measured using a thermocouple T.

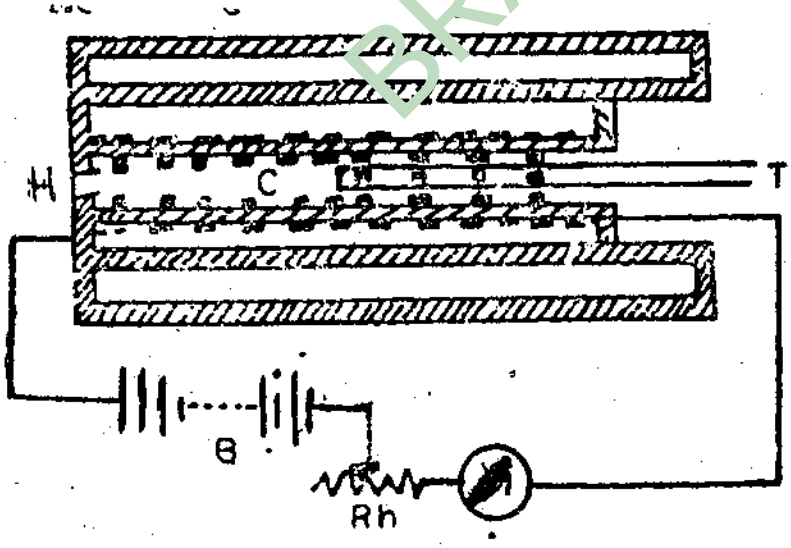


Fig. 1.2 Wiens Black Body

1.4 TOTAL RADIATION FROM BLACK BODY - STEFAN-BOLTZMANN LAW

By annalysing the experimental data of Tyndall and Dulong and Petit, Stefan in the year 1879 deduced emperically that the total radiation from any heated body is prportional to the fourth power of its absolute temperature. In 1884, Boltzmann gave a theoretical proof of the law based on thermodynamic considerations. The law which goes by the

names of the two scientists is known as Stefan - Boltzmann law and is applicable to emission from black bodies only. The law may be states as follows.

When a black body at absolute temperature T is surrounded by another black body at absolute temperature T_0, then the amount of energy E lost per second per square centimeter of the former is given by

$$E = \sigma\left(T^4 - T_0^4\right) \qquad 1.3$$

where σ is known as Stefan's constant. Its value is given by

$$5.67x10^{-5} ergs\ s^{-1}cm^{-2}K^{-4}\ or\ 5.67x10^{-8} W/m^2K^4$$

Lummer and Pringsheim investigated the emission from a black body over the temperature range 100^0 to $1260^0 C$ and found the law to hold good within the experimental error. Assuming the Sun as a black body Stefan's Boltzman law can be used to find the surface temperature of the Sun.

1.5 DISTRIBUTION OF ENERGY IN THE BLACK BODY SPECTRUM-WIEN'S LAW

A black body at a temperature emits radiation of different wavelengths. The energy distributed in different wavelengths is different. Based on thermodynamic principles Wien attempted to explain how energy is distributed over different wavelengths.

To find the distribution of energy in the thermal spectrum Wien considered an enclosure full of black body radiation expanding adiabatically with a velocity small compared with the velocity of light. He proved by thermodynamic reasoning that after adiabatic expansion the enclosure was found still full of radiation but was characteristic of a new temperature.

During the adiabatic expansion, as the perfectly reflecting walls of the enclosure move outwards the wavelengths of all the waves incident upon the walls change due to Doppler effect. If the radius of the enclosure changes from r to r_1 then wavelength λ of the radiation changes to λ_1 satisfying the following relation

$$\frac{\lambda}{\lambda_1} = \frac{r}{r_1} \qquad 1.4$$

Thus the wavelength increases in the same ratio as the radius of the adiabatically expanding spherical enclosure. By evaluting the work done by the radiation pressure during the adiabatic expansion and applying Stefan-Boltzmann law and using the relation (1.4) Wien deduced the following relation

$$\frac{T}{T_1} = \frac{\lambda_1}{\lambda}$$

$$or \lambda_1 T_1 = \lambda T \qquad 1.5$$

So λT = Constant 1.6

Eq. 1.6 is known as Wien's first displacement law. Accordingly, when the radiation of a particular wavelength corresponding to a definite temperature is adiabatically altered to another wavelength, the temperature changes in the inverse ratio. On raising the temperature of a black body the ordinates of the energy curve move towards shorter wavelengths by an amount such that the product λT is a constant.

Wien derived a relation connecting the emissive power (E_λ) of a black body and its temperature T and is given by

$$E_\lambda T^{-5} = \text{Constant} \qquad 1.7$$

The above equation is known as Wien's second displancement law, which states that the monochromatic emissive power of a black body varies directly as the fifth power of the absolute temperature..

Combining Eqs. (1.6) and (1.7) we get

$$E_\lambda \lambda^5 = \text{Constant} \qquad 1.8$$

Wien supposed that $E_\lambda \lambda^5$ should be some function of (λT).

Thus

$$E_\lambda \lambda^5 = Af(\lambda T) \qquad 1.9$$

Where A is an absolute constant. The distribution of energy in the spectrum of a black body radiation can be expressed as

$$E_\lambda = A\lambda^{-5} f(\lambda T) \qquad 1.10$$

Eq. (1.10) does not represent the actual distribution of energy in the spectrum of a black body since it contains the unknown function $f(\lambda T)$. To obtain the form of the function $f(\lambda T)$, Wien made the following assumptions regarding the emission of radiation from a black body.

1. The radiator is considered as a hollow vessel filled with a gas mixture capable of absorbing and emitting raditions of all wavelength.
2. Each molecule of the gas emits only a single wagelength which depends on its velocity

3. The energy of radiation of wavelength lying between λ and $\lambda + d\lambda$ is proportional to

a) number of molecules vibrating within the periods corresponding to the wavelength with in the limits λ and $\lambda + d\lambda$. and

b) a function of molecular velocity.

By applying Maxwell's law for the distribution of velocities and the principles of equipartition of kinetic energy Wien obtained a form of $f(\lambda T)$ and deduced the radiation formula.

$$E_\lambda = C_1 \lambda^{-5} \exp(-C_2/\lambda T) \qquad 1.11$$

Where C_1 and C_2 are constants.

The distribution of energy among the wavelengths of radiation emitted by a black body was investigated experimentally by Lummer and Pringsheim in 1899 and Wien's law was subjected to experimental verification. A carbon plug situated in an electrically heated carbon tube was used as the source of radiation. The tube was thermally insulated by surrounding it with coaxial tubes of fireclay and asbestos. A current o nitrogen was passed over the carbon tube to

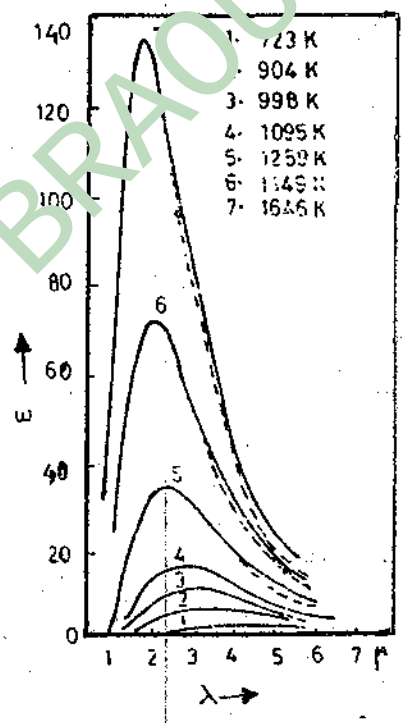


Fig. 1.3 The Energy distribution curves

E_λ = Emissive power

λ = Wave length

prevent it from oxidation at high temepratures. A thermocouple was used to measure the temperature. The radiation spectrum from the carbon tube was produced by refraction through a prism of Flourspar. Fluorspar is transparent to infrared radiation (heat radiation)

and its two strong aborption bands are outside the region of investigation. Corrections were applied to convert the prismatic spectrum into a normal one. The radiation after dispersion was focussed using silvered concave mirrors and the energy in the various parts of the spectrum was measured by means of a linear bolometer. The energy of distribution in the spectrum for termperatures between 621 and 1646 ^{0}K was obtained. The energy distribution curves obtained are shown in Fig. 1.3.

The emission is maximum at a particular wavelength (λ_m) at any given temperature and λ_m shifts to lower wavelengths as the temperature is increased. $\lambda_m T$ is found to be constant (0.2898 cm ^{0}k) in accordance with Wien's displacement law. The total radiation emitted per second at any temperature is given by the area enclosed between the curve and the X-axis and is found to be proportional to T^4 in accordance with Stefan-Boltzmann law. Within the limits of experimental error $E_m T^{-5}$, where E_m corresponds to the maximum energy radiated at given temperature at λ_m, was found to be constant obeying Wien's displacement law. However the formula given by Eq. (1.11) does not represent the variation of radiation energy at higher wavelengths accurately while the law accounts well at shorter wavelengths. As per Eq. (1.11) when $\lambda = 0$ or ∞, $E_\lambda \to 0$. Thus no energy is emitted by a wave of infinite wavelength or zero wavelength. The expermental curve as shown in Fig.1.3 does not pass through the origin contradicting the Wien distribution law. Also when $T \to \infty$, $E_\lambda = A\lambda^{-5}$ which is a finite quantity. This is wrong in view of Stefan-Boltzmann law. According to Stefan-Boltzmann law $E_\lambda = \sigma T^4$, as $T \to \infty$, $E_\lambda \to \infty$. These results indicate that the assumptions based on which the distribution law was derived may be defective some where and must be rectified. This problem was tackled by Rayleigh and Jeans in 1900 which is discussed below.

1.6 DISTRIBUTION OF ENERGY IN THE BLACK BODY RAYLEIGH-JEANS' LAW

By the time Rayleigh and Jeans took up the problem of black body radiation, Maxwell established the fact that light is an electromagnetic radiation with electric and magnetic vectors vibrating in mutually perpendicular directions and perpendicular to the direction of propagation. This concept of radiation was applied to black body radiation by Rayleigh and Jeans. According to the electromagnetic theory a black body emits radiation of continously variable wavelengths from zero to infinity. The radiation suffers multiple reflection inside the cavity giving rise to standing wave patterns. Each mode of vibration represents a standing waves. As per the laws of probability. the number of modes of vibration lying between λ and $\lambda + d\lambda$ is given by $8\pi\lambda^{-4}d_\lambda$. According to the principle of equipartition of energy the total energy per mode of vibration is given by kT where k is called the Boltzmanns constant and T Represents the absolute temperature of the system. Hence the total energy of the system consisting of $8\pi\lambda^{-4}d\lambda$ modes of vibration lying within the range λ and $\lambda + d\lambda$ is given by

$$E_\lambda d_\lambda = 8\pi\lambda^{-4}d\lambda kT \qquad 1.12$$

$$\text{or } E_\lambda = 8\pi\lambda^{-4}kT \qquad 1.13$$

$$\text{Since } C = \nu\lambda \qquad 1.14$$

Where C represents the velocity of light and ν represents the frequency of the mode, we can express Eq. 1.13 as

$$E_\nu = \frac{8\pi\nu^4 kT}{e^4}$$

Eq. 1.13 and 1.15 represent Rayleigh-Jeans' law :

Rayleigh-Jeans law was found to hold good with experimental results for long wavelengths and high temperatures, When $T \to \infty$, $E_\lambda \to \infty$, whereas in Wien's relation E_λ is finite when $T \to \infty$. Rayleigh Jeans law does not fit in with experimental data at low temperatures and short wavelengths. As per Eq. 1.13 at any given temperature when $\lambda \to 0$, $E_\lambda \to \infty$ which is against the experimental observation. This is known as ultraviolet catastrophy.

The total energy E can be given by

$$E = \int E_\lambda d\lambda = 8\pi kT \int_0^\infty \lambda^{-4} d\lambda \qquad 1.16$$

$$E = 8\pi kT \left[\frac{1}{3\lambda^3}\right]_0^\infty \qquad 1.17$$

$$E = 8\pi kT \left[\frac{1}{0^3} - \frac{1}{\infty^3}\right] \to 8\pi kT\alpha \to \infty \qquad 1.18$$

As per Rayleigh-Jeans law the total energy at any given temperature comes out to be infinity whereas it should be finite except as absolute zero it becomes zero.

Thus we find Rayleigh-Jeans' law to be successful in explaining the energy distribution in black body spectrum only at high temperatures and higher wavelengths.

Check Your Progress

Rayleigh Jeans law does not hold good at _____ temperatures and _____ wavelengths.

1.7 DISTRIBUTION OF ENERGY IN THE BLACK BODY SPECTRUM-PLANCK'S LAW

We have seen in the previous sections that Wien's law can account satisfactorily the energy distribution in the black body spectrum only at short wavelengths. Rayleigh-Jean's law accounts well the black body spectrum only at higher wavelengths. Neither of these two laws could account satisfactorily the distribution of energy in the complete spectrum of the black body. These observations indicate that the assumptions made regarding the radiation emitted by a black body based on which the Wien's and Rayleigh-Jeans law have been derived must be defective. In the year 1901 Max Planck put forward a new concept namely quantum concept regarding the radiation emitted by a black body and derived an expression for the radiation law that satisfactorily accounts for the experimentally observed

spectrum throughout the range.

Planck argued that the classical concept of continuous energy changes might be wrong and proposed that energy changes could take place discontinuously and discretely as integral multiples of a small unit of energy which is called quantum. It is assumed that the enclosure of an experimental black body is filled with linear electromagnetic oscillators. Radiation emitted by these oscillators will eventually fill the enclosure and a statistical equilibrium will be obtained at some definite temperature. Under these conditions radiation will be both emitted and absorbed by each oscillator. There exists a definite ratio between the density of radiation of any given frequency and average energy of the oscillators of that frequency.

It is assumed that the linear oscillators can vibrate with integral energy values $0,\varepsilon,2\varepsilon,3\varepsilon....n\varepsilon$ where ε represents the energy of elementary quantum. Thus if ν represents the frequency of the oscillator its energy E can be given by

$$E = nh\nu \qquad 1.19$$

here $n = 0,1,2,...n$

Where h is called the Planck's constant. Its value is 0.920 x 0.34JS. The energy of the elementary quantum $E = h\nu$.

The number of modes of vibration of the oscillators within the range of wavelengths λ and $\lambda + d\lambda$ according to classical statistics is given by $8\pi\lambda^{-4}d\lambda$. The total energy emitted by the oscillators lying in the wavelength range λ and $\lambda + d\lambda$ given by

$$E_\lambda d\lambda = 8\pi\lambda^{-4}d\lambda\overline{E} \qquad 1.20$$

where $\overline{E}$ represents the average energy per oscillator. $\overline{E}$ has been evaluated by Planck applying the laws of probability. We can write

$$\overline{E} = \frac{\sum_{n=0}^{\infty} Ee^{-E/kT}}{\sum_{n-0}^{\infty} e^{-E/kT}} \qquad 1.21$$

where $\sum_{n=0}^{\infty} Ee^{-E/kT}$ represents the total energy of all the oscillators and

$\sum_{n-0}^{\infty} e^{-E/kT}$ represents the total number of all oscillators. Substituting the value of E from Eq. 1.19 in Eq. 1.21 we get

$$\overline{E} = \frac{\sum_{n=0}^{\infty} nhve^{-nh\nu/kT}}{\sum_{n-0}^{\infty} e^{-nh\nu/kT}} \qquad 1.22$$

Put $\alpha = 1/kT$ in Eq. 1.22, we get

$$\overline{E} = \frac{\sum_{n=0}^{\infty} nhve^{-nhv\alpha}}{\sum_{n=0}^{\infty} e^{-nhv\alpha}} \qquad 1.23$$

Eq. 1.23 can be rewritten as

$$\overline{E} = -\frac{d}{d\alpha}\left[\log \sum_{n=0}^{\infty} e^{-nhv\alpha}\right]$$

Let us try to evaluate the sum $\sum_{n=0}^{\infty} e^{-nhv\alpha}$

$$\sum_{n=0}^{\infty} e^{-nhv\alpha} = 1 + e^{-\alpha hv} + e^{-2\alpha hv} + e^{-3\alpha hv} + \ldots$$

Let $x = e^{-\alpha hv}$ then

$$\sum_{n=0}^{\infty} e^{-nx} = 1 + x + x^2 + x^3 + \ldots x^n$$

Since $(1-x)^{-1} = 1 + x + x^2 + x^3 \ldots x^n$

We get

$$\sum_{n=0}^{\infty} e^{-n\alpha hv} = \left(1 - e^{-\alpha hv}\right)^{-1}$$

Now

$$\overline{E} = -\frac{d}{d\alpha}\left[\log\left(1 - e^{-\alpha hv}\right)^{-1}\right.$$

$$\overline{E} = -\frac{1}{\left(1 - e^{-\alpha hv}\right)^{-1}}(-1)\left[1 - e^{-\alpha hv}\right]^{(-2)}\left[(-1)e^{-\alpha hv}(-hv)\right]$$

Simplifying

$$\overline{E} = -\frac{hv}{\left(e^{hv\alpha} - 1\right)}$$

Substituting for $\alpha = 1/kT$ we get

$$\overline{E} = \frac{h\nu}{\left(e^{h\nu/kT} - 1\right)} \qquad 1.24$$

Using Eq. 1.24 in Eq. 1.20 we get

$$E_\lambda d\lambda = 8\pi\lambda^{-4}\overline{E}d\lambda = 8\pi\lambda^{-4}d\lambda\frac{h\nu}{\left(e^{h\nu/kT} - 1\right)}$$

or

$$E_\lambda = 8\pi\lambda^{-4}\frac{h\nu}{\left(e^{h\nu/kT} - 1\right)}$$

Using $c = \nu\lambda$ we get

$$E_\lambda = \frac{8\pi\lambda^{-4}\, hc/\lambda}{\left(e^{hc/\lambda kT} - 1\right)} = \frac{8\pi\lambda^{-5}ch}{e^{hc/\lambda kT} - 1} \qquad 1.25$$

Also we can write

$$E_\lambda = \frac{8\pi(c/\nu)^{-4}h\nu}{\left(e^{h\nu/kT} - 1\right)} = \frac{8\pi h\nu^3}{c^3}\frac{1}{\left(e^{h\nu/kT} - 1\right)} \qquad 1.26$$

Eqs. 1.25 and 1.26 represent the Planck's radiation formulae. Which fit experimental results very well. Both Wien's and Rayleigh-Jeans' formulae can be obtained from Plack's law.

(i) Deduction of Wien's law

For shorter wavelengths $e^{hc/\lambda kT} >> 1$ and hence 1 in the denominator of Eq. 1.25 can be neglected. Hence

$$E_\lambda = \frac{8\pi\lambda^{-5}hc}{e^{hc/\lambda kT}} = 8\pi\lambda^{-5}hce^{-hc/\lambda kT}$$

This relation is similar to the Wien's law Eq. 1.11, that is

$$E_\lambda = -c_1\lambda^{-5}e^{-C_2/\lambda KT}$$

Where $C_1 = 8\pi hC$ and $C_2 = hc/K$

(ii) Deducting of Raleigh-Jeans law

When λ is large, we can write

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda kT} \qquad 1.27$$

Using Eq. 1.27 in Eq. 1.25, we get

$$E = \frac{8\pi\lambda^{-5}hc}{1 + \frac{hc}{\lambda kT} - 1} = \frac{8\pi\lambda^{-5}hc\lambda kT}{hc}$$

$$E = 8\pi\lambda^{-4}kT$$

This is the same as Rayleigh-Jeans' law

Worked Example 1

Starting from Planck's law obtain the Wien's displacement law.

When $\lambda = \lambda_m, E_\lambda$ becomes maximum. Hence

$\frac{d}{d\lambda}(E_\lambda) = 0$ for a given temperature T.

Hence

$$E_\lambda = 8\pi\lambda^{-5}hc\frac{1}{\left(e^{hc/\lambda kT} - 1\right)}$$

$$\frac{d}{d\lambda}(E_\lambda) = 8\pi hc\frac{d}{d\lambda}\left[\frac{\lambda^{-5}}{\left(e^{hc/\lambda kT} - 1\right)}\right]$$

$$= 8\pi hc\frac{d}{d\lambda}\left[\lambda^{-5}\left(e^{hc/\lambda kT} - 1\right)^{-1}\right]$$

$$= 8\pi hc\left[\left(e^{hc/\lambda kT}-1\right)^{-1}(-5)\lambda^{-6}+\lambda^{-5}(-1)\right.$$

$$\left.\left(e^{hc/\lambda kT}-1\right)^{-2}e^{hc/\lambda kT}(-)\frac{hc}{kT\lambda^2}\right]$$

When $\lambda=\lambda m;\ \frac{d}{d\lambda}(E_\lambda)=0$. Hence

$$8\pi hc\left[-5\lambda^{-6}\left(e^{hc/\lambda kT}-1\right)^{-1}+\frac{hc}{kT\lambda^7}e^{hc/\lambda kT}\left(e^{hc/\lambda kT}-1\right)^{-2}\right]=0$$

or

$$5\lambda^{-6}=\frac{e^{hc/\lambda kT}hc}{\left(e^{hc/\lambda kT}-1\right)\lambda^7 kT}$$

or

$$\left(e^{hc/\lambda kT}-1\right)=\frac{hce^{hc/\lambda kT}}{5k\lambda T}$$

or

$$e^{hc/\lambda kT}\left(1-\frac{hc}{5k\lambda T}\right)=1$$

The above equation is of the form $e^x\left(1-\frac{x}{5}\right)=1$ where $x=\frac{hc}{\lambda kT}$

The roots are x = 0 or

$$1-x/5=\frac{1}{e^x}$$

or

$$x=5-\frac{5}{e^x}\text{ or }x=4.965$$

$x=0$ is absured since $\lambda\to\infty$. Since $\lambda=\lambda_{max}=\lambda_m$ we take $x=4.965$

Hence

$$\frac{hc}{\lambda_m kT}=4.965$$

or

$$\lambda_m T = \frac{hc}{k(4.9650)} = \text{Constant. This is Wien's displacement law.}$$

Worked Example 2

Obtain Stefan-Boltzmann law based on Plack's law.

The total energy emitted per unit area per second by a black body at temperature T is given by

$$E_T = \int_0^\infty E_v dv = \int_0^\infty \frac{8\pi h v^3}{e^3} \left(e^{hv/kT} - 1\right)$$

$$E_T = \frac{8\pi h^3}{C^3} \int_0^\infty \frac{v^3 dv}{e^{hv/kT} - 1}$$

Put $hv/kT = x$

$dv = \frac{kT}{h} dx$ and $v = \frac{kT}{h} x$, the above equation

Then

$$E_T = \frac{8\pi h}{C^3} \int_0^\infty \frac{k^3 T^3 x^3}{h^3} \frac{kT}{h} dx \left(\frac{1}{e^x - 1}\right)$$

$$E_T = \frac{8\pi k^4 T^4}{h^3 c^3} \int_0^\infty \frac{x^3}{\left(e^x - 1\right)} dx = A \int_0^\infty x^3 dx \left(e^x - 1\right)^{-1} \text{ where } A = \frac{8\pi k^4 T^4}{h^3 c^3}$$

$$E_T = A \int_0^\infty x^3 dx \left(e^{-x} + e^{-2x} + e^{-3x} + \ldots\right)$$

Since $\int_0^\infty x^3 e^{-rx} dx = \frac{6}{r^4}$

$$E_T = A6\sum_{n=0}^{\infty}\frac{1}{r^4} = A6\left(\frac{\pi^4}{90}\right)$$

$$E_T = \frac{8\pi k^4 T^4}{h^3 c^3}\frac{6\pi^4}{90} = \left(\frac{48\pi^5 k^4}{90h^3 c^3}\right)T^4$$

$$\therefore E_T = CT^4$$

Where C is a constant.

This is Stefan-Boltzmann law.

Worked Example 3

The maximum in the energy distribution curve of solar radiation occurs at $\lambda = 4700\,\overset{o}{A}$ Considering sun to radiate like a black body determine the surface temperature of the sun.

The Wien's displacement law derived from Planck's law as per worked example 1 is given

$$\lambda_{max}T = \frac{hc}{k(4.965)}$$

$$T = \frac{hc}{k(4.965)\lambda_{max}}$$

$$T = \frac{6.626\times10^{-34}\,Js\times3\times10^{8}\,m}{4.700\times10^{-10}\,m\times1.3805\times10^{-23}\,jK^{-1}\times4.965}$$

$$T = 6170^0\,K$$

Worked Example 4

The average amount of solar radiation the earth receives is $2.03\times10^4\,j/m^2$ /-min. If the earth re-emits this radiation in the form of black body radiation, determine the temperature of the earth.

According to Stefan-Boltzmann relation

$$E = \sigma T^4$$

Where σ is hte Stefan-Boltzmann constant.

$$\therefore T = \left(\frac{E}{\sigma}\right)^{1/4}$$

E represents the energy emitted by the black body from its surface of unit area per second.

$$E = 2.03 \times 10^4 Jm^{-2} \min^{-1} = \frac{2.03 \times 10^4 Jm^{-2} Sec^{-1}}{60}$$

$$E = 338.3 Jm^{-2} s^1$$

The Stefan-Boltzmann constant

$$\sigma = 5.67 \times 10^{-8} Wm^{-2} K^{-4}$$

or

$$\sigma = 5.67 \times 10^{-8} Js^{-1} m^{-2} K^{-4}$$

$$\therefore T = \left(\frac{E}{\sigma}\right)^4 = \left[\frac{338.3 Jm^{-2} s^{-1}}{5.67 \times 10^{-8} Js^{-1} m^{-2} K^{-4}}\right]^{\frac{1}{4}}$$

$$T = (59.66)^{1/4} 10^2 K$$

$$T = 278^0 K$$

1.8 SUMMARY

A perfect black body is one which absorbs completely all the radiation falling on it and reflects or transmits no radiation from it. Lampblack and platinum black are the nearest approaches to the ideal black body.

Prevost theory of exchanges states that substances at all finite temperatures emit radiant energy which increases with increase in temperature and is not affected by the presence of surroundings. The rise or fall in temperature of body is due to its exchange of radiant energy with surrounding bodies.

Eirchhoff's law states that any given temperature for any wavelength, the ratio of the emissive power of a body to its absorptive power at that temperature for that wavelength is constant and is equal to the emissive power of a perfectly black body. Stefan-Boltzmann law states that the energy radiated by a black body per sec per square centimeter is proportional to the fourth power of its temperature in absolute scale. According to Wien's displacement laws, (i) the wavelength at which the emissive power of a black body is maximun is inversely proportional to its temperature and (ii) the monochromatic emissive power of black body varies directly as the fifth power of the absolute temperature. Wien's radiation law is given by

$$E_\lambda = C_1 \lambda^{-5} \exp\left(-C_2/\lambda T k\right)$$

when E_λ represents the emissive power of the black body at the wavelength λ and temperature T^0K, C_1 and C_2 are constants. Rayleigh-Jeans radiation law is given by

$$E_\lambda = 8\pi\lambda^{-4} kT$$

Where k is the Boltzmann constant.

Wien's law holds good with the experimental results only at shorter wavelengths whereas Rayliegh-Jean's law explains the experimental results, that is the energy distribution in the spectrum of black body radiation, only at higher wavelengths.

Planck introduced the quantum concept regarding the energy possessed by a cavity oscillator. He proposed that the cavity oscillator cannot accommodate all possible energies but can have energy $n\ \varepsilon$ where in n is an integer and $\varepsilon = h\nu$. h represents the Planck constant and ν the frequency of the oscilator.

According to Planck's theory the average energy possessed by a cavity oscillator is given by

$$\overline{E} = \frac{h\nu}{\left[e^{h\nu/\lambda T} - 1\right]}$$

Planck's radiation formula is given by

$$E_\lambda = \frac{8\pi\lambda^{-5}}{\exp\left[\left(hc/\lambda kT\right) - 1\right]}$$

Where c represents the velocity of light.

Planck's law explains satisfactorily the experimentally observed black body spectrum. As limiting cases the Wien's law, Rayleigh-Jean's law and stefan-Boltzmann law can be deduced from Planck's law.

1.9 MODEL ANSWERS

Check Your Progress

Rayleigh Jeans' law does not hold good at low temperature and short wavelength.

1.10 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Discuss Wien's law for the distribution of energy in the radiation spectrum of a black body. Give an account of its experimental validity.

2. Derive Rayleigh-Jeans' law. Discuss its experimental validity.

3. Derive Planck's radiation law. Discuss how the Wein's law and Rayleigh-Jeans' law can be obtained from Plank's law as limiting cases.

II. Answer the following questions in brief.

1. Define and explain prevost's theory.
2. Define and explain Stefan-Boltzmann theory.
3. How to formulate a black body for experimental observations - Explain.

III. Solve the following problems.

1. The maximum in the energy distribution curve of radiation emitted by a black body occurs at $\lambda = 3500\,\text{Å}$. Determine the surface temperature of the black body.

(Ans : $8285^0 K$)

2. The maximum temperature in the fire ball resulting from nuclear bomb explosion is found to be $10^{8o} K$. Determine the wavelength at which radiation emission is maximum. Determine the corresponding quantum energy.

(Ans : $0.290\,\text{Å}$, $\varepsilon = 42.8 KeV$)

3. The intensity of radiation emitted by a black body is found to be maximum at $\lambda = 6000\,\text{Å}$. The temperature of the body is raised until its radiation energy emission is doubled. Determine the wavelength at which the intensity of radiation is a maximum.

(Ans : $\lambda_{max} = 5000e\,\text{Å}$)

UNIT–2 : PHOTO ELECTRIC EFFECT

Contents

2.1 AIMS AND OBJECTIVES

In this unit photo electric effect and compton effect are explained on the basis of quantum nature of radiation.

After going through this unit.

- you will be able to calculate the threshold frequency and also the work function of the metal emitting photo electrons.
- you will infer that compton shift depends only on the angle of scattering and is independent of the energy of incident photon and nature of the material.

2.2 INTRODUCTION

The development of the modern concept of light quanta namely photons began with Planck's ideas concerning heat radiation. Indeed, Newton proposed the hypothesis of light corpuscules, but the facts which he cited to support this view were later explained successfully by Fresnel based on wave theory of light proposed by Huygens. The particle nature of light got revived at the beginning of this century with new experimental observation. In this unit we shall study the two phenomena namely photoelectric effect and compton effect, the interpretation of which played a significant role in the development of quantum theory.

2.3 PHOTO ELECTRIC EFFECT

The phenomenon of emission of electrons by metalic surfaces when bombarded by light is called photoelectric effect. The photoelectric effect was discovered by Heinrich Hertz in 1887. The apparatus used by Hertz in its simplest form shown in Fig. 2.1.

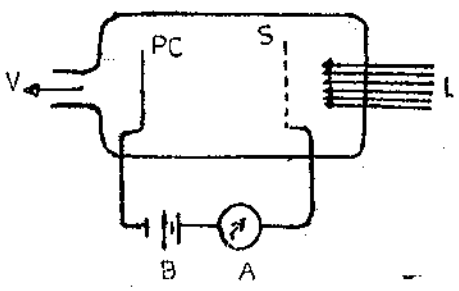


Fig. 2.1 Photoelectric effect

The glass tube contains a polished metal electrode called photocathode and a second electrode in the form of perforated metal Plate. Two electrodes are maintained at a potential difference of a few volts. The second electrode is positive with respect to the photocathode. When ultraviolet light passing through the perforated second electrode is incident on the inner surface of the photo cathode, a current is observed to flow through the tube. This phenomenon is called photelectric effect which persisted even when the tube is evacuated to very low pressures indicating that the current carriers are not geseous ions. Experiments carriedout by applying a magnetic field to the region between the photo cathode and the second electrode indicated that the current consisted of the flow of negatively charged particles. Thomson's discovery of the existence of electrons suggested that the negatively charged particles of the photoelectric effect could also be electrons. This hypothesis was confirmed by Lenard in 1900 by measuring the e/m ratio of the photo electric particles which was found to be the same as that for electrons. The experiments of Lenard also demonstrated certain-properties of the photo electric effect which were very difficult to understand based on the principles of classical physics.

The experimental observation of photoelectric effects are as follows.

1. For a given material, there is a minimum photon energy $h\nu_0$ characteristic of the material. No photo electrons would be released by radiation of frequency less than ν_0 called the threshold frequency.
2. The Kinetic energies of the emitted photoelectrons show an energy distribution that ranges from zero to a sharp energy maximum K_{max}. The value of K_{max} depends upon the incident photon energy and the work function of material. K_{max} in independent of light intensity.
3. The photoelectric current is directly proportional to the light intensity.
4. The photo electric emission phenomenon is instantaneons i.e. there is no time lag between the irradiation and the emission.
5. The phenomenon is a random phenomenon at any particular instant only one among millions of atoms of the metal will emit the photo-electron. The classical electromagnetic wave theory of radiation is found incapable of explaining the above mentioned experimental facts. For a satisfactory explanation of these facts we have to resort to an altogether new idea known as the "Quantum theory" of radiation.

Check Your Progress - I

Photo electrons will be released if the frequency of the incident radiation is _____ than the threshold frequency.

2.4 PHOTO ELECTRIC EFFECT-EINSTEIN'S EXPLANATION

Einstein proposed quantum theory of photoelectric effect in 1905 which was closely related to the quantum theory of black body radiation put forth by Max Planck. Einstein considered that the Planck's requirement that the energy content of the electromagnetic waves of frequency ν in a radiant source can only be, 0, $h\nu$ or $2h\nu$ or $3h\nu$ ..... or nh, implied that in the process of going from energy state $nh\nu$ to another energy state (n-1) $h\nu$, the source would emit a burst of emitted energy, is initially localized in a small volume of space and remains localized as it moves away from the source with velocity C instead of spreadingout in the manner characteristic of waves. He also assumed that the energy content E of the bundle called quantum of energy is related to the frequency $h\nu$ by the

following equantion.

$$E = h\nu \qquad 2.1$$

Einstein also assumed that in the photoelectric effect a quantum is completely absorbed by an electron in the photocathode.

To arrive at the photoelectric equation derived by Einstein, consider the box model of a metal crystal as illustrated in Fig. 2.2.

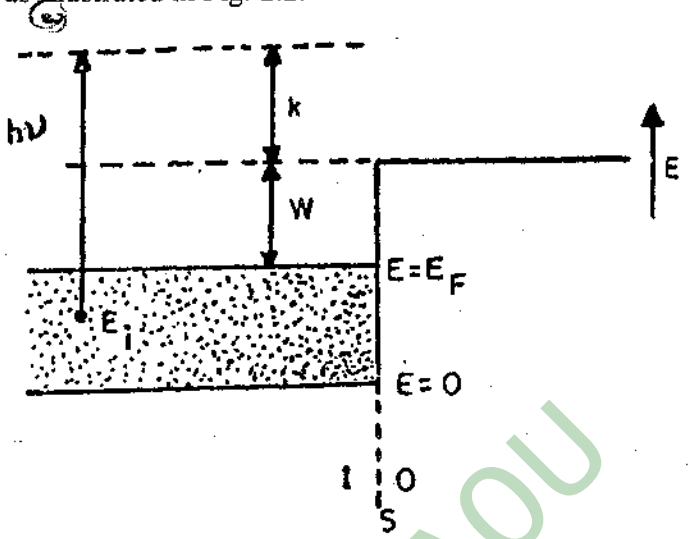


Fig. 2.2 Box model of a metal Crystal indicating Photo electric emission
I - Inside, O - Outside, S - Crystal Surface.

The necessary condition for photo electric emission to take place is $E > E_F + W$ where E_F represents the Fermi-energy of the electrons that are at the surface. W represents the work function of the metal defined as the amount of energy that must exceed E_F so that the electron barely escapes from the metal surface. When an electron having an energy E_i inside the (crystal) metal absorbs a photon and escapes, then its kinetic energy outside the metal surface is given by

$$K = (E_i + h\nu) - (E_F + W) \qquad 2.2$$

For photons of given frequency, the kinetic energy of the electron is largest when its internal energy is largest. The maximum value of E_i is E_F. Hence the maximum kinetic energy the electron emitted is given by

$$K_{max} = (E_F + h\nu) - (E_F + W) \qquad 2.3$$

or

$$K_{max} = (h\nu - W) \qquad 2.4$$

$$\text{or } \frac{1}{2}mv^2 = h\nu - W$$

Eq. 2.4 is called Einstein's photoelectric equation.

All the known features of the photoelectric emission can be explained on the basis of the above (2.4) equation as described below :

1) the velocity of the photoelectron is directly proportional to the frequency of radiation.
2) the velocity of the photoelectron is independent of the intensity of radiation.
3) there exists a threshold frequency which varies with the nature of the emitters.
4) there is no time lag in the process.

According to the above equation, the rate of emission of photo electrons will be proportional to the flow of quanta incident upon the photo cathode. Thus photocurrent is proportional to the intensity of incident electromagnetic radiation in agreement with Lenard' observation. Eq. 2.4 also indicates that K_{max} does not depend on intensity of incident light but depends on the frequency. This prediction was tested in 1916 by Millikan who measured K_{max} for radiation in the frequency range $6 \times 10^{14} Hz$ to $12 \times 10^{14} Hz$. The variation of K_{max} with frequency is shown in Fig. 2.3.

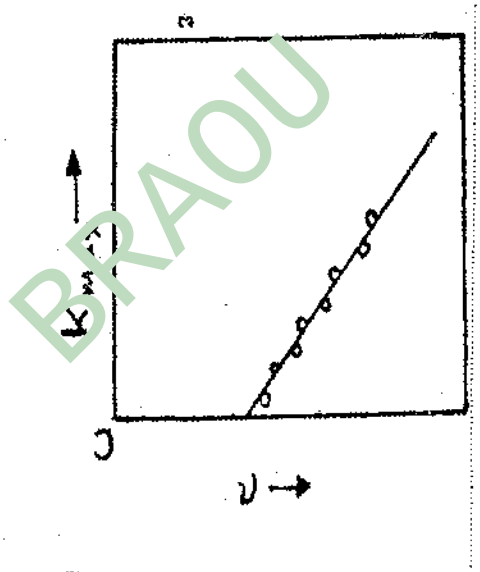


Fig. 2.3 K_{max} versus frequency ν for Photoelectrons

In the Fig. 2.3 the intercept on the X-axis gives W/h (the threshold frequency ν_0) and the value of h determined from photoelectric experiment and with the aid of Einstein's theory agreed to within better than 0.5% of the value determined by fitting Planck's theory to the experimental black body spectrum.

Worked Example 1

An experimental set up shown in Fig. 2.4 is used to measure the stopping potential V_0 which must be applied between photocathode and collector so as to reduce the photoelectron current to zero. The data obtained is as follows. For a copper surface irradiated by light of wavelength $\lambda = 1849 \overset{o}{A}$ the stopping Potential $V_0 = 2.75V$. Calculate the threshold frequency ν_0, the work function W and the maximum kinetic energy of the photoelectrons.

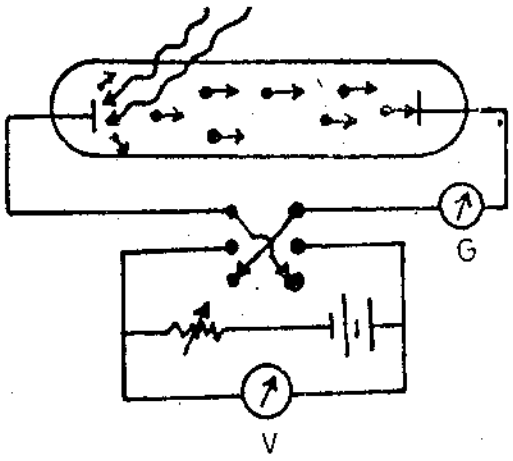


Fig. 2.4 Experimental set up to measure stopping potential in photoelectric effect

According to Einstein's photoelectric equation

$$K_{max} = (h\nu - W)$$

Also $K_{max} = \frac{1}{2}m(v^2)_{max}$

If V_0 represents the stopping potential then

$$eVo = \frac{1}{2}m(v^2)_{max} = K_{max} = 2.75eV.$$

$$\therefore eVo = h\nu - W$$

$$\therefore W = h\nu - eV_0 = \frac{hc}{\lambda} - eV_0$$

$$W = \frac{\left(6.625\times10^{-34}\,Js\right)\left(3\times10^{8}\,ms^{-1}\right)}{1849\times10^{-10}\,m} - 2.75eV$$

$$W = 1.075\times10^{-18}J - 2.75eV$$

Since $1eV = 1.602\times10^{-19}J$

$$W = \frac{1.075\times10^{-18}J}{1.602\times10^{-19}}eV - 2.75eV$$

$$W = 6.71eV - 2.75eV = 3.96eV$$

If the threshold frequency is v_0 then

$$hv_0 = W$$

$$\therefore v_0 = \frac{W}{h} = \frac{(3.96eV)1.602\times10^{-19}/J(eV)^{-1}}{6.625\times10^{-34}Js}$$

$$v_0 = \frac{3.96\times1.602\times10^{-19}}{6.625\times10^{-34}}$$

$$v_0 = 0.9575\times10^{15}Hz$$

or

$$\therefore v_0 = 9.575\times10^{14}Hz$$

Worked Example 2

In a photoelectric experiment the stopping pontential is found to be 0.18v when photocathode is irradiated by light of wave length 5461 $\mathring{A}$. When the same photocathode is irradiated by light of wavelength 1849 $\mathring{A}$, the stopping potential was found to be $4.64\,V$. Determine the Plack's constant assuming the charge of an electron to be $1.602\times10^{-19}C$.

According to Einstein's equation

$$K_{max} = hv - W = eV$$

Where V represents the stopping potential.

As per the data given in the problem.

$\frac{hc}{\lambda} - W = eV$ becomes

$$\frac{h3\times10^{8}}{5461\times10^{-10}} - W = e(0.18eV) = 0.18e$$

$$\frac{h3\times10^{8}}{1849\times10^{-10}}-W=e(4.64eV)=4.64e$$

$$\frac{h}{10^{-10}}\left[3\times10^{8}\right]\left[\frac{1}{1849}-\frac{1}{5461}\right]=e(4.64-0.18)$$

$$=+4.46\times(e)=+4.46\times1.602\times10^{-19}$$

$$h=\frac{4.46\times1.602\times10^{-19}cV\times10^{-10}}{3\times10^{8}\left[\frac{1}{1849}-\frac{1}{5461}\right]}$$

$$h=6.64\times10^{-34}Js$$

2.5 COMPTON EFFECT

A.H. Compton in 1923 discovered that, when a beam of X-rays of well defined wavelength λ_0 is scattered by a metal foil, the scattered radiation at an angle θ to the incident radiation was found to contain a component of well defined wavelength λ_θ which is longer than λ_0. This phenomenon is called Compton effect. Compton effect demonstrates more vividly the photon aspect of electromagnetic radiation.

Compton's experimental set up is illustrated schematically in Fig. 2.5. X-rays from the X-ray tube T were made to strike graphite block which get scattered in all directions. The target of the X-ray tube was molybdenum. The K_α wave length is $0.708A^0$ This monochromatic radiation was employed in the scattering experiments.

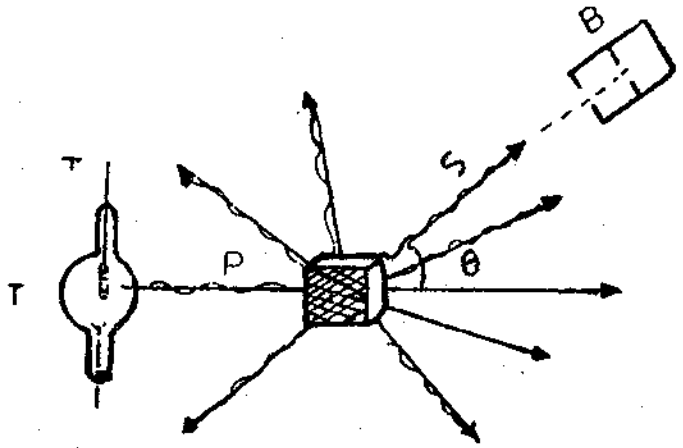


Fig. 2.5 Schemetic diagram of Compton experiment
T - Radiation tube, P - Primary time, S - Secondary line,
θ - Scattering angle, B - Bragg Spectrometer

Compton studies the wavelength of the scattered X-rays as a function of angle of scattering θ, using Bragg crystal spectrograph. He found that the rays coming from the scatterer at any particular angle, θ, except at $\theta = 0$, consist of two components of different wavelengths. One component was found to have the same wavelength λ_0 of incident radiation and the other component had a wavelength λ_0 greater than λ_θ. These two components λ_0 and λ_θ correspond to primary line (P) and modified line (M) respectively. In the forward direction where $\theta = 0^0$ there was no M line. The rays had wavelength λ_0 only. The wavelength of the M line was found to increase with the increase in the angle of scattering. Typical data for different angles of scattering is illustrated in Fig. 2.6

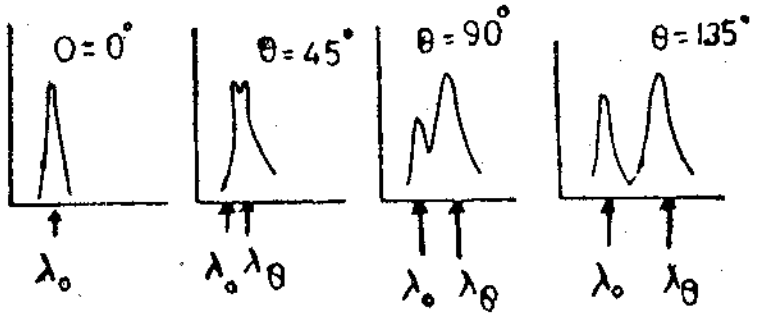


Fig. 2.6 Wavelength Spectra of X-ray quanta scattered at various angles from graphite.

$\Delta\lambda_\theta = (\lambda_\theta - \lambda_0)$ which represents the shift in the wavelength of incident X-rays as a result of scattering was found to increase with θ and is independent λ_0 and the nature of the scattering material.

According to the classical theory of electromagnetic waves when a wave of frequency ν_0 interacts with electrons in matter, its oscillating electric field drives the electrons in forced vibrations at the same frequency. These vibrating electrons then reradiate the absorbed energy, in the form of electromagnetic waves in all directions with the same frequency ν_0. Hence classical theory can not account for the observed modified line in the Compton effect. The existence of modified line M in the scattered radiation was accounted by Compton making use of quantum theory of light.

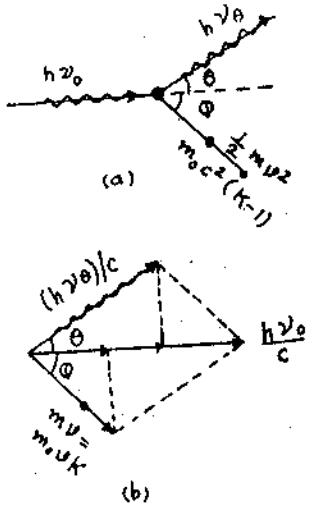


Fig. 2.7 Collision between X-ray Photon and free electron in Compton effect

Compton proposed that a single X-ray photon acting like a material particle, may collide with a free electron and recoil as through it were a perfectly elastic sphere shown in Fig. 2.7a.

Applying the law of conservation of energy to the collision, Compton assumed that the energy $\frac{1}{2}mv^2$ imparted to the recoiling electron must be supplied by the incident X-ray quantum hv_0. Having lost this energy the X-ray moves off in some new direction with a lower frequency v_θ and energy hv_θ. Applying the principle of conservation of energy to the collision process we get

$$hv_0 = hv_\theta + \tfrac{1}{2}mv^2 \qquad 2.5$$

Where m represents the mass of the electron moving with velocity v. Since in most cases the velocity of the recoiling electron is near the velocity of light c the relativistic equation has to be used. Hence

$$\frac{1}{2}mv^2 = mc^2 - m_oc^2 = \frac{m_oc^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} - m_o \qquad 2.6$$

$$m_oc^2 = \left[\frac{1}{\left(1-v^2/c^2\right)} - 1\right] \qquad 2.7$$

$$= m_oc^2(k-1) \qquad 2.8$$

where $K = \left(1-v^2/c^2\right)^{-1/2}$ 2.9

In the above equations m_0 represents the rest mass of the electron. Eq. (2.5) can be rewritten as

$$hv_o = hv_\theta + m_oc^2(k-1) \qquad 2.10$$

Considering the X-ray photon as particle which carried energy E given by

$$E = mc^2 = hv \text{ or}$$

$$E = hv = \frac{hc}{\lambda}$$

$$mc.c = hv$$

$$mc = \frac{hv}{c}$$

$$p = \frac{hv}{c} = \frac{E}{c} \qquad 2.11$$

Since photon is considererd as a particle and since all photons travel with the velocity of light the momentum of the photon is mc. The rest mass of the photon is zero.

We can write for its momentum p as

$$p = \frac{E}{c} = \frac{h}{\lambda} = \frac{hv}{c} \qquad 2.12$$

Since momentum is a vector quantity we can construct a vector diagram as shown in Fig. 2.7 b. The momentum of the photon before collision is $\frac{hv_o}{c}$ and after collision it is $\frac{hv\theta}{c}$. The momentun of the electron is m_o or no Kv. Appling the principle of conservation of momentum we get in the direction of incident X-ray beam (horizontal direction)

$$\frac{hv_o}{c} = \frac{hv\theta}{c}\cos\theta + m_o Kv\cos\phi \qquad 2.13$$

In the vertical direction

$$\frac{hv_\theta}{c}\sin\theta = m_o Kv\sin\phi \qquad 2.14$$

In the Eqs. 2.10, 2.13 and 2.14, the quantities we know are v_o and the fundamental constants c, h, and m_o. There are four unknowns namely v_θ, θ,ϕ and K and we have only three independent equations. The best way is to eliminate θ and K and obtain a relation between v_θ and θ. To do this let us assume

$$\frac{hv_o}{c} = \frac{hv_\theta}{c}\cos\theta + m_o Kv\cos\phi \qquad (1)$$

$$= \frac{hv_\theta}{c}Cos\theta + mvCos\phi$$

(as $m_o K = m$

$$0 = \frac{hv_\theta}{c} Sin\theta - mvSin\theta \qquad (2)$$

Taking equation 1 and 2

$$mvcCos\theta = hv_o - hv_\theta Cos\theta$$

$$= h(v_0 - v_\theta Cos\theta)$$

$$mvcSin\theta = hv_\theta Sin\theta$$

Squaring and adding

$$m^2v^2c^2 = h^2(v_o - v_\theta Cos\theta)^2 + h^2{v_\theta}^2 Sin^2\theta$$

$$= h^2{v_o}^2 + h^2{v_\theta}^2 Cos^2\theta - 2h^2v_o v_\theta + h^2{v_\theta}^2 Sin^2\theta$$

$$= h^2(v_o^2 + v_\theta^2 - 2v_\theta v_o Cos\theta) \qquad (3)$$

We have

$$hv_o = hv_\theta + m_o c^2(k-1)$$

$$hv_o = hv_\theta + m_o kc^2 - m_o c^2$$

$$= hv_\theta + mc^2 - m_o c^2$$

$$mc^2 = h(v_o - v_\theta) + m_o c^2$$

$$m^2c^4 = [h(v_o - v_\theta) + m_o c^2]^2$$

$$m^2c^4 = h^2(v_o - v_\theta)^2 + m_o^2c^4 + 2m_o c^2 h(v_o - v_\theta)$$

$$= h^2(v_0^2 + v_\theta^2 - 2vv_\theta) + m_o^2c^4 + 2hm_o c^2(v_o - v_\theta) \qquad (4)$$

Subtracting 3 from 4

$$m^2c^2\left(c^2-v^2\right)=-2h^2v_ov_\theta(1-\cos\theta)+2h(v_o-v_\theta)m_oc^2+m_o^2c^4$$

We have $m=m_ok$

$$=\frac{mo}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

Substitute in $m^2c^2\left(c^2-v^2\right)$

$$\frac{mo^2}{\left(1-\frac{v^2}{c^2}\right)}c^2\left(c^2-v^2\right)=\frac{m_o^2c^4\left(c^2-v^2\right)}{\frac{c^2-v^2}{c^2}}$$

$$=m_o^2c^4$$

So $m_o^2c^4=-2h^2v_ov_\theta(1-\cos\theta)+2h(v_o-v_\theta)m_oc^2+m_o^2c^4$

$$=2h^2v_0^2(1-\cos\theta)=2h(v_o-v_\theta)m_oc^2$$

$$(v_o-v_\theta)m_oc^2=hv_ov_\theta(1-\cos\theta)$$

$$\frac{v_o-v_\theta}{v_\theta}=\frac{hv_o(1-\cos\theta)}{m_oc^2}$$

$$\left(\frac{v_o}{v_\theta}-1\right)=\frac{hv_o}{m_oc\lambda_o}(1-\cos\theta) \qquad 2.15$$

$$\left(\frac{\lambda_\theta}{\lambda_o}-1\right)=\frac{h}{m_oc\lambda_o}(1-\cos\theta) \qquad 2.16$$

$$\frac{\lambda_\theta - \lambda_o}{\lambda_o} = \frac{h}{m_o c \lambda_o}(1 - \cos\theta)$$

or

$$(\lambda_\theta - \lambda_o) = \Delta\lambda_\theta = \frac{h}{m_o c}(1 - \cos\theta) \qquad \ldots 2.17$$

Eq. 2.17 represents the Compton wavelength shift $\Delta\lambda_\theta$ of the modified line in the X-rays scattered through an angle θ. $\Delta\lambda_\theta$ depends only on θ and not on any other factors like λ_o, nature of the scatterer etc.

When θ = 90

$$\Delta\lambda_\theta = \frac{h}{m_o c} = \frac{6.626 \times 10^{-34} Js}{(9.109 \times 10^{-31} kg)(2.998 \times 10^{8} ms^{-1})}$$

$\Delta\lambda_\theta = 2.426 \times 10^{-12} m$ or $0.024\ \mathring{A}$. This value is in good agreement with the shift observed experimentally at an angle $\theta = 90^\circ$. $\frac{h}{m_o c}$ is called Compton wavelength. It is the wavelength of the photon whose energy is equal to the rest energy of the electron.

The Compton equation does not give any information regarding the existence of unmodified line in the scattered radiation. In the theory of Compton effect we have assumed that the electrons which took part in the scattering process are free electrons and are ejected out from the atom. The unmodified line is due to the interaction of incident quanta with bound electrons. The bound electrons do not receive any energy from the photon and hence there is no change in the wavelength of the scattered photon.

Worked example 3

Monochromatic X-rays of wavelength. $0.708\ \mathring{A}$ are scattered by graphite material. Find the wavelength of the scattered radiation which undergoes Compton scattering at an angle of 90°. Also determine the maximum kinetic energy of the recoil electrons.

$$\lambda_\theta - \lambda_o = \frac{h}{m_o c}(1 - \cos\theta)$$

As per the data when $\theta = 90$

$$\lambda_\theta = \lambda_o + h/m_o c = 0.708\ \mathring{A} + 0.024\ \mathring{A}$$

$$\lambda_\theta = 0.732\,\mathring{A}$$

The energy of recoil is given by

$$E = h\nu_o - h\nu_\theta = h\nu_o - \frac{hc}{\lambda_\theta}$$

$$E = h\nu_o - \frac{hc}{\lambda_o + \frac{h}{m_o c}(1-\cos\theta)}$$

$$E = h\nu_o - \frac{hc/\lambda_o}{1 + \frac{h}{m_o c\lambda_o}(1-\cos\theta)}$$

$$E = h\nu_o - \frac{h\nu_o}{\left[1 + \frac{h}{m_o c}(1-\cos\theta)\right]}$$

$$E = h\nu_o\left[1 - \frac{1}{1 + \frac{h\nu_o}{m_o c^2}(1-\cos\theta)}\right]$$

or

$$E = h\nu_o\left[\frac{\left(h\nu_o/m_o c^2\right)(1-\cos\theta)}{1 + \frac{h\nu_o}{m_o c^2}(1-\cos\theta)}\right]$$

E is maximum when $\theta = \pi$

$$E = hv_o\left[\frac{2hv_o/m_oc^2}{1+2hv_o/m_oc^2}\right]$$

$$hv_o = \frac{hc}{\lambda_o} = \frac{6.626\times10^{-34}Js\left(3\times10^8 ms^{-1}\right)}{\left(0.708\times10^{-10}m\right)}\times\frac{1}{1.602\times10^{-19}J(eV)^{-1}}$$

$$hv_o = \frac{6.626\times10^{-26}\times3}{0.708\times1.602\times10^{-29}}eV$$

$$hv_o = 17.53\times10^3 eV$$

Also

$$m_oc^2 = \left(9.109\times10^{-31}kg\right)\left(3\times10^8 ms^{-1}\right)^2$$

$$m_oc^2 = 81.981\times10^{-15}kgm^2s^{-2}$$

$$m_oc^2 = 81.981\times10^{-15}kgms^{-2}.m$$

$$m_oc^2 = 81.981\times10^{-15}J$$

$$m_oc^2 = 81.981\times10^{-15}(J)\frac{1}{1.602\times10^{-19}J\left(eV^{-1}\right)}$$

$$m_oc^2 = 51.17\times10^4 eV$$

$$m_oc^2 = 0.512\times10^6 eV$$

$$\therefore E = 17.53\times10^3\left[\frac{2\left(17.53\times10^3 eV\right)}{1+\frac{2\left(17.53\times10^3 eV\right)}{0.512\times10^6 eV}}\right]eV$$

$$E = 17.53\times10^{3}\left[\frac{68.48\times10^{-3}}{1+68.48\times10^{-3}}\right]eV$$

$$E = 17.53\times10^{3}\left[\frac{0.06848}{1.06848}\right]eV$$

$$E = 1.124keV.$$

Worked Example - 4

X-rays of 0.15 meV energy are incident on a target and undergo. Compton scattering. Determine the energy of the radiation scattered by a free electron at an angle of 60^{o}. Also determine the energy of recoil of the electron.

We have

$$\lambda_{\theta} - \lambda_{o} = \frac{h}{m_{o}c}(1-\cos\theta)$$

Since $E = h\nu = \frac{hc}{\lambda}$

$$\therefore \frac{1}{\lambda} = \frac{E}{hc} \text{ or } \lambda = \frac{hc}{E}$$

$$\therefore \frac{hc}{E_{\theta}} - \frac{hc}{E_{o}} = \frac{h}{m_{o}c}(1-\cos\theta)$$

or

$$\frac{1}{E_{\theta}} - \frac{1}{E_{o}} = \frac{1}{m_{o}c^{2}}(1-\cos\theta)$$

Using the data given in the problem

$$\frac{1}{E_{\theta}} = \frac{1}{0.15} + \frac{1}{0.512}(1-\cos60^{o}) = \frac{1}{0.15} + \frac{(1-0.5)}{0.512}$$

$$\frac{1}{E_\theta} = (6.666 + 0.9766) = \frac{1}{MeV}$$

$$\therefore E_\theta = \frac{1}{7.64327} = 0.131 MeV$$

The energy of the scattered X-ray photon = 0.131 MeV.

The kinetic energy of the recoil electron.

$$E = 0.150 - 0.131 = 0.019 MeV.$$

2.6 SUMMARY

This emission of electrons by metal foils when bombarded by electromagnetic radiation is called phtoelectric effect. This effect was discovered by Heinrich Hertz in 1887. Einstein gave an explanation for the phenomenon of photoelectric effect based on Planck's ideas of the quantum nature of radiation.

The maximum Kinetic energy of the photoelectrons emitted from a metal surface having work function W when irradiated by electromagnetic radiation of frequency ν is given by

$$k_{max} = (h\nu - W)$$

This relation is called Einstein's photoelectric effect are

(i) there exists a threshold frequency ν_o for any material and no photoelectrons will be released if the frequency of the incident radiation is less than ν_o.

(ii) the maximum kinetic energy of the photoelectrons depends on the frequency of the indicent photon and workfunction of the photocathode and is independent of the intensity of the radiation.

(iii) the photocurrent is directly proportional to the light intensity, above the threshold frequency.

Compton effect is associated with scattering X-rays by materials. According to this effect when an X-ray photon collidies with a free electron of the atoms of the material, it transfers part of its energy to the electron and gets scattered. The transfer of energy from the photon to the electron manifests itself is a change in wavelength of the scattered photon. The change in wavelength $\Delta\lambda_\theta$ is given by $\lambda_\theta - \lambda_c = \frac{h}{m_o c}(1 - \cos\theta)$ where θ represents the angle of scattering.

The Compton shift $\Delta\lambda_\theta$ depends only on the angle of scattering and is independent of the energy of incident photon and nature of the material.

Both the photoelectric effect and Compton effect demonstrate the particle nature of electromagnetic radiation. The quantum concept is well established by these two experimental observations which can not be explained based on classical theories.

2.7 MODEL ANSWERS

Check Your Progress

Photo electrons will be released if the frequency of the incident radiation is less than the threshold frequency.

2.8 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. What is Photo Electric Effect? Derive Einstein's Photoelectric equation and explain how this equation explain the experimental facts about photoelectric phenonmenon.

2. What is Compton effect? Derive an expression for Compton shift when an X-ray beam undergoes scattering.

II. Solve the following problems.

1. Determine the stopping voltage required for photoelectrons emitted from a surface having a work function of 2.2 eV. When the electrons are produced by mercury light of wavelength $\lambda = 2537\,\mathring{A}$.

   (Ans : 5.5V)

2. The photoelectric threshold of sodium is $5420\,\mathring{A}$. Determine the maximum velocity of photoelectrons released when sodium is irradiated by light of wavelength $4861\,\mathring{A}$

   (Ans : $4.07 \times 10^5 ms^{-1}$)

3. A freshly cleaned surface of cesium is illuminated with monochromatic light of various wavelengths (λ) and the stopping potentials (V_6) for the wavelength are determined and the date are as follows.

λ in $\mathring{A}$	5896	5461	4358	4077	3663	3303	2830	2537
V_o in volts	0.12	0.28	0.86	1.08	1.39	1.78	2.20	2.92

   By means of graph verify Einstein's photoelectric equation. Also determine the work function of cesium and Plank's constant.

   (Ans : W = 2.04 eV; h = 6.65 x 10^{-34} Js)

4. X-rays of 0.1 MeV energy undergo compton scattering by a target. Determine the energy of radiation scattered at an angle of 45° with the direction of incidence. Determine the energy of recoil electron.

   (Ans : 0.0946 MeV; 0.0054 MeV)

5. X-rays of wavelength $0.612\,\mathring{A}$ are scattered by electrons in Compton collision. Determine the wavelength of the scattered radiation observed at an angle of 40° with the direction of incidence.

   (Ans : 0.618 A°)

UNIT–3 ; MATTER WAVES

Contents

3.1 AIMS AND OBJECTIVES

In this unit the dual nature of light is explained and also the concept of waves associated with material particles is developed.

3.2 INTRODUCTION

In this unit we shall study the diffraction of electrons

3.3 THE DUAL NATURE OF LIGHT

The seventeeth century witnessed the birth of two theories of light; the corpuscular theory propounded by Isaac Newton and the wave theory propounded by Christian Huygens. According to Newton, a source of light discharges streams of particles which constitute a beam of light. The beam suffers reflection and refraction. According to Huygens light energy from a source is propogated in all directions in the form of waves. The waves also suffer reflection and refraction. Both the theories were in vogue in England and Europe.

There was a means of deciding which theory was correct. For, according to the corpusular theory the velocity of light in an optically denser medium like water or glass should be greater than the velocity in vacuum or air. The wave theory demands that the velocity of light in optically denser media should be less than the velocity in air. A determination of the velocities of light in-different media could have settled the issue. But the method of determining the velocity of light could not be formulated due to certain paractical difficulties and the issue had to wait till the velocities were determined.

Yet, in the begining of the 19th century Thomas Yound performed an experiment the results of which could be explained on the wave model only. A beam of light from a single source was split into two beams and they were superposed on each other. There was interference between the beams resulting in the formation of alternately bright and dark bands. A particle model could never explan this result and yet the corpuscular theory was not rejected.

In the middle of the 19th century Jean Bernard Leon Foucault succeed in determining the velocities of light in air and in water. Water is an optically denser medium. The velocity of light in water was found to be lesser than the velocity in air. It was thought that this result heralded the death knell of the corpuscular theory. Thus the controversy seemed to

have ended finally in favour of the wave theory.

After a few decades the corpuscular model had to be revived in another form. Rays of light of higher frequency, like the ultra-violet and X-rays, while interacting with matter produced effects which could not be explained on the wave model. They were the photoelectric and Compton effects. The corpuscular model had to be revived to explain these effects. Thus, the electromagnetic radiation displays the wave nature in some experiments and the corpuscular nature in some others. Instead of cogitating on the question: "Is radiation particle-like or wave-like" the dual nature had to be accepted.

3.4 THE CONCEPT OF MATTER WAVES

This turn of events gave rise to a pertinent question : "when radiation sometimes display the characteristics of particles, ie., matter, why not matter like wise sometimes display the characteristics of waves?" Generally, scientists have belief in the symmetry of Nature. So taking up the question seriously L. de Broglie developed a thought process which culminated in the postulation of 'matter waves'.

A photon of energy E has a frequency ν given by

$$E = h\nu \qquad 3.1$$

According to the theory of relativity, the energy E, the rest mass m_o and the momentum p of a particle are related by

$$(E/c)^2 = p^2 + m_o^2 c^2$$

A photon has no rest mass, So $m_o = 0$

Hency

$$(E/c)^2 = p^2$$

or

$$E/c = p$$

Substituting the values of E from eqn. 3.1 we have

$$\frac{h\nu}{c} = p$$

or

$$\frac{h}{\lambda} = p \qquad \text{since } c = \nu\lambda$$

Which is in accordance with the results of Compton effect.

or $$\lambda = \frac{h}{p} = \frac{h}{mv}$$

From the above equation the wavelength associated with a beam of electrons of kinetic energy 50 eV can be calculated. The velocity of the electrons can be found the relation.

$$K.E. = K = \frac{1}{2}mv^2$$

it yields

$$v = \sqrt{2K/m}$$

$$= \sqrt{\frac{2 \times 50 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= 4.078 \times 10^6 m/Sec.$$

The momentum is given by

$$p = mv$$

$$= 9.1 \times 10^{-31} \times 4.078 \times 10^6$$

$$= 3.711 \times 10^{-24}$$

The associated wave length λ is given by

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{3.711 \times 10^{-24}}$$

$$= 1.778 \text{Å}$$

This is suggestive of the wavelength of X-rays. This is also of the same order as the spacing of atoms in crystals.

3.5 EXPERIMENT OF DAVISSON AND GERMER

The similarity between X-rays and de Broglie matter waves suggested that the method of experimenting on the diffraction of X-ray could be extended to matter waves. In 1927 D.J. Davisson and L.H. Germer performed an experiment using a beam of electrons instead of X-rays. Their apparatus is shown in Fig.3.1. A tungsten filament. F is heated electrically. The emitted elecrons are accelerated by a variable potential difference V between the filament and the plate, so that the electrons emerge with a kinetic energy eV, in the form of a fine parallel pencil. This arrangement acts as an electron gun. The beam travels in free space and falls normally on single crystal of nickel at C. The apparatus is enclosed in high vacuum chamber. The reflected electrons are collected by a deterctor D, which is a Faraday cylinder. This cylinder is surrounded by another cylinder to which is applied a retarding potential.

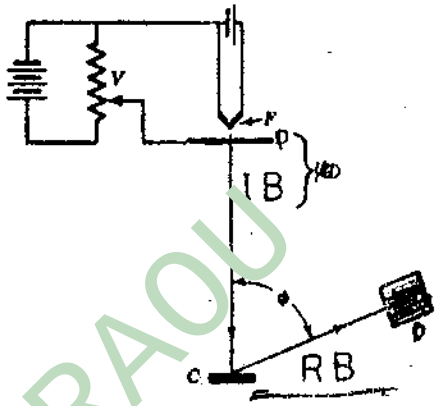


Fig. 3.1 The arrangement of the appartus of Davisson and Germer IB - Incident Beam, F - Filament, RB - Reflected Beam, C - Single Crystal, D - Detector, P - Plate.

This is to allow into the cylinder only those electrons whose velocity after reflection is of the same order as the velocity before reflection. This also prevents electrons of lower energy from entering D. The detector is set at a particular angle ϕ to the incident beam and the readings of the intensity of the reflected beam is noted for different values of V. Fig. 3.2 shows that the most intense beam occurs for values of $\phi = 50^{\circ}$ and V = 54 volts.

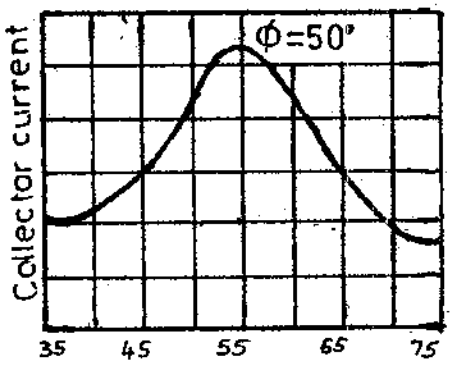


Fig. 3.2 The relation between the collector current of Fig. 3.1 and the Kinetic energy of the incident electrons at angle 50°

On the assumption that the electron beam has a wavelength given by de Broglie relation, $\lambda = h/p$, all strong reflections can be explained.

The wave length calculated from this formula for 54 eV comes out to be $1.64\,\mathring{A}$. This wavelength is of the order of the wavelengths of X-rays and X-rays have already given us information about the spacing of the atomic planes in nickel crystal. In the experiment the scattering angle is 50°. Hence the angle of incidence and reflection should each be 25°. It is evident from Fig. 3.3 that the reflecting planes in the crystal are inclined at 25° to the surface of the crystal. The distance between the successive reflecting planes is obtained as

$$d = 2.15 \sin 25^{o}$$

$$= 0.915\,\mathring{A}$$

From the Bragg formula

$$\lambda = 2d \sin\theta$$

We have

$$\lambda = 2 \times 0.915 \sin 65^{o}$$

$$= 1.67\,\mathring{A}.$$

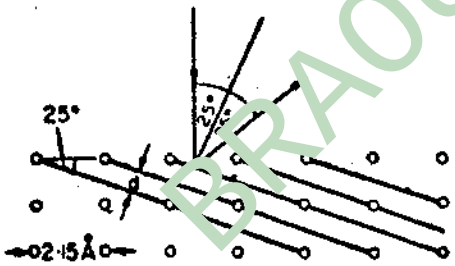


Fig. 3.3 Reflection of electrons from the crystal planes of a nickel crystal

The close agreeement between the calculated or predicted and observed wavelength of the electron beam clearly establishes the validity of de Broglie matter waves.

3.6 THE EXPERIMENT OF THOMSON AND REID

An year after Davisson and Germer performed their experiment on electron beams, **G.P. Thomson and A. Reid** hit upon a method of using high energy electrons producted in a discharge tube B maintained at low pressure and subjected to voltages lying between 10,000 and 60,000 volts. Their apparantus is shown in Fig. 3.4 - The electron beam was restricted to a narrow pencil by means of approprite stops. The thin electron beam is then passed through a metallic foil F. The foil is made up of innumerable microscopic crystals. But, they were arranged in a random fashion Yet, some of them were so oriented as to give reflection according to the Bragg formula. The thickness of the foil was about 10^{-6} cm and it was specially prepared. A very high vacuum was maintained in B. The ring pattern produced was photographed. A number of metals like gold, platinum and aluminium were used. By measuring the angular.

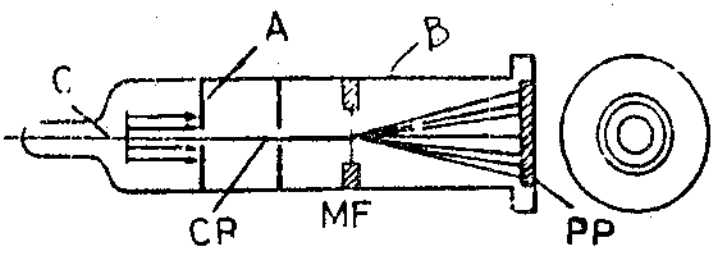


Fig. 3.4 The experimental arrangment of Thomoson and Reid id.
PP - Photographic Plate; MF - Metal Foil; CR - Cathode Rays; A - Anode;
C - Cathode B - Discharge tube

diameter of the rings, Thomson calculated the wavelength of the wave associated with electrons. There was perfect aggreement between the sizes of the crystal unit cell obtained by electron diffraction and by X-rays. For example, in the case of gold the electron diffraction value was 4.08 Å . While the value obtained from X-rays was 4.06 Å .

3.7 THE PRINCIPLE OF COMPLEMENTARITY

At the first sight the wave-particle duality seems to be irreconcilable. A wave is extended and a particle is localised; a wave has a wavelength and velocity inspace a particle has mass and velocity when it moves. These characteristics appear to be quite contrary to one another. And yet we have to accept that the electromagnetic waves sometimes display the properties of particles, and photons., Electrons are particles and they sometimes show the properties of waves. How to eventually reconcile two apparently irreconcilable aspects?

By experience we know that energy is transported in form of waves or particles. Consider the disturbance caused on the still surface of a pond when a stone is dropped. Waves clearly appear. Energy is transported in all directions on the surface of water by the movement of the surface. In this case the phenomenon is actually visualised. The wave model is used to explain the characteristics of energy transmission. The wavelengths of the waves could be computed. The particle model has obviously no place here.

Consider a tennis ball in flight as the play goes on. At every instant it has mass, velocity and position. It requires lots of energy to play tennis and hit the ball in the desired direction. In this case energy is transported by a particle though it is big. The particle model excelently describes the flight characteristics of the ball. The wave model cannot be applied here.

So far our experiences are clear cut. In both the cases there was not an iota of doubt as to which model should be chosen to fully describe and explain the observed phenomenon. In both the cases the objects are macroscopic and do not require our imagination to be unduly stretched.

Sound is a form of energy. In our attempts to explain the transmission of sound the wave model is applied, though the waves are not visible as in the case of water waves. The idea of water waves comes to our rescue. Like water waves, sound waves are reflected, refracted, diffracted and suffer interference. These effects clearly drive us to choose the wave model.

In the kinetic theory of gases, a gas is supposed to be an aggregate of molecules, invisible, but could be treated as spheres that are always in motion within the confines of a vessel. Each has mass, velocity and position at a given instant. The concepts of temperature and pressure are explained in terms of the kinetic energy and momentum of the molecules. The molecules are supposed to move with all sorts of velocities and the statistical result bears out the fact. The particle model is the only suitable one to explain the behaviour of gas. The wave model has no place here.

Light is a form of energy, electromagnetic in nature. A light wave has a frequency, wavelength and velocity. In terms of which model is the propagation of light to be explained? Fortunately or unfortunately both the wave and particle models are used to fully explain the characteriestics of light. It is interesting to note that the corpuscular model proposed by Newton and the wave model proposed by Huygens were both in vogue for a pretty long time till the velocity of light in air and in water were determined by about the middle of the 19th century. Then the wave model scored: but its victory was shortlived.

With the advent of the photoelectric and Compton effects, the particle model had to be revived with as much gusto as the wave model. For, in the two above mentioned effects the electromagnetic waves behave as if they are particles. The so called particle was christened PHOTON. Photons carry energy. The frequency of the wave decides the amount of energy carried by a photon. If ν is the frequency of the wave and E the energy of its photon, they are related by

$$E = h\nu$$

Where h is Planck's constant. The higher the frequency of the wave the greater is the energy of its photon. Comparatively high frequencies are capable of revealing the particle nature. In such cases the wave model does not function. It lies suppressed in favour of the particle model.

It is obvious that depending upon the result of an experiments the appropriate model is to be chosen. In any case one model is to be accepted and the other rejected.

The "ultraviolet catastrophe" which is said to have paved the way for the emergence of the quantum theory is a pointer in this direction. In relating the energy with the frequency of radiation emitted by a hot body, classical physics arrived at results not compatible with observations. It was only the quantum concept of radiation that could explain the results in full. It may not be a far fetched idea to think that as the frequency of the electromagnetic wave increases the energy gets more and more localised, so that in the ultraviolet and higher frequency regions the particle model gains validity, while in regions of lower frequencies the wave model becomes appropriate. Just as the wave model cannot be applied to higher frequencies, the particle model cannot be applied to lower frequencies. They are mutually exclusive. Thus, depending upon the results of an experiments only one model is to be chosen to the exclusion of the other.

The electron, which in many experiments behaves as a particle is said to have waves associated with it. They are known as "matter waves". In the early stages of the idea of a wave model applicable to electrons was not even imagined. Many experiments revealed their wave characteristics by undergoing diffraction similar to light waves. Then arose the inevitable necessity for turning towards the wave model. Thus matter waves have come to be accepted.

In the case of electrons each time only one model becomes operative; for example, the particle model is general and the wave model is used in the case of diffraction.

Thus, duality is inherent in waves and particles. This duality surfaces in the case of subatomic particles and the lightest atoms, for, in other cases the associated wavelengths are so small as to elude measurement. It is enough to remember that only one model works at a time. As a crude analogy consider the case of coins. A coin has a head and a tail on either side. When it is tossed and comes to rest either the head or the tail is seen, sometimes the head and at other times the tail. Both the head and the tail of a coin never appear at the same instant. One is exclusive of the other. But, this situation does not baffle us, because we are able to see the head and the tail alternately by holding the coin in the hand. And so the question "how can a coin have a head and a tail?" does not even strike us.

The wave and particle models are not at cross roads, they are only complementary.

3.8 SUMMARY

The corpuscular theory and wave theory of light which existed together for a pretty long time ultimately lead to the acceptance of the dual nature of electromagnetic radiation.

This dual aspect of electromagnetic radiation gave rise to the concept of matter waves by de Broglie, He showed that material particles have waves associated with them and derivaed a relation between the momentum of the particle and the wavelength of the wave associated with it.

Davission and Germer performed an experiment by allowing a beam of electrons to fall normally on a nickel crystal and observed the intensity of the diffracted beam at different angels and for different energies at electrons. The results of the expeiment showed that the value of the measured wavelength was in close agreement with the value calculated theoretically.

Thomson and Reid performed experiments with high energy electrons and found that the observed value and the calculated value of the wavelength of the associated wave were in close agreement.

The principle of complementarity shows that though the concepts of wave and particle are mutually exclusive, both waves and particles under certain conditions are capable of revealing their complementary nature.

3.9 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Trace the development of the concept of matter waves
2. Describe an experimental in which the wavelength of the waves associated with electrons were determined.
3. Discuss the concept of complementary proposed by bohr.

II. Answer the following questions briefly.

1. What is meant by the de Broglie matter waves

2. What are the features of the experiment of Thomson and Reid?

II. Solve the following problems.

1. A body of mass 1 gram moves with aspeed of 3 m/sec. calculate wavelength associated with it.

   (Ans : 1.1×10^{31} m)

2. A proton is accelerated by a potential difference of 2000 volts calculate the wavelength associated with it.

   (Ans : 0.64×10^{-12} m)

3. Calculate the wavelength with 5 kilovolt electron beam

   (Ans : 0.169×10^{-10} m)

4. An electron and a photon each have a wavelength of $4\,\mathring{A}$ calculate their moment.

   (Ans : 1.65×10^{-24} kg-m/sec)

5. Find the speed of a proton whose de Broglie wavelength is $0.08\,\mathring{A}$.

   (Ans : 4.94×10^{4} m/sec)

UNIT–4 : GROUP AND PHASE VELOCITIES

Contents

4.1 AIMS AND OBJECTIVES

In this unit you are going to study different velocities in matter waves. After going through this unit you will be able to derive relationship between different types of velocities in matter waves.

4.2 INTRODUCTION

We had studied that a de Broglie wave is associated with a moving particle. Since the de Bronglie wave is associated with a moving partice the wave travels with the same velocity as that of the particle. The "wave velocity" also known as "phase velocity" is given by

$$w = \nu\lambda \qquad 4.1$$

When ν and λ are the frequency and the wavelength of the wave respectively.

But $\nu = \frac{E}{h}$

Where E is the energy of the wave and h is the plank's constant. According to mass energy relations.

Also $E = mc^2$

Where c is the velocity of light and m is the mass of the particle.

Combing these two equations

$$\nu = \frac{mc^2}{h} \qquad 4.2$$

Substituting equation 4.2 in equations 4.1

$$w = \frac{mc^2\lambda}{h}$$

Using the de Bronglie relations $\lambda = \frac{h}{mv}$

$$W = \frac{mc^2}{h} \times \frac{h}{mv}$$

or $W = \frac{c^2}{v}$

From this equation two conclusions can be draw

1. The wave velocity and particle velocity are different.
2. Since the particle velocity is always less than c, wave velocity w will be greater thanc. This controdicts the fundamental postulate of the theory of relatioity.

For avoiding this difficulty the concepts of wave velocity or phase velocity and group velocity are made more clear.

4.3 CONCEPTS OF DIFFERENT VELOCITIES IN MATTER WAVES

The wave associated with a moving particle corresponds to a wave packet or a group of waves. Each component of the wave propogates with a definite velocity called the wave velocity or phase velocity. When this distubance consists of number of component waves each travelling slightly with different velocities, the resultant velocity is that of the group, is known as group velocity. We will try to desire a relation between particle velocity, group velocity and phase velocity, group velocity.

4.4 RELATIONSHIP BETWEEN DIFFERENT VELOCITIES

4.4.1 Relationship between group velocity and particle velocity.

Let v, g and w be the velocities of moving particle, group of waves and a wave comprising the group respectively. Let λ be the wavelength of the individual wave.

The group velocity of a system consisting of number of waves is found to be equal to

$$g = w - \lambda \frac{dw}{d\lambda} \qquad 4.3$$

In the free space $\frac{dw}{d\lambda} = 0$ and $g = w$

But in the dispensive medium, $g < w$ and hence $\frac{dw}{d\lambda}$ is positive.

Multiplying and dividing the right hand side of the equation 4, 3 with λ^2, it can be written as,

$$g = \lambda^2\left(\frac{w}{\lambda^2} - \frac{1}{\lambda}\frac{dw}{d\lambda}\right)$$

$$\text{or} \quad g = -\lambda^2\left(\frac{1}{\lambda}\frac{dw}{d\lambda} - \frac{w}{\lambda^2}\right) \qquad 4.4$$

Using the principle of differentiation, equ. 4.4 can be written as

$$g = -\lambda^2 \frac{d}{d\lambda}\left(\frac{w}{\lambda}\right)$$

But $\frac{w}{\lambda} = v$

$$\therefore g = -\lambda^2\left(\frac{dv}{d\lambda}\right)$$

$$\text{or} \quad \frac{1}{g} = -\frac{1}{\lambda^2}\left(\frac{d\lambda}{dv}\right)$$

$$= \frac{d}{dv}\left(\frac{1}{\lambda}\right) \qquad 4.5$$

If m, v and E denote the mass, potential energy and the total energy of the particle respectively, then the expression for kinetic Energy can be written as

$$\frac{1}{2}mv^2 = E - V$$

or $v = \left[\frac{2}{m}(E-V)\right]^{\frac{1}{2}}$ 4.6

The momentum p can be written as

$$p = mv\left[2m(E-V)\right]^{\frac{1}{2}} \quad \boxed{4.7}$$

According to de Broglie relations

$$\lambda = \frac{h}{mv}$$

Substituting for mv from equation 4.7 equ. and rearranging the terms

The above equation can be written as

$$\frac{1}{\lambda} = \frac{[2m(E-v)]^{1/2}}{h}$$

Substituting for $\frac{1}{\lambda}$ in equation 4.5

$$\frac{1}{g} = \frac{d}{dv}\frac{[2m(E-V)]^{1/2}}{h}$$

$$= \frac{1}{h}\frac{d}{dv}[2m(E-V)]^{1/2}$$

Since $E = hv$

$$\frac{1}{g} = \frac{1}{h}\frac{d}{dv}[2m(hv-V)]^{1/2}$$

$$= \frac{1}{h}\frac{1}{2}[2m(hv-V)]^{-1/2},\ 2mh$$

$$= m[2m(hv-V)]^{-1/2}$$

$$= \left[\frac{m}{2(hv - V)}\right]^{1/2}$$

$$= \left[\frac{m}{2(E - V)}\right]^{1/2}$$

$= \frac{1}{v}$ from equation 4.6

or $g = v$.

That is the group velocity of a deBroglie wave group associated with a moving particle is the same as the particle velocity.

The wave velocity corresponds to the velocity of the individual particle, comprising the group and the group velocity is the velocity with which the energy travels.

4.4.2 Relation between Group Velocity and the Phase Velocity.

This relation holds good for non relativistic free particles, that is for the particle whose velocity is not camparable to the velocity of light. Let W and g be the wave or phase velocity and group velocities for a non relativistic free particle of mars m. If λ and v are the wavelength and frequency of the wave respectively then the wave velocity.

$$w = v\lambda \qquad 4.8$$

According to de Broglie by hypothesis

$$\lambda = \frac{h}{mg} \qquad 4.9$$

The energy E of the free particle can be written as its kinetic energy $\frac{1}{2}mg^2$. We know that $E = hv$.

$$\therefore v = \frac{E}{h}$$

$$\text{or } v = \frac{1}{2}\frac{mg^2}{h} \qquad 4.10$$

Substituting equations 4.9 and 4.10 in equation 4.8

$$W = \frac{1}{2}\frac{mg^2}{h} \cdot \frac{h}{mg}$$

$$= \frac{1}{2}g$$

That is the wave velocity or phase velocity is half of the group velocity.

4.5 SUMMARY

The concepts of wave velocity, group velocity, were explained using de Bronglie concept of matter waves. The relations between group velocity, particle velocity and group velocity, phase velocity were derived.

4.6 MODEL EXAMINATION QUESTIONS

I. Answer the following question in detail.

1. With the help of the de Broglie equation derive the relation between phase velocity and particle velocity.

II. Answer the following question in brief.

1. Show that wave velocity is half of the group velocity.

UNIT–5 : UNCERTAINITY PRINCIPLE

Contents

5.1 AIMS AND OBJECTIVES

In this unit you will study Heisenbarg's uncertainity principle and its consequences. After going through this unit.

- you will be able to observe that the uncertainity principle finds a place only in operation of sub atomic particles.
- you will be able to know that, with the help of this principle the probable position of the electron can be known.

5.2 INTRODUCTION

In the sub atomic state, two physical quantities like position and momentum of a particle cannot be determined with the same accuracy. This is clearly explained in this unit.

5.3 UNCERTAINITY PRINCIPLE

It looks surprising that a principle under the caption uncertainity principle should have found a respectable place in science. But there is no reasons for surprise. The uncertainity principle operates in the microworld only. Classical physics has all along been dealing with macro objects. When the atom was subjected to severe examination certain events made it necessary to develop a new vision to understand the characteristics of atoms and subatomic particles. Two new theories, the quantum theory and the relativity theory were proponded to succesfully meet the situation. The uncertainty principle is applicable to the micro world only. In the macro world classical physics still holds good. The predictability of the eclipses of the sun and the moon and the successful launching of satellite are clear indication of its validity.

The need for the principle arose during the investigation of the structure of the atom. Consider the case of an electron in motion. It has to be observed through a microscope. The observation has many aspects. First, the limitations of the microscope have to be taken into account. Certain conditions have to be satisfied before two points, closely situated, are seen as two separate points. Due to the wave nature of light, the image of a point in a microscope does not appear as a point but as a small disc. The two nearby points can be seen separately only when the disc images do not overlap. Suppose a light of wavelength λ is used to illuminate the points. The smallest distance between the points for which the images appear distinctly can be calculated. Suppose that a particle is observed under the microscope Fig. 5.1 at P. Join P to the end points of the diameter of the lens. Let the angle subtended by these lines be α. If there be two points at P and their images are to be

distinctly seen, by optical considerations it can be shown that the minimum distance between the points is given $\lambda/\sin\alpha$ where λ is the wavelength of the light used for illumination.

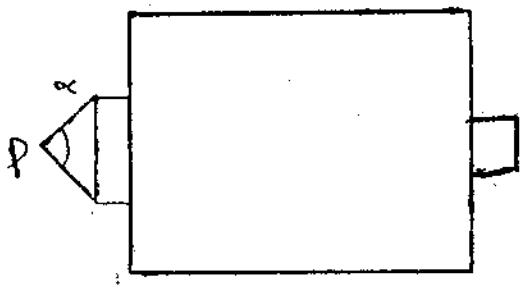


Fig. 5.1 Particle viewed through a microscope

The smaller the wavelength the smaller could be distance between the points for a clear view. Thus the proximity of the points can be increased by decreasing the wavelength of the light used.

Now, turning to a particle at P, it cannot be asserted that the particle lies exactly at P, since its image is a small disc. It can be anywhere near P. So, in trying to locate the exact position of the particle by observing its image we are faced with a certain amount of uncertainty, the magnitude of which is given by

$$\lambda/\sin\alpha$$

Now, suppose that the point at P is replaced by an electron of mass m moving with a velocity v in the field of vision of the microscope along the X-axis. Ofcourse the electron is illuminated. Then a photon of the illuminating light falls upon the electron, gets deflected by it and enters the eye of the observer. This is necessary for the electron to become visible. Suppose the image appears to be at P. That does not mean that the electron is exactly at P. But as stated earlier, it should be near about P within a short interval Δx, defined by

$$\Delta x = \lambda/\sin\alpha$$

Where Δx indicates the range of uncertainty in the position of the electron.

Considering the momentum of the electron, let us suppose that prior to the collision with the photon of the incident light, that is before the image is seen, its mass and velocity were most accurately determined, so that the momentum p_x was definitely known. But the moment it collided with the electron there was an exchange of momentum between the two. Hence, the momentum after collision will not be the same as the momentum collision. From the theory of the photoelectric effect it is known that a photon of fre v and wavelength λ, has an energy hv and a momentum h/λ. (For, $E = mc^2 = pc = hv$. Hence $p = hv/c = h/\lambda$). It can be shown from the principles of conservation of energy and of momentum that the momentum of the electron after the collision will not be p_x, but lies within the limits of

$$P_x - \frac{h}{\lambda}\frac{\sin\alpha}{2} \text{ and } P_x + \frac{h}{\lambda}\frac{\sin\alpha}{2}$$

So, the range of magnitude within which the momentum could be found is

$$\Delta px = \frac{h}{\lambda}\sin\alpha$$

This indicates the uncertainty in the momentum of the electron after collision. By multiplying the uncertainties in poisition and momentum we get

$$\Delta x \Delta P_x = \frac{h}{\sin\alpha}\frac{h}{\lambda}\sin\alpha$$

$$= h$$

The value of uncertainty cannot be less than the Planck's constant that is $\Delta x \cdot \Delta P_x = h$ or $\Delta x \cdot \Delta P_x > h$ or $\Delta x \cdot \Delta P_x \geq h$. The point to be noted is that the refining of the microscope or the method of observation fail to have any influence on reducing the uncertainty. For, if light of higher frequency is used to minimise the uncertainty in position of the electron, the corresponding photon being more energetic delivers a bigger flow on the electron so that the momentum after observation becomes more uncertain. If light of a lower frequency is used, the uncertainty in momentum becomes less, but, due to the longer wavelength of the light used the position of the electron becomes more uncertain. As the accuracy of measurement of one quantity, say, the position, is increased more and more, the accuracy of the other quantity, the momentum, becomes less and less. This is a clear indication of the fact that the accurate values of position and momentum cannot be obtained simultaneously.

5.4 CONSEQUENCES OF UNCERTAINTY PRINCIPLE

Similar uncertainties arise in the case of the measurement of the energy of a particle at a certain instant. It can be shown that if ΔE be the uncertainty in energy at Δt, the uncertainty in time, in accordance with the uncertainty principle

$$\Delta E \cdot \Delta t \sim h$$

Theoretically uncertainty is ubiquitous, but, practically it is confined to the micro world of subatomic particles. It is possible to determine the conditions under which uncertainty surfaces. Consider the relation

$$\Delta x \cdot \Delta P_x \geq h$$

Writing mv_x for the momentum, we have

$$\Delta x \cdot \Delta m(\mathrm{v})_x \geq h$$

The smaller the value of m, the greater will be the uncertainty of the position and the velocity.

Example

A bullet of mass 100 gm travels with a velocity of 200 m/s. It is accurate to 0.02%. Find the accuracy with which its position can be located.

The momentum of the bullet = p = mv

$$= (0.1) \times (200) = 20 \text{ kg-m}$$

The uncertainty in its momentum is

$$\Delta P = 0.0002 \times 20 = 4 \times 10^{-3}\ kg - m/\text{sec}$$

$$\Delta x = \frac{6.6 \times 10^{-34}}{4 \times 10^{-3}}$$

$$= 1.65 \times 10^{-31} \text{ meter.}$$

The uncertainty in position is of the order of 10^{-31} meter and this is so unimaginably small that we can understand how a good shikari always scores a hit, inspite of the operation of the uncertainty principle. But in the case of a particle like an electron, its mass being quite minute, the uncertainty becomes quite large as could be seen from the following example.

Example : An electron travels with a speed of 200 m/s. and has an accuracy to 0.02%. What is the accuracy with which the electron can be located?

The momentum of the electron is

$$P = mV = \left(9.1 \times 10^{-31}\right) \times \left(200 ms^{-1}\right)$$

$$= 1.8 \times 10^{-28}\ kg.ms$$

For the uncertainty in momentum we have

$$\Delta p = \left(0.0002\right) \times \left(1.8 \times 10^{-28}\right) = 3.6 \times 10^{-32}$$

Hence, the uncertainty in position is

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{3.6 \times 10^{-32}}$$

$= 1.83 \times 10^{-2}$ meter.

$= 1.83$cm.

On account of the incredibly small mass of the electron the uncertainty in position has assumed a respectable value, a value that should be taken into account. Further, the quantum uncertainties are due to the dual nature of energy and matter.

5.5 SUMMARY

Heisenberg's uncertainty principle operates in the world of subatomic particles. To observe an electron it should be illuminated. The illumination in the form of photons disturbs the electron and so its position and momentum cannot be determined accurately at the same time. Thus the measurements become uncertain. The product of the uncertainty in position and the uncertainty in momentum is never smaller than the Planck's constant h. Uncertainty appears in the measurement of any pair of quantities like energy and time. This principle is confined to the subatomic world and it has no application in the macroworld.

5.6 MODEL EXAMINATION QUESTIONS

I. Answer the following question in detail.

1. Explain and discuss the broad features of the principle of uncertainty.

II. Answer the following questions briefly.

1. What are the limitations of the principle of uncertainty?

2. How the uncertainty arises in the case of subatomic particles.

III. Solve the following problems.

1. A microscope is used to locate an electron in an atom within a distance of 0.2A. Find the uncertainty in the momentum of the electron.

   (Ans : 3.3 x 10^{-23} Kg-m/sec.)

2. In trying to measure the position and velocity simultaneously of an electron, the velocity in the X-direction is measured accurate to 10^{-4} cm/sec. Find the limit of accuracy with which it can be located along the X-axis.

   (Ans : 7.6 x 10^{2} meter.)

3. A particle travels along the X-direction with an energy $E = \frac{1}{2}mv^2$. Supposing that its X-coordinate is uncertain Δx, shows that

   $$\Delta E, \Delta t \geq h$$

   if $\Delta P_x, \Delta x \geq h$ and $\Delta t = \Delta x / v$

UNIT–6 : SCHROEDINGER WAVE EQUATIONS

Contents

6.1 AIMS AND OBJECTIVES

In this unit Schroedinger's wave equations (time independent and time dependent) were derived.

After going through unit you will be able to derive

- Schroedinger's time independent wave equation.
- Schroedinger's time dependent wave equation, and
- Explain eigen values and eigen functions.

6.2 INTRODUCTION

In this unit you will study the Schroedinger's wave equation and the concepts of eigen values and eigen functions.

6.3 SCHROEDINGER WAVE EQUATION

The quantum or wave machanics has its birth when the problem of the atom could not be adequately solved by the then existing semi-emperical theory of Niels Bohr. The basic concept in formulating quantum mechanics is the wavelike behaviour and the resulting wavelike properties of particles.

With the aid of different mathematical tools Born, Heisenberg, Schroedinger and other have interpreted the theories of quantum mechanics. When Born and Heisenberg selected matrix for their interpretation, the mathematical interpretation given by Schroedinger is based on replacing the equations of motion of classical mechnics with a wave equation, which described the same properties of matter in a certain wave process in the sense of deBrogile theory. Thus, Schroedinger's wave equation plays the maximum role in solving many of the problems in atomic physics.

6.3.1 Time Independent Wave Equation

Consider a particle moving freely along the positive X-direction. Let the mass of the particle be m and its speed v. Then it has only kinetic energy. So, its total energy E is given by

$$E = K = \frac{1}{2}m(v)^2 = p^2/2m \text{ (since p=mv)}$$

Now that the momemtum and energy are defined in terms of

$$P = h/\lambda \text{ and } E = h\nu$$

the wave associated with the particle should be monochromatic... Let its frequency be v and wavelength be λ. Such a travelling wave can be represented by a function φ, such that

$$\Psi_1 = A_1 \cos 2\pi\left\{\left(\frac{x}{\lambda}\right) - \nu t\right\}$$

The reflected wave travelling in the opposite direction will be represented by

$$\Psi_2 = A_1 \cos 2\pi\left\{\left(\frac{x}{\lambda}\right) + \nu t\right\}$$

when these are superposed the result is stationary waves represented by

$$\Psi = \Psi_1 + \Psi_2 = 2ACos(2\pi x/\lambda)Cos(2\pi\nu t) \qquad 6.1$$

This is a product of two terms the spatial dependent term

$$\Psi(x) = 2ACos\,2\pi x/\lambda$$

and a time-dependent term $f(t) = ACos(2\nu t)$. Let us confine our attention to the spatial dependent term only. The partial deriviation of $\varphi(x,t)$ is taken withrespect to x only.

We have

$$\frac{d^2\Psi}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \Psi$$

Since $\Psi(x,t) = \Psi(x)f(t)$ We have

$$\frac{d^2\Psi}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \Psi = -4\pi^2\left(\frac{p^2}{h^2}\right)\Psi \qquad 6.2$$

Equation 8.2 is developed for a free particle. But schroedinger assumed that this equation equally describe the wave behaviour of a particle subjected certain constraints. The particle can also possess potential energy V, in which case the total energy E is given by

$$E = K + V = \left(p^2/2m\right) + V$$

Hence

$$p^2 = 2m(E - V) \qquad 6.3$$

Substituting this value in Eqn. 8.2 we get

$$\frac{h^2}{2m}\frac{d^2\Psi}{dx^2} + (E - V)\Psi = 0 \qquad 6.4$$

The above equation is time-independent Schroedinger equation. It is one-dimensional. This can be extended to three dimensions. This is called the time independent wave equation.

6.3.2 Time Dependent Wave Equation

To derive this equation the fundamental wave equation is taken as

$$\Psi = A\exp\left[2\pi i\left(\frac{x}{\lambda} - vt\right)\right]$$

Here v and λ are the frequency and wavelength of the wave.

Since $E = hv = 2\pi\hbar v$ and $\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$, the above equation for Ψ can be written as

$$\Psi = A\exp\left[\frac{i}{h}(p_x - Et)\right] \qquad 6.5$$

Here A represents the amptitude of the wave, p, E, are the momentum and total energy, of the particle differentiating equation 6.5 twice with respect to x,

$$\frac{\partial\Psi}{dx} = \frac{ipA}{h}\exp\left[\frac{i}{\hbar}(p_x - Et)\right]$$

$$\frac{d^2\Psi}{dx^2} = \frac{i^2p^2}{\hbar^2}A\exp\left[\frac{i}{\hbar}(p_x - Et)\right]$$

$$= -\frac{p^2}{h^2}\Psi \qquad 6.6$$

Since $i^2 = -1$ and $A\exp\left[\frac{1}{\hbar}(p_x - Et)\right] = \Psi$

Differentiating equation 6.5 with respect to t,

$$\frac{d\Psi}{dt} = -\frac{iE}{\hbar} A\exp\left[\frac{1}{\hbar}(p_x - Et)\right]$$

$$= -\frac{iE}{\hbar}\Psi \qquad 6.7$$

Here again $\Psi = A\exp\left[\frac{i}{\hbar}(p_x - Et)\right]$

In the above two equations $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t}$ are known as partial differentials.

Total energy E of the particle can be written as sum of the potential energy v and Kinetic energy $\frac{p^2}{Z^m}$ where v is a function of x and time t.

(i.e.) $E = \frac{p^2}{2m} + V \qquad 6.8$

Multiplying both sides of the equation 6.8 with Ψ,

$$E_\Psi = \frac{p^2\Psi}{2m} + V\Psi \qquad 6.9$$

From equation 6.6 and 6.7

$$E_\Psi = -\frac{\hbar}{i}\frac{\partial\Psi}{\partial t}$$

and $p^2\Psi = -\hbar^2\frac{\partial^2\Psi}{\partial x^2}$

Substituting for $E\Psi$ and $P^2\Psi$ in equation 6.9,

$$-\frac{\hbar}{i}\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

or $$-i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2 4}{\partial x^2}+V\Psi \qquad 6.10$$

Since $-\frac{1}{i}=i$

Equatin 6.10 is the time dependent form of the Schroedinger wave equation.

6.4 EIGEN VALUES AND EIGEN FUNCTIONS

The values of energy E_n for which Schroedinger's steady state equation can be solved are known as "eigen values" and the corresponding wave functions are known as eigen functions. These are also known as proper or characteristic values and proper or characteristic functions.

These are explained with examples in the unit 8 while dealing with the application of wave equation.

6.5 SUMMARY

In this unit time independent and time dependent were derived and the concept about the eigen values and eigen functions was explained.

6.6 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in 30 lines each.

1. Derive the time independent Schroedinger's wave equation.

2. Derive the time dependent Schroedinger's wave equation.

UNIT–7 : SIGNIFICANCE OF WAVE FUNCTIONS

Contents

7.1 AIMS AND OBJECTIVES

This unit deals with the Significance of wave functions. After going through this unit you will be able to explain different types of interpretations to explain the significance of wave functions given by Millikan, Schroedinger and Born.

7.2 INTRODUCTION

Understanding wave function and its interpretation in a proper way is very important in solving different types of problems using wave mechanics Scientists gave different types of interpretations for the wave function. Three of them were found to be much useful to explain the experimental facts in physics.

7.3 SIGNIFICANCE OF WAVE FUNCTIONS

7.3.1 Millikan's Interpretation

According to Millikan wave function Ψ is an ordinary mathematical quantity introduced to fecilitate computation to explain certain experimental results like electron diffraction experiment by G.P. Thompson calculation of intersity of the waves scattered by a crystal in the direction of the detector etc., In such cases without understanding the significance of the mathematical quantity, they cannot be explained.

7.3.2 Schroedinger's Interpretation

Schroedinger explained the significance of wave function φ using the concept of charge density. In any Electro Magnetic wave system, if A is the amplitude of the wave, then the energy permit volume can be given by A^2. Hence

$$\left.\begin{array}{l}\text{The number of Photons per unit volume} \\ \text{or Photon density}\end{array}\right\} = \frac{\text{Energy per unit volume}}{\text{Energy of one photon}}$$

$$= \frac{A^2}{h\nu}$$

$\therefore$ Photon density is αA^2.

If Ψ is the amplitude of the matter waves at any point in space, then the particle density at that point may be taken as proportional to the square of the absolute value of Ψ, ie $|\Psi|^2$. If this is multiplied by the charge of the particle change density is obtained. Since charge is constant Ψ^2 becomes the measure of the charge density. In wave mechanics use different types of wave functions including functions with complex variables. In general $\Psi\Psi$ is written for $|\Psi|^2$ where Ψ^* is the complex conjugate of Ψ. In other words $|\Psi|^2$ is proportional to the number of particles crossing unit area per second. Which gives the particle density. This argument was found to give satisfactory results when wave mechanics was applied to directional distribution of photoelectrons, intensity distribution in compton scattering, the stable states of Bohr atoms, emission of spectral lines etc. If Ψ refers to a single particle, then the particle density is appreciably different from zero within some fimte region and that region is called the wave packet. Then the position of the particle within the wave packet could not be explained properly.

7.3.3 Born's Interpretation

The above idea suggested by Schroedinger was explained more elaborately by Max Born by giving a new idea and it was further elobarated in detail by Heisenberg, Bohn and Dirac. According to Max Born $|\Psi|^2$ does not measure the particle density at any point, but gives the probability of finding the particle at that point at any given moment. The value of the square of the wave functions may be real or imaginary depending upon the nature of wave function. Since the probability of finding a particle in space at a given point is real, it is taken as $|\Psi|^2$ or $\Psi^*\Psi$. More exactly the probability of the particle being in a volume element dxdydz is $|\Psi|^2$ dxdydz. The total probability of finding the particle some where within the volume dxdydz is found by intergrating $|\Psi|^2$ over the space, and it is written as $\iiint|\Psi|^2 dxdydz$

It should be equal to unity since the probability is unity. Ψ satisfying this requirement is said to be normalized. This type of interpretation could explain all the physical problems and hence it is accepted at present.

7.4 SUMMARY

The significance of wave functions was explained clearly.

According to Millikan it is a mathematical quantity introduced to explain experimental results.

According to schroedinger $|\Psi|^2$ given the particle density.

According to Max Born and his coworkers $|\Psi|^2 dxdydz$ given the probability of finding the particle within the region dxdydz. This is acceptable interpretation now a days.

7.5 MODEL EXAMINATION QUESTIONS

I. Answer the following question in detail.

1. Explain the different interpretations of the wave function Ψ.

II. Answer the following question briefly.

1. Give the Schrodinger's concept of charge density.

UNIT–8 : APPLICATIONS OF SCHROEDINGER'S WAVE EQUATION

Contents

8.1 AIMS AND OBJECTIVES

In this unit two main applications of Schroedinger's wave equations were discussed. After studying this unit you will be able to explain the energies of different states for a particle in a box and the concept of penetration of a particle though potential barrier.

8.2 INTRODUCTION

The Schroedinger equation is applied to different problems in physics to explain the experimentally observed results which could not be explained with the help of classical concept. Two main examples are taken in this unit. They are particle moving in a box and a particle penetrating through a potential barrier.

8.3 PARTICLE IN A BOX

Consider a particle of mass in moving inside a box of dimenision L along the X-direction. The particle is moving back and forth between the walls of the box. The box has insurmontable potential barrier at x = 0 and at x = L. The potential energy V of the particle is infinite at both sides of the box and zero from x = 0 to L that is within the box. In the mathematical way the same conditions can be written as

$$V = 0 \text{ for } 0 < x < L$$

$$V = \infty \text{ for } x \leq 0 \text{ and for } x \geq L$$

Since the particle cannot exist outside the box and so its wave function $\Psi = 0$ for $x \leq 0$ and for $x \geq L$. We will find the behaviour of the wave function Ψ within the box.

The Schroedinger's wave equation can be written as

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\varphi = 0 \qquad 8.1$$

Where E is the energy of the particle and h is the plank's constent.

Where C and D are constants. $e^{\beta x}$ represents as wave travelling in the positive x direction, i.e the wave transmitted into the region (ii) . $e^{-\beta x}$ represents a wave travelling in the -ve x direction i.e the reflected wave.

The Schroedinger wave equations in the third region can be written as

$$\frac{d^2\psi_3}{dx^2}+\frac{2mE}{\hbar^2}\psi_3=0 \qquad 8.12$$

ψ_3 is the wave function corresponding to the particle in region (iii)

Since $\alpha^2=\frac{2mE}{\hbar^2}$

The above equation can be written as

$$\frac{d^2\psi_3}{dx^2}+\alpha^2\psi_3=0$$

The general solution of this equation can be written as

$$\psi_3=Fe^{i\alpha x}+Ge^{-i\alpha x} \qquad 8.13$$

where F and G are contants. $e^{i\alpha x}$ represents the wave travelling in positive x direction and $e^{-i\alpha x}$ represents the wave travelling in the negative x direction. As there is no obstruction the wave cannot be reflected back. Hence $Ge^{-i\alpha x}=0$.

$$\therefore \psi_3=Fe^{i\alpha x} \qquad 8.14$$

The contents A, B, C, D, F can be determined using the following boundary conditions.

In order that ψ and $\frac{d\psi}{dx}$ be continuous at x = 0 the conditions are

$$\psi_1(at\,x=0)=\psi_2(at\,x=0)$$

$$\frac{d\psi_1}{dx}(at\,x=0)=\frac{d\psi_2}{dx}(at\,x=0)$$

Hence equations 8.9 and 8.11 can be written as

$$A + B = C + D$$

$$i\alpha A - i\alpha B = \beta C - \beta D$$

Solving these equations for A and B

$$A = \left(1 - \frac{i\beta}{\alpha}\right)\frac{C}{2} + \left(1 + \frac{i\beta}{\alpha}\right)\frac{D}{2} \qquad 8.15$$

$$B = \left(1 + \frac{i\beta}{\alpha}\right)\frac{C}{2} + \left(1 - \frac{i\beta}{\alpha}\right)\frac{D}{2}$$

Again in order that ψ and $\frac{d\psi}{dx}$ be continuous at $x = a$, the conditions are

$$\psi_2(at\ x = a) = \psi_3(at\ x = a)$$

$$\frac{d\psi_2}{dx}(at\ x = a) = \frac{d\psi_3}{dx}(at\ x = a)$$

Hence the equations 8.11 and 8.14 can be written as

$$Ce^{\beta a} + De^{-\beta a} = Fe^{i\alpha a}$$

$$\beta Ce^{\beta a} - D\beta e^{-\beta a} = i\alpha Fe^{i\alpha a}$$

Solving these equations for C and D

$$C = \left(1 + \frac{i\alpha}{\beta}\right)\frac{F}{2}e^{-\beta a}e^{i\alpha a}$$

or $$C = \left(1 + \frac{i\alpha}{\beta}\right)\frac{F}{2}e^{(i\alpha - \beta)a}$$

$$D = \left(1 - \frac{i\alpha}{\beta}\right)\frac{F}{2}e^{(i\alpha + \beta)a} \qquad 8.16$$

The normalised wave functions of the particle can be written as

$$\psi_n = \sqrt{\frac{2}{L}} Sin \frac{n\pi x}{L} \qquad 8.7$$

The graphical study of the wave functions ψ and the probility distribution ψ^2 against the given boundary conditions gives some information about the particle in a box. Fig. 8.1 shows the wave functions for the first three stationary states of a particle in box and the probability distribution functions. Max Born suggested that the quantity ψ^2 at any particular point is a measure of the probabilities and that the particle will be found in the volume element dv at any instant is $\psi^2 dv$. From the Fig. 8.1, the following points can be drawn.

1) ψ can be both positive or negative, but ψ^2 is always positive.
2) The probabality is always zero at the boundaries irrespective of the value n. When n= 1; the probability of locating the particle at a midpoint between the walls is maximum. at n = z, this probabity is zero at the mid point.

8.4 POTENTIAL BARRIER

If a particle is impinging on the barrier with an energy less than the height of th potential barrier it need not necesserily be reflected by the barrier. There is always a probability that it may cross the protential barrier and continues its forward motion. This probability of crossing the barrier is called Tunnel effect.

Consider a potential barrier of height V_o shown in Fig. 8.2. Let the particle of mass 'm' having a total energy of E travel from left to right, In the region x=0 to x=a $V_o > E$. According to classical mechanics since $E < V_o$ the particle coming from the left would be reflected. Because of the wave nature of matter, there is some probability that the particle penetrates though the barrier to the otherside of the barrier but actually the particle escapes to the other side of the barrier.

Let us divide the barrier diagram 8.2 into three regions.

In the region (i) the Schroedingers

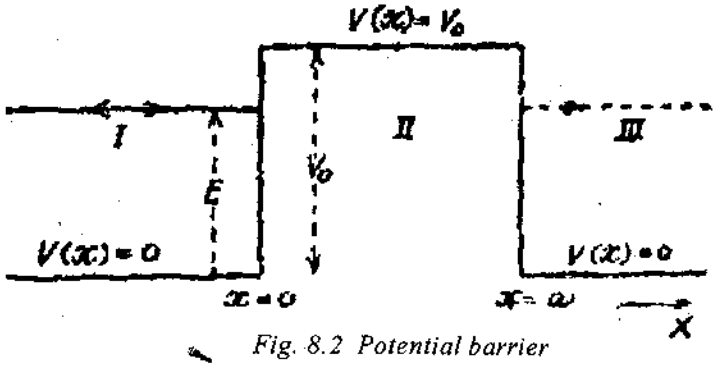


Fig. 8.2 Potential barrier

wave equation can be writtens,

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \qquad 8.8$$

where $\hbar = \frac{h}{2\pi}$

ψ is the wave function corresponding the penticle in region (i)

Let $\frac{2mE}{\hbar^2} = \alpha^2$

Hence eqn. 8.8 can be written as

$$\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0$$

The general solution of this equation can be written as

$$\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x} \qquad 8.9$$

where A and B are constants. $e^{i\alpha x}$ represents a wave travelling in the positive x directions i.e incident wave. $e^{-i\alpha x}$ represents a wave travelling in the negative x direction, that is the reflected wave.

The Schroedinger wave equation in the second region can be written as

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(E - V_o)\psi_2 = 0 \qquad 8.10$$

ψ_2 is the wave function corresponding to the penticle in region (ii)

Let $\frac{2m}{\hbar^2}(V_o - E) = \beta^2$

Eqn. 8.10 can be written as

$$\frac{d^2\psi_2}{dx^2} - \beta^2\psi_2 = 0$$

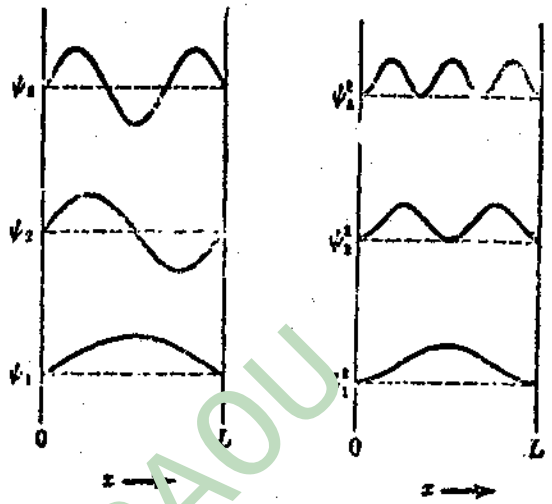
The general solution of this equation can be written as

$$\psi_2 = Ce^{\beta x} + De^{-\beta x} \qquad 8.11$$

$$\therefore \frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad 8.6$$

For each value of n there exists an energy level and the corresponding wave functions are given by eqn. 8.5. Each value of energy E_n is known as eigen value the corresponding



a) Wave function for the first three stationery states, b) probability distribution

wave function ψ_n is known as Eigen function. From the equation 8.6 it is known that particle in a box can have descrete energy values. It is very important to note that it cannot have zero energy.

It is quite clear that the particle exists only within the box. For normalised wave functions

$$\int_0^L \psi^* \psi dx = 1$$

$$\text{or} \quad \int_0^L A^2 Sin^2 \frac{n\pi x}{L} = 1$$

After intergrating we get

$$A^2 \frac{L}{2} = 1 \qquad \text{or} \quad A = \sqrt{\frac{2}{L}}$$

Let $\frac{8\pi^2 m}{\hbar^2}E = \alpha^2$

Then Equ. 8.1 can be written as

$$\frac{d^2\Psi}{dx^2} + \alpha^2\Psi = 0 \qquad 8.2$$

The general solution of this equation can be written as

$$\Psi = ASin\alpha x + BCos\alpha x \qquad 8.3$$

To evalutate the constant A and B boundary conditions are to be used. These boundary conditions are

$\Psi = o$ at $x = 0$ and $x = L$

Applying the first boundary condition that $\varphi = o$ at $x = 0$, equ. 8.3 can be written as

$0 = BCos0$

i.e. B = 0.

$$\therefore \Psi = ASin\alpha x \qquad 8.4$$

Applying second condition that $\Psi = o$ at $x = L$, equ. 8.4 can be written as

$0 = ASin\alpha L$

Since $A \neq 0 \quad \alpha L = n\pi$ or $\alpha = \frac{n\pi}{L}$

$$\therefore \Psi_n(x) = ASin\frac{n\pi x}{L} \qquad 8.5$$

Since $\alpha = \frac{n\pi}{L}; \quad \alpha^2 = \frac{n^2\pi^2}{L^2}$

But $\alpha^2 = \frac{8\pi^2 mE}{h^2}$

If the barrier is thick, we may neglect C to a first approximation. Then eqn. 8.15 can be written as

$$A=\left(1+\frac{i\beta}{\alpha}\right)\frac{D}{2}$$

Substituting for D from equations 8.16

$$A=\left(1+\frac{i\alpha}{\beta}\right)\left(1-\frac{i\alpha}{\beta}\right)\frac{F}{4}e^{i\alpha a}e^{\beta a} \qquad 8.17$$

Here our aim to find the ratio of the intensity of the transmitted wave to that of incident wave. that is transmion coeffieient T.

$$\therefore T=\frac{|F|^2}{|A|^2}$$

or
$$T=\frac{|F|^2}{|A|^2}=\frac{16e^{-2\beta a}}{\left[1+\left(\frac{\beta}{\alpha}\right)^2\right]\left[1+\left(\frac{\alpha}{\beta}\right)^2\right]}$$

This expression can be obtained easily by multiplying equations 8.17 with its own complex comjugate.

This result shows that there is a small probability for an object to penetrate a potential barrier. But according to classical theory it cannot even enter into the barrier.

The probability decreases as the thickness of the barrier increases. The transmission probability is a function of energy.

The reflection Coefficient R,

$$R=\frac{|D|^2}{|A|^2}$$

For calculating this C cannot be neglected as we had done earlied in calculating in T.

This barrier pentration problem has a number of application in physics, specially α decay in nuclear physics.

8.5 SUMMARY

In this unit two applications of Schroedingers wave equation were discussed.

In the case of a particle moving in a box expressions for the Energies and for Corresponding wave functions are derived. They are known as eigen values and eigen function.

In the case of a particle impinging on a potential barrier, it is proved that their exists a finite probabality for the particle through the potential barrier. This could not be explained by classical physics. An expression for the Transmission coefficient is also derived.

8.6 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in 30 lines.

1. Show that a particle impinging a potential barrier can penetrate through barrier.

2. Derive the expression for eigen values and eigen functions in a case of particle moving in box along x direction.

BLOCK-2 : SPECTROSCOPY

UNIT–9 : BOHR'S THEORY AND LINE SPECTRA

Contents

9.1 AIMS AND OBJECTIVES

In this unit we study the line spectra and fine structure of the Hydrogen atom with the help of Bohr's theory, Somefeld's theory and vector atom model. After going through this unit

- you will be able to estimate the energy of the electron in any Bohr orbit.
- you will be able to explain the fine structure of the hydrogen lines.
- you will be able to explain an experiment to determine critical potentials.

9.2 INTRODUCTION

The distribution of energy in the continuous spectrum of a black body could not be explained by classical theories until Max Planck introduced the revolutionary concept of radiation quanta. A similar situation existed at the beginning of this century regarding the line spectra of atoms and molecules. From the accurate data available on the wavelengths of the line spectra, certain relation were discovered empirically between the frequencies of various lines in the spectra of certain elements. These relations pointed out that the origin of characteristic line spectra might be due to some fundamental mechanism which is common to all atoms. The line spectra due to hydrogen has been studied extensively and the developments regarding the structure of the atom led Bohr in 1913 to propose a theory of the atomic structure. In this unit we study the development of empirical laws of spectral series and the Bohr's theory in explaining the line spectra due to the hydrogen atom. Further we are going to study the vector atom model and also a method to determine the critical potentials.

9.3 LINE SPECTRA

A typical apparatus that may be employed to measure atomic spectra is shown in Fig. 9.1. The source consists of an electric discharge passing through a region containing a monoatomic gas. Because of collision in atoms with electrons and with other atoms some of the atoms in the discharge tube may go to an energy state which is higher than the energy possessed by the atom in the normal state. In returning to their normal energy state these atoms give up

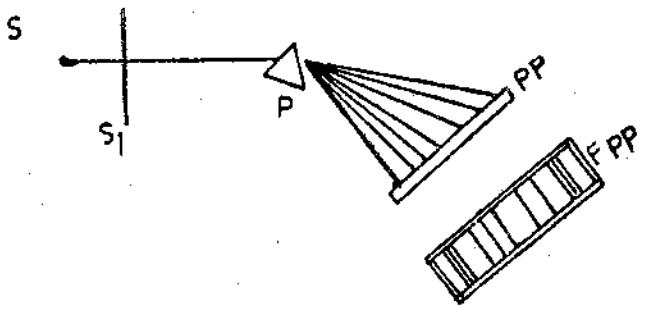


Fig. 9.1 Apparatus to measure atomic Spectra
S - Source, S_1 - Slit, P - Prism, PP - Photo Graphic Plate, FPP - Front View of PP

their excess energy by emitting electromagnetic radiation. This radiation after collimation by the slit passes through the prism and is recorded on a photographic plate. The electromagnetic radiation emitted by a free atom is found to be concentrated at a number of discrete wavelengths. Each of these wavelength components is called a line because of the line which it produces on the photographic plate. Investigations on the spectra emitted by different kinds of atoms indicated that each atom has its own characteristic spectrum.

Liveing and Dewar in 1880 observed certain similarities in the spectra of alkali metals. They observed successive pairs of lines in the arc spectrum of sodium to be alternately sharp and diffuse and were closely crowded together toward the short-wavelength end of the spectrum. In 1883 Harteley found that the difference in frequency between the components of a multiplet (doublet or triplet) in a particular spectrum were the same for all similar multiplets of lines in that spectrum.

The spectrum of hydrogen is relatively simple. Fig. 9.2 represents the hydrogen spectrum in the visible region. The wavelength of the lines is represented in terms of units of Angstrom ($\mathring{A}$) which is equal to

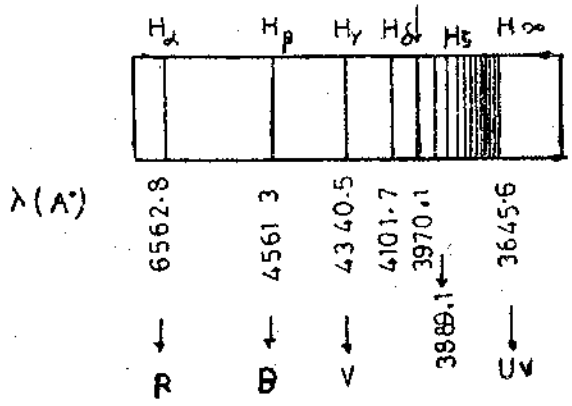


Fig. 9.2 Visible part of hydrozen Spectrum

10^{-10}m, named after Angstrom who was responsible for the earliest precise spectroscopic measurements. The spacing between adjacent lines in wavelengths continuously decreases with decreasing wavelength of the lines. Thus the series of lines converges to the series limit as 3645.6 $\mathring{A}$. In the year 1885 Balmer putforth an empirical formula to represent the wavelengths of the hydrogen spectrum. The relation is as follows.

$$\lambda = 3646 \frac{n^2}{n^2 - 4} \overset{o}{\text{A}}$$

With n = 3 for H_α, n = 4 for H_β, n = 5 for H_γ etc. Balmer could predict the wavelengths of the first nine lines of the series known at that time to better than 1 part in 1000. A search for similar empirical laws that would apply to series of lines emitted by other elements was mostly carried out by Rydberg. He found it convenient to represent the empirical formulae interms of wave number $K = \frac{1}{\lambda}$ of the lines. In terms of this quantity Blamer's formula can be written as

$$K = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

n = 3,4,5 ...

Where R_H is called Rydberg constant for hydrogen. Its value in terms of recent spectroscopic data is known to be

$$R_H = (109677.576 \pm 0.012) cm^{-1}$$

For alkali elements the formulae have the general nature as

$$K = R\left[\frac{1}{(m-a)^2} - \frac{1}{(n-b)^2} \right]$$

Where R represents the Rydberg constant for the particular (atom) element, a and b are constants for the particular series. m is an integer which is fixed for the particular series and n is a variable integer. To within about 0.05% the Rydberg constant has the same value for all elements eventhough it shows a slight systematic increase with increasing atomic weight.

The remarkable properties possessed by spectral series indicate the existence of a simple and universal mechanism within atoms by which the spectra are emitted. It terms of classical theories it was found difficult to explain the spectra with observed features. The developments in the understanding of the structure of the atom at the end of the nineteenth century like disovery of electron in 1897, experiments of Rutherford confirming the existence of nucleus which consists of all the mass of the atom and being positively charged, the quantum concept of radiation putforth by Max Planck and Einstein's explanation of photoelectric effect in terms of quanta of radiation possessing discrete energy values led Bohr to propose the structure of the atom and there by explain the hydrogen spectra. In the following section we study the Bohr theory of hydrogen atom in detail.

9.4 BOHR'S THEORY, ENERGY LEVELS OF THE HYDROGEN ATOM

In the year 1913 Neils Bohr proposed a theory of the hydrogen atom. With this theory, Bohr gave a satisfactory explanation of Balmer series of hydrogen. Bohr proposed

the following postulates in developing his theory.

1. An electron in an atom moves in a circular orbit about the nucleus under the influence of Coulomb attraction between the electron and the nucleus obeying the laws of classical mechanics.
2. Instead of infinity of orbits which would be possible according to classical mechanics it is assumed that the electron moves in an orbit for which its orbital angular momentum L is an integral multiple of $h/2\pi$ where h represents Planck's constant.
3. Eventhough the electron is constantly accelerating an electron moving in such an allowed orbit does not radiate electromagnetic energy. Its total energy E remains constant. These allowed orbits are called stationary orbits.
4. Electromagnetic energy is emitted if an electron initially moving in an orbit of total energy E_i discontinuously changes its motion so that it moves in an orbit of total energy E_γ. The frequency of the radiation emitted is given by $\frac{E_i - E_\gamma}{h}$.

The first postulate embodies some of the ideals concerning the stability of an atom. The second postulate introduces quantzation of energy of the system. The third postulates accounts for the stabiltity of an electron executing circular motion which is against the classical electromagnetic theory. The fourth postulate is closely related to the Einstein's postulate that the frequency of a quantum of electromagnetic radiation is equal to the energy carried by the quantum divided by Planck's constant.

The validity of Bohr's postulates can be found by comparing the predictions that can be derived based on the postulates with the results of experiment.

Consider an atom consisting of a nucleus of charge +Ze and mass M and a single electron of charge -e mass m. Since mass of an electron is negligible compared with the nucleus we shall assume that the nucleus is at rest. As dipicted in Fig. 9.3.

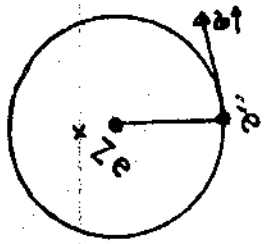


Fig. 9.3 Motion of electron in a circular orbit around the nucleus

the condition for mechanical stabiltiy of the electron is that coulomb force acting on the electron must be equal to the centrifugal force and is given by

$$\frac{K_o Ze^2}{r^2} = \frac{mv^2}{r} \qquad 9.1$$

Where v represent the velocity of the electron on its orbit and r is the radius of the orbit K_o represents the permittivity of free space, $K_o = 9 \times 10^9 Nm^2C^{-2}$. Since the force acting on the electron is entirely radial the orbital angular momentum of the electron must be a constant and is given by

L = mvr

As per postulate 2

$$L = mvr = \frac{nh}{2\pi} = n\hbar \qquad 9.2$$

n = 1,2,3, ...

Where $\hbar = h/2\pi$

From Eq. 9.2

$$v = \frac{n\hbar}{mr} \qquad 9.3$$

Substituting Eq. 9.3 in Eq. 9.1, we get

$$K_o Ze^2 = mrv^2 = mr\frac{n^2\hbar^2}{m^2r^2} = \frac{n^2\hbar^2}{mr}$$

Therefore

$$r = \frac{n^2h^2}{mZe^2K_o} \qquad 9.4$$

n = 1,2,3,....

and

$$v = \frac{n\hbar}{mr} = \frac{n\hbar mZe^2K_o}{mn^2\hbar^2} = \frac{Ze^2K_o}{n\hbar} \qquad 9.5$$

Eq. 9.4 indicates that the radius of the orbit is proportional to the square of quantum principal number n. It is called the quantum number. When n = 1 for hydrogen atom.

$$r_1 = \frac{\hbar^2}{mZe^2} = \frac{h^2}{4\pi^2mZe^2k_o}$$

Substituting the values of various parameters, we get

$$r_1 = \frac{\left(6.626\times10^{-34}Js\right)^2}{4\pi^2\left(9.1\times10^{-31}kg\right)(1)\left(-1.6\times10^{-19}C\right)^2\left(9\times10^{9}Nm^2C^{-2}\right)}$$

$$r_1 = 5.28\times10^{-11}m$$

The radius of the second orbit r_2 is 4 times greater than r_1 and the radius of the third orbit is 9 times greater than r_1 and so on.

When n = 1 the speed of the electron as per Eq. 9.5 comes out as (1/137)°C where c is the velocity of light. In the second orbit $v = \frac{1}{2}\left(\frac{c}{137}\right)$, in the third orbit $v = \frac{1}{3}\left(\frac{c}{137}\right)$ and so on. The fact that the velocity of the electron is less than one percent of the velocity of light justifies the use of classical mechanics instead of relativistic mechanics in Bohr's theory.

To calculate the total energy of an atomic electron moving in one of the allowed orbits let us define the potential energy to be zero when the electron is at infinite distance from the nucleus. The potential energy V at any distance r can be obtained by integrating the energy imparted to the electron by the coulomb force acting from ∞ to r. Hence

$$V = \int_{\infty}^{r} \frac{K_o Ze^2}{r^2} dr = -\frac{K_o Ze^2}{r}$$

The potential energy is negative because the coulomb force is attractive. The kinetic energy of the electron T is given by

$$T = \frac{1}{2}mv^2 = \frac{K_o Ze^2}{2r} \quad \text{(Using equation 9.1)}$$

The total energy E is given by

$$E = T + V = \frac{K_o Ze^2}{2r} - \frac{K_o Ze^2}{r} = -\frac{K_o Ze^2}{2r} = -T$$

Using the value of r we get

$$E = \frac{mz^2e^4K_o^2}{2\hbar^2n^2} \qquad 9.6$$

n = 1,2,3 ....

Eq. 9.6 indicates the quantization of total energy of the electron. We can write for hydrogen atom.

$$E = -\frac{R}{n^2} \qquad 9.7$$

Where

$$R = \frac{2\pi^2 m e^4 K_o^2}{h^2} = 2.179 \times 10^{-18} J$$

Based on Eq. 9.7 the energy level diagram for hydrogen atom can be obtained and is shown in Fig. 9.4

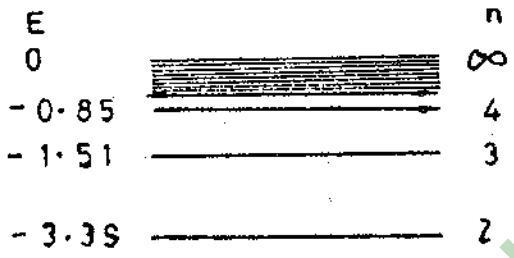


Fig. 9.4 Energy level diagram for hydrogen atom

The lowest (most negative) allowed value of the total energy occurs for n = 1. As n increases the total energy of the quantum states becomes less negative. E approaches zero as n tends to infinity. Since the state of lowest total energy is the most stable state for the electron, the normal state of the electron in one electron atom is the state for which n=1.

To calculate the frequency of the radiation emitted when an electron jumps from an initial state characterized by the quantum number n_i to a final state characterized by the quantum number n_f then

$$v = \frac{\left(E_i - E_f\right)}{h} = \frac{-mZ^2e^4K_o^2}{h2\hbar^2n_i^2} + \frac{mZ^2e^4K_o^2}{h2\hbar^2n_f^2}$$

$$v = \frac{2\pi^2K_o^2mZ^2e^4}{h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Since

$$\frac{v}{c} = \frac{1}{\lambda} = K$$

We get

$$K = \frac{2\pi^2 K_o^2 m Z^2 e^4}{ch^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

or

$$K = R_\infty Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where

$$R_\infty = \frac{2\pi^2 K_o^2 m e^4}{ch^3}$$

and n_i and n_f are integers.

The predictions of Bohr's theory are contained in Eqs. 9.6 and 9.8. Let us analyse the emission of electromagnetic radiation by one electron Bohr atom it terms of these equations.

1. The normal state of an atom will be the state for which the electron has lowest energy. This is the state for which n = 1. It is called the ground state of the atom.
2. In an electrical discharge the atom receives energy due to collisions. The electron makes a transition to a state of higher energy. This state is called the excited state for which n > 1.
3. The atom emits its excess energy and returns to the ground state. In each transition electromagnetic energy is emitted with a wave number which depends on the energy lost by the electron and is given by 9.8
4. If the very large number of excitation and deexcitation processes which take place during the measurement of an atomic spectrum all possible transitions occur subject to the restriction that $n_i > n_f$. For hydrogen atom Z = 1, Then

$$K = R_\infty Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R_\infty\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If $n_f = 2$, $n_i > n_f = n$ say n then

$$K = R_\infty\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad 9.9$$

Where n = 3,4,5 ....

Eq. 9.9 is identical with the formula for Balmer series of the hydrogen spectrum. According to Bohr's theory other series also exist. Series eith

$n_f = 1, n_i = 2,3,4$ .... are called Lyman series

$n_f = 2, n_i = 3,4,5$ .... are called Balmer series

$n_f = 3, n_i = 4,5,6$ .... are called Paschen series

$n_f = 4, n_i = 5,6,7$ .... are called Brackett series,

and

$n_f = 5, n_i = 6,7,8$ .... are called Pfund series.

These series are illustrated in terms of the energy level diagram in Fig. 9.5

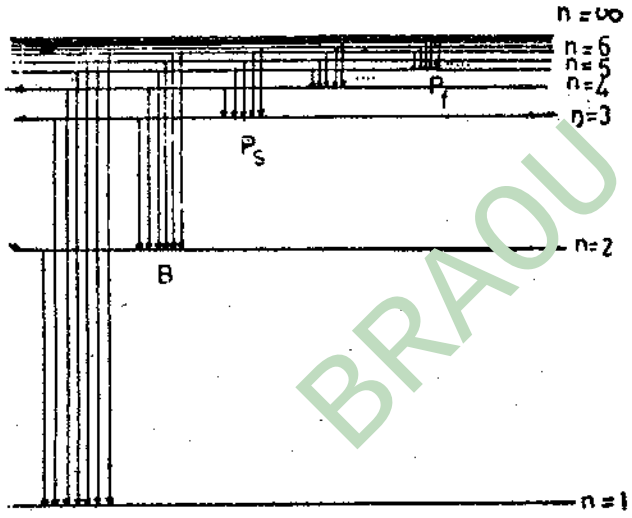


Fig. 9.5 Energy level diagram of hydrogen atom illustrating the Production of Spectral Series

The wave numbers of the lines of these are fitted very accurately by the Eq. 9.8 using appropriate values of n_f . This is the greatest success of Bohr's theory. The success of Bohr's theory lies in the prediction of Lyman, Brackett and Pfund series which have not been discovered at the time the theory was developed by Bohr. These series later were observed experimentally by Lyman, Brackett and Pfund which go by their names.

Check Your Progress

Radius of the Bohr's orbit is _____ to the _____ number.

Worked Example - 1

Determine the energy of the electron in the first orbit in a hydrogen atom

$$E_1 = \frac{-R}{n^2} = -\frac{2.179 \times 10^{-18} J}{1^2}$$

$$= \frac{-2.179 \times 10^{-18} J 1eV}{13.6 \times 10^{-19} J} = -13.6eV$$

Worked Example - 2

Determine the wavelength of the first spectral line of singly ionized Helium atom that corresponds to the first line in the hydrogen Balmer series.

We have

$$K = \frac{2\pi^2 K_o^2 m Z^2 e^4}{ch^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$K = R_\infty Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

As per the problem

$$K = R_\infty Z^2 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

Here Z = 2

$$\therefore \quad R_\infty = \frac{2\pi^2 m e^4 K_o^2}{h^3 c}$$

$$R_\infty = \frac{2(22/7)^2 \left(9.1 \times 10^{31} kg\right)\left(-1.6 \times 10^{19} c\right)^4 \left(9 \times 10^9\right)^2 Nm^2 c^2}{6.62 \times 10^{34} Js\left(3 \times 10^8\right) ms^1}$$

$$R_\infty = 1.0973731 \times 10^7 m^{-1}$$

$$K = (2)^2\left(\frac{1}{4} - \frac{1}{9}\right) 1.0973731 \times 10^7 m^{-1}$$

$$K = 4(5/36)1.0973731 \times m^{-1}$$

$$\lambda = \frac{1}{K} = \frac{9}{5}\left(\frac{1}{1.0973731 \times 10^{7}}\right)m = 1640.3 \times 10^{-10} m$$

$$\lambda = 1640.3 \times 10^{-10} m$$

Worked Example - 3

Calculate the wavelength of the second spectral line of the Brackett series of hydrogen

For Brackett series we have

$$K = \frac{1}{\lambda} = R_{\infty}\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$$

Where in = 5,6,7...

For the second line of the series, n = 6

$$K = \frac{1}{\lambda} = R_{\infty}\left(\frac{1}{4^2} - \frac{1}{6^2}\right)$$

$$K = \frac{1}{\lambda} = 1.0973731 \times 10^{7}\left(\frac{36-16}{36 \times 16}\right)$$

$$K = \frac{1}{\lambda} = 1.0973731 \times 10^{7}\,\frac{20}{30 \times 16} m^{-1}$$

$$\therefore \quad \lambda = \frac{36 \times 16}{20 \times 1.0973731 \times 10^{7}}$$

$$\lambda = 26244 \times 10^{-10} m$$

9.5 SOMMERFELD'S THEORY

According to Bohr's Theory, hydrogen spectral lines have well defined wavelength. But experiments with high resolution spectrographs showed that each of the H_{α}, H_{β} and H_{γ} lines consists of several very closed lines packed together. Michelson found that H_{α} lines consists of two close components which are separated by a wavelength difference of

$0.13\,\mathring{A}$. These are known as fine structure spectral lines which could not be explained by Bohr's theory. To explain this Sommerfeld gave two modifications to the Bahr's theory.

1. Electrons revolve round the nucleus in elliptic orbits. The circular orbits of Bohr is a special case of this.
2. The velocity of the electron moving in an elliptic orbit varies considerably at different parts of the orbit. This causes the relaticistic variations of the mars of the moving electron. Hence this is known as the relativistic model of the atom.

Let us consider an electron moving in an elliptical orbit around the nucleus. Its position at any instant is given by its position r, the distance between the electron and one of the force of the ellipse, also known as the radi i vector and ϕ the angle made by the vector with the X axis or the major axis of the ellipse. According to the Bohr's quantum condition the momentum associated with these two quantities is to be quantized following equations.

$$\oint P_\phi d\phi = n_\phi h \qquad 9.10$$

$$\oint P_r dr = n_r h \qquad 9.11$$

Here n_ϕ and n_r are known as azimuthal and radial quantum numbers. This azimttial quantum number is also known as angular quantum number. These will take any positive integers and their sum should be equal to n Which is the principal quantum number.

To determine the allowed elliptical orbits, the above two equations are to be evaluated.

Consider equ. 9.10

$$\oint_0^{2\pi} P_\phi d\phi = n_\phi h$$

or $\quad 2\pi P_\phi = n_\phi h$

or $\quad P_\phi = \frac{h}{2\pi} n_\phi \qquad 9.12$

Let us integrate eqn. 9.11

$$\oint P_r dr = n_r h$$

After integrating

$$\frac{n_\phi h}{\left(1-\epsilon^2\right)^{1/2}} - n_\phi h = n_r h$$

The detailed steps of integration are beyond the scope of this book

Here E is the eccenticity of the ellipse.

$$\frac{n_\phi}{\left(1-\epsilon^2\right)^{1/2}} = n_r + n_\phi$$

$$\text{or } \left(1-\epsilon^2\right) = \frac{n_\phi^2}{n_r^2} \qquad 9.13$$

This is the condition to be satisfied for the allowed elliptical orbits. For an ellipse $1-E=\frac{b^2}{a^2}$ where b and a are semi minor and semi major axes herply.

$$\therefore \frac{b^2}{a^2} = \frac{n_\phi^2}{n^2} \qquad \text{or} \qquad \frac{b}{a} = \frac{n_\phi}{n}$$

If n_ϕ = n then b = a or ϵ = 0.

That is it represent, a circle. n_ϕ cannot be equal to zero. If it is zero the ellipse is transfound into a stragic line. Passing through the nucleus. Since b is always less than 'a', n_ϕ cannot be greater 1.

Hence for a gives value, of n, n_ϕ can assume n different values with different eccenticities. Let us examine this with respect to examples.

For the first orbit n = 1 since $n_r + n_\phi = 1$ and $n_\phi = 0$ $n_\phi = 1$. The first orbit is a circle. With n = 2 n_ϕ = 1 or 2. That is out of two orbits, one is circular and the other is elliptical. Similarly for the other values of n also. These are the allowed orbits. These are represented by 2_2 and 2_1. Here 2 represents the value of n and the subscript 2 and 1 represent the azimuthal quantum number n_ϕ.

The azimenthal quantum numbers can be represented by the letters s, p, d, f etc. Now we calculate the total energy also.

The total energy T of an electron is the sum of the potential energy and kinetic energy. Which can be written as

$$E = -\frac{Ze^2K_o}{r} + \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right]$$

The first term represents the potential energy, and the second term represents the kinetic energy. $\left(\frac{dr}{dt}\right)$ represents the radial component of the velocity and $r\left(\frac{d\phi}{dt}\right)$ represents the transverse comparents of the velocity. On evaluating this, it gives,

$$E_n = -\left(\frac{2\pi^2 K_o^2 m Z^2 e^4}{h^2}\right)\frac{1}{n^2}$$

which is similar to the expression for energy in Bohr's theory. This could not explain the fine structure of the spectial lines. In Bohr's theory we had seen that the velocity of the electron is $\frac{1}{137}c$. Sommenfeld modified his theory taking into account, the variation of mass with velocity. He showed that the relativistic equation describing the path of an election as

$$\frac{1}{r} = \frac{1+\epsilon Cos\psi}{a\left(1-\epsilon^2\right)}\phi$$

where $\psi^2 = 1 - \frac{Ze^2K_o^2}{p^2c^2}$

using this relativistic concept, the expression for total energy is, derived as

$$E_n = \frac{2\pi^2 K_0^2 m Z^2 e^4}{h^2 n^2} - \frac{2\pi^2 m Z^4 e^4 \alpha^2}{h^2}\left(\frac{n}{n_\varphi} - \frac{3}{4}\right)\frac{1}{n^4} \qquad 9.14$$

Where $\alpha^2 = \frac{16\pi^2 K_0^2}{2ch} \approx \frac{1}{137}$

α is the dimensionless quantity and is known as the fine structure constant. The first term in equation 9.14 is the energy of the electron in the orbit with principal quantum number n. The second term which depends on n_φ represents the Sommerfelds relativistic correction. The correction gives the splitting of the energy levels of the atom corresponding to n permitted values of n_φ.

9.6 ENERGY LEVELS OF HYDROGEN ATOM

H_α line results due to the transition of an electron from n = 3 to n = 2 state of the hydrogen atom. For n = 3 there are three possible levels corresponding to the three values of n_φ. That is n_φ = 1,2 and three. Simlarly 2 possible levels for n = 2. Theoretically, the six possible transitions are $3_3 \to 2_2$, $3_3 \to 2_1$, $3_2 \to 2_2$, $3_2 \to 2_1$, $3_1 \to 2_2$, $3_1 \to 2_1$. Using the selection rule $\Delta n_\varphi = \pm 1$ the allowed transitions are $3_3 \to 2_2$, $3_2 \to 2_1$ and $3_1 \to 2_2$.

Other transitions are forbidden.

In the Fig. 9.6, the allowed transitions are shown as solid lines and the forbidden transitions are shown as broken lines. This is found to be in good agreement with the experimentally observed results.

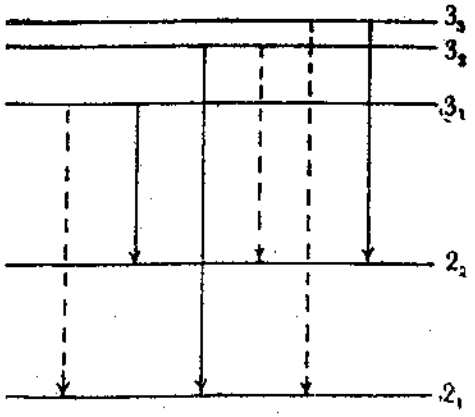


Fig. 9.6

Sommenfeld's theory could not predict the number of fine structure lines in all the cases. It could not explain the complex spectra in alkali metals. It could not explain the arrangement of electrons in each orbit. It could explain the observed splitting of spectral lines due to the application of magnetic or electric field to the source of the spectrum. It could not explain the intensity of the spectral lines also. To explain the above experimental results, a new model known as vector atom model was introduced. The two main characteristics of this model are spatial quantizations and spinning of an electron.

9.7 VECTOR ATOM MODEL

a) Spartial Quantization

According to Bohr's theory the orbits are quantized in magictude (ie size and form) only. But quantum theory insists on the quantization of orientation of the orbits in space or simply the direction. For this purpose the direction of the applied, magnetic field applied to the atom is taken as the reference axis. The projection of the quantized orbits on the field direction must also be quantized. Only such orientations are permitted. This principle of space quantization could explain the splitting of the spectral lines under the application of magnetic and electric field (Zeeman effect, Stark effect). There are confirmed by experiments also.

b) Spinning of electron

To explain the fine structure Uhlenback and Goudsmit introduced the concept of spinning electron. According to this, an electron spins about an axis of its own in addition to orbital motion. Quantum theory insists on the quantization of spin also. Hence a new quantum number s is introduced.

Since both the orbital and spin motions are quantized in magnitude and direction they are known as quantized vectors and hence this model is known as vector atom model.

The quantum numbers asociated with the vector atom model will be discussed here one after one.

1. Principal quantum number n :- This represents the number of shell as in Bohr's atom model. It can take only integral values (ie) n = 1,2,3 ....
2. Orbital quantum number l :- This can take any integer values including zero. This is similar to the azimuthal quantum number n_ϕ in the Sommerfeld's theory. If n = 3, l can take values 0,1 and 2. If l = 0. it represents selection, if l = 1 it represents p electron, if l = 2, it represents d-electron and soon. The orbital angular momentum P_l of the electron will be $P_l = \frac{lh}{2\pi}$. But according to wave mechanics, it is $\sqrt{l(l+1)} \cdot \frac{h}{2\pi}$.
3. Spin quantum number s :- The magnitude of this quantum number is always $\frac{1}{2}$. The spin angular momentum P_s of the electron will $bep_s = \frac{sh}{2\pi}$. But according to wave mechanics, it is $\sqrt{s(s+1)} \cdot \frac{h}{2\pi}$.
4. Total angular momentum quantum number j :- The resultant regular momentum of the electron due to both orbital and spin motion is the total anular momentum. This is the numerical value of the vector sum of l and s; ie $j = l \pm \frac{1}{2}$, plus sign indicates that s vector is parallel to l vector and minus sign indicates that they are antiparallel. That is if l = 1 and s = $\frac{1}{2}$; j = $\frac{3}{2}$ or $\frac{1}{2}$. The total angular momentum P_j of the electron is given by $P_j = \frac{jh}{2\pi}$. But according wave mechanics $P_j = \sqrt{\frac{j(j+1)h}{2\pi}}$.

   To explain the splitting of spectral lines we consider more number of quantum numbers and these arise, when a magnetic field is applied to the atom.
5. Magnetic orbital quantum number m_l :- The projection of the orbital quantum number 'l' on the magnetic field direction is known as magnetic orbital quantum number. It takes $(2l+1)$ values, (i.e) from $-l$ to $+l$ including zero. If l=2, m_l can take five values namely -2, -1,0, +1 and +2. That is l vector can take only fine orientations around the magnetic field direction as shown in the Figure 9.7. Hence this is known as spatial quantization.

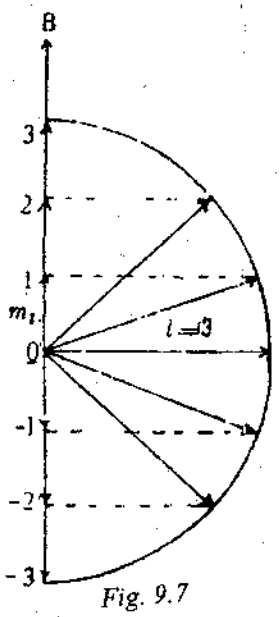


Fig. 9.7

6. Magnetic Spin quantum number, m_s :- This the projection of the spin vector s, along the direction of the magnetic field. It can assume only two values $+\frac{1}{2}$ or $-\frac{1}{2}$ with respect to the magnetic field. These spin vectors may be parallel or antiparallel as shown in the figure 9.8

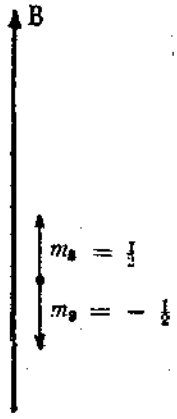


Fig. 9.8

7. Magnetic total angular momentum quantum number m_j :- This is the projection of the total angular momentum vector j on the direction of the magnetic field. Since $j = l \pm \frac{1}{2}$, m_j can have only odd half integral values. It can have (2j+1) values (ie) from -j to +j excluding zero as shown in the Fig. 9.9

Fig. 9.9

The state of any electron and the arrangement of electrons in different orbits on the basis of quantum numbers n, l, m_l and m_s. Using the pauli's exclusion principle will be discussed in detail in the next unit.

9.8 FRANK HERTZ EXPERIMENT

According to Bohr's theory an atom will be in the ground state when the electron revolves in the first orbit. For an atom to radiate energy, the electron from a ground state should be raised to a higher energy state. This process is known as excitation. If the excitations is high such that the electron is knocked out of the atom then the process is known as ionization. The potentials that are necessary for the above two processes are known as critical potentials. Excitation potential is also known as radiation potential or resonance potential. These potentials are expressed as electron volts.

In the case of the hydrogen atom there is only one ionization potential but it can have several excilation potentials. In the case of atoms with more number of electrons. there can be more than one ionization potential. The energy required to remove an outermost electron is known as first ionization potential. The energy required to remove a second electron, which is more attached to the mucleus is known as second ionization potential. This is greater than the first ionization potential. Helium has two ionization potential, at

24.5 and 78.6 electron volts. These critical potentials can be determined by an experiment devised by Frank and Hertz.

Frank-Hertz experiment : The apparatus used by Frank and Hertz is shown in Fig 9.10. They studied the critical potentials for Mercury vapour.

The apparatus consists of a glass tube T which is filled with a gas, whose critical potentials are to be determined at a pressure of 1mm of Mercury.

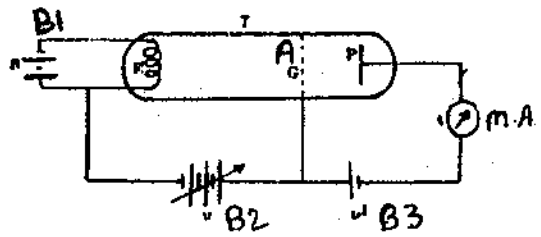


Fig. 9.10

Electrons are produced by heating the filament F with the help of a battery B_1. These electrons are accelerated towards a perforated plate A; known as Grid by giving a variable postive voltage with respect to F. The electrons are collected by the plate P which is given a small variable negative voltage from a battery B_3. That is the electons having energy greater than this small voltage can reach the plate P. A milliammeter is connected in the plate circuit to know the plate current. The voltage of the battery B_3 is kept constant at a very low value. The voltage of the battery B_2 is increased slowly and the plate current is noted. A graph drawn between the accelarating voltage and collector current as shown in Fig. 9.11

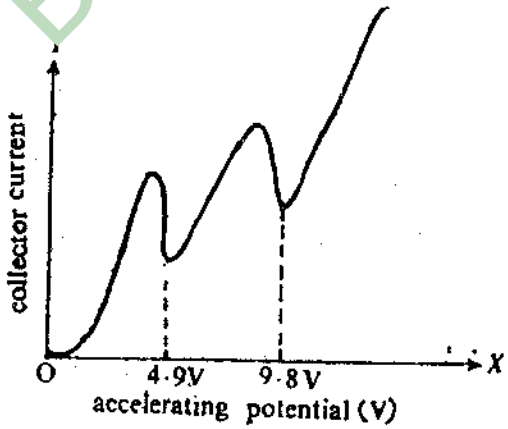


Fig. 9.11

From this graph the following conclusion can be drawn.

1. Upto a certain accelerating potential no collector current is observed (ie) upto the potential given by the battery B_3.
2. The collector current continuously increases to a certain value and suddenly falls to a low value and again increases.The first decrease in the collector current is observed at 4.9V in this. The electrons passing through the mercury vapour do not loose energy and reaches the collector till certain value of accelerating potential. At this value the electron collide with the mercury atoms loosing energy due to the collision and cannot reachy the plate P. Hence decrease in the collector current is

observed. This potential is know as first ionzation potential.

3. If the collector voltage is further increaseing again another decrease in the current is observed which is know as second critical potential at 9.8V. These values are found to be in good agreement with the calculated values. The main draw-back in this experiement is one cannot distingmish between excitations and ionization potentials.

9.9 SUMMARY

The wavelengths of the atomic spectra can be represented in the form of an empirical formula. For hydrogen spectra the Balmer series can be represented by

$$\nu = \frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

n = 3,4,5 ....

R_H is called Rydberg constant for hydrogen. Its value is given by (10967757.6+1.2)m^{-1}.

Bohr's theory of hydrogen atom explains successfully the hydrogen spectra i.e. Balmer series. It also could predict the existence of other series which have been observed by Lyman, Paschen, Brackett, and Pfund and these series are called by names of these discoverers.

Bohr proposed the following postulates which combine both classical and quantum concepts in developing the theory of the hydrogen atom.

(i) An electron revolves round the nucleus in circular orbits under the influence of Coulomb attraction between the nucleus and electron.

(ii) The orbital angular momentum of the electron is given by $nh/2\pi$ where n = 1,2,3 .... and h is Planck's constant.

(iii) The electron does not radiate energy as long as it moves in the allowed orbits. These orbits are called stationary orbits.

(iv) Electromagnetic energy will be radiated when the electron jumps from a higher orbit or energy E_i to a lower orbit of energy E_f. The frequency of radiation emitted is given by $\nu = \left(E_i - E_f\right)/h$

Sommenfeld's theory could not explain fine structure of spectral lines in Hydrogen atom. With the help of seven quantum numbers mainly with four a new model of the atom known as vector atom model was developed and fine structure was explained successfully. Critical potentials in Mercury were determined using Frank Hertz experiment.

The radius of the Bohr orbit is proportional to the principle quantum number n. The velocity of the electron in the Bohr orbit is inversely proportional to the principle quantum number. The radius of the first Bohr orbit is 5.28 x 10^{-9} m and the velocity of the electron in this orbit is (1/137) C where C represents the velocity of light.

The energy of the electron in any Bohr orbit is given by

$$E = -\frac{mz^2e^4k_o}{2h^2}\frac{1}{n^2}$$

n = 1,2 ....

Where m = 9.1 x 10^{-31} kg = mass of the electron

e = -1.6 x 10^{-19} C

$$k_o = 9\times10^9\, Nm^2C^{-2}$$

$$h = 6.626\times10^{-34}\, Js$$

z = Charge of the nucleus

According to Bohr's theory the wave number of spectral line of the hydrogen atom can be given by

$$v = \frac{1}{\lambda} = R_\infty\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Where $R_\infty = \left(\frac{me^4k_o}{411ch^3}\right)$

$n_f = 1, n_i = 2,3,4$ .... Lyman series

$n_f = 2, n_i = 3,4,5$ .... Balmer series

$n_f = 3, n_i = 4,5,6$ .... Paschen series

$n_f = 4, n_i = 5,6,7$ .... Brackett series,

$n_f = 5, n_i = 6,7,8$ .... Pfund series.

9.10 MODEL ANSWERS

Radius of the Bohr's orbit is proportional to the Principle Quantum Number.

9.11 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Give an account of the main characteristics of line spectra and empirical laws that represent the wavelength of a spectral lines.

2. Discuss Bohr's theory and obtain expressions for energy level of hydrogen atom and the wavelength of spectral line. Discuss the importance of Bohr's theory.

3. Explain vector atom model and discuss how it could explain the fine structure of spectral lines.

4. Describe the Frank Experiment for the determination of critical potentials.

II. Solve the following problems.

1. Evaluate the minimum energy required to excite the first line in the Lyman series of hydrogen.

2. Determine the energy of μ -meson existing in the lowest energy level if the mesonic atom consists of a proton and a μ -meson of mass $207\, m_e$ where m_e represent of the mass of an electron given by 9.1×10^{-31} kg.

(Ans : -2.53)

UNIT–10 : QUANTUM NUMBER

Contents

10.1 AIMS AND OBJECTIVES

In this unit Bohr's correspondence principle and Pauli's exclusion principle are discussed. After going through this unit

- You will be able to specify the state of an electron in an atom completely by the quantum numbers.
- You will be able to discuss the periodic table of elements in the light of Pauli's exclusion principle.
- You will be able to understand the molecular spectra.

10.2 INTRODUCTION

In this unit we shall study the Bohr's correspondence principle and Pauli's exclusion principle.

10.3 CORRESPONDENCE PRINCIPLE

Bohr proposed the theory of hydrogen atom combining both classical and quantum concepts. When the quantum number n becomes very large, successive values of the allowed energy values appear almost continuous. Similarly when the spectral lines approach the series limit, they crowed together (at $n = \infty$) so that the spectrum appears almost continuous. This behaviour led Bohr to postulate the correspondence principle. This principle states that the predictions of the quantum theory for the behaviour of an atomic system must correspond to the predictions of classical theory, when the quantum theory is taken to the classical limit. The classical limit for the quantum theory is the situation of transitions between nearby states having very large principal quantum number and it can be shown that the quantum and classical theories do give identical predictions for the radiated frequency in this limit. Bohr used the corresponding principle to deduce what selection rules apply to transitions between particular states in atoms.

Worked Example - 1

Show that the angular frequency of revolution of an electron in its orbit is given by Bohr theory as $\omega = \pi mz^2e^4/2E_o^3h^3n^3$. Hence, show that when n is large, the frequency of revolution $\omega/2\pi$ is equal to the frequency of radiation emitted in the transistion of an

electron from state $n_2 = n+1$ to state $n_1 = n$. Comment on the significance of the result.

Solution :

From Bohr's theory the angular frequency of revolution is given by

$$mr^2\omega = \frac{nh}{2\pi}$$

where

$$r = \frac{n^2h^2 \in_o}{\pi me^2 Z}$$

$$\therefore \omega = \frac{nh}{2\pi m}\left(\frac{\pi me^2 Z}{n^2h^2 \in_o}\right)^2 = \frac{\pi mz^2e^4}{2 \in_o^{\ 2} h^3n^3}$$

Frequency of revolution $\nu = \frac{\omega}{2\pi} = \frac{mz^2e^4}{4 \in^2 h^3 n^3}$

The frequency of emitted radiation for the transistion $n+1 \to n$ is given by

$$\nu = \frac{mz^2e^4}{8 \in_o^2 h^3}\left[\ \frac{1}{n^2} - \frac{1}{(n+1)^2}\ \right]$$

$$= \frac{mz^2e^4}{8 \in_o^2 h^3}\frac{(2n\_1)}{n^2(n+1)^2}$$

For sufficiently large n

$$\frac{(2n\_1)}{n^2(n+1)^2} \cong \frac{2n}{n^4} \cong \frac{2}{n^3}$$

and therefore

$$\nu \cong \frac{mz^2e^4}{4 \in_o^2 h^3n^3} \cong \frac{\omega}{2\pi}$$

Classical electromagnetic theory predicts that the frequency of the emitted radiation is equal to the frequency of revolution, and in the limit of transition between nearby states having very large principle quantum numbers the classical theory and Bohr theory yield the same results. Thus, in this respect, the theories satisfy the correspondence principle.

10.4 THE QUANTUM NUMBERS

As few months after Bohr published a report stating his phenonmenal success in explaining the hydrogen spectrum with circular orbits. Sommerfeld extended the theory to include elliptical orbits as well. Because these orbits played such an important role in later developments in atomic structure, they deserve some attention here.

The net result of Sommerfeld's theory showed that the electron in any one of the allowed energy levels of a hydrogen atom may move in any one of the number of orbits. For each energy level n = 1, n = 2, n = 3, etc., there are n possible orbits. Diagrams of the allowed orbits for the first three energy levels as shown in Fig. 10.1.

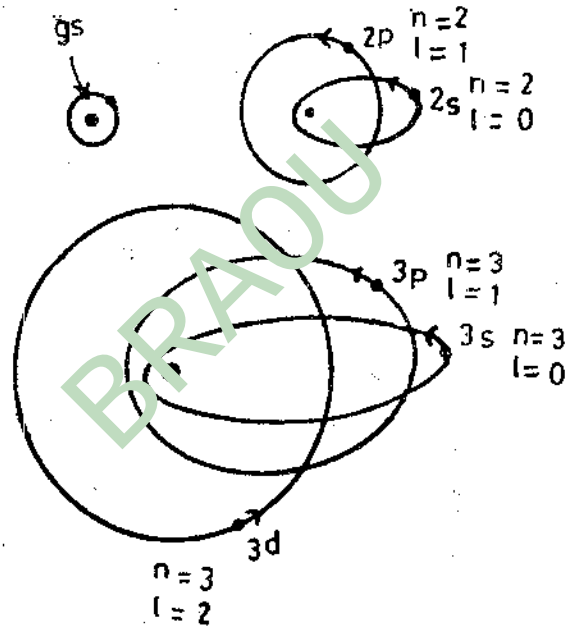


Fig. 10.1 Electron Orbitals for the hydrogen atom according to Bohr-Sommerfeld theory gs-ground state

For n = 3, for example, there are three orbits, with designations $l = 2$, $l = 1$, $l = 0$. The diameter of the circular orbit is given by Bohr's theory and this is just equal to the major axes of the two elliptical orbits. The minor axes are 2/3 and 1/3 of the major axis.

It is common practice to assign to the l value as follows :

l=0	l=1	l=2	l=3	l=4	l=5	l=6	
s		p	d	f	g	h	i

According to this system, the circular orbit with the principal quantum number n = 3 and $l = 2$ is designated by 3d while the ellptical orbit n = 2 and $l = 0$ is designated by 2s

etc. n is the principal quantum number and l is orbital quantum number. The orbital angular momentum is given by

$$p_l = l\frac{h}{2\pi} \qquad 10.1$$

The angular momenturm P_l is frequently called the mechanical moment.

If the spectral lines of hydrogen, lithium, sodium and potassium are observed under high magnification, each series member if found to be a double line (Fig. 10.2). These closely spaced doublets arise from the fact that all electrons are spinning.

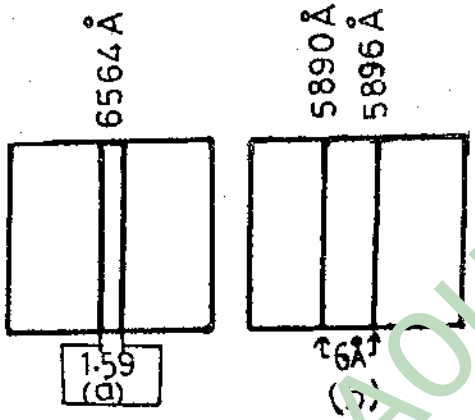


Fig. 10.2 Highly magnified Photograph showing doublet Structure of
(a) The first member of Balmer Series of hydrogen atom
(b) The first member of the Principal Series of Sodium (the yellow line)

The single electron in the hydrogen atom, which is responsible for the observed spectrum, is spinning around its own axis as it moves in an orbit around the nucleus. Similarly the single valance electron in all sodium atoms is spinning as shown in Fig.10.3. This spinning of electrons in atoms was first proposed by Goudsmit and

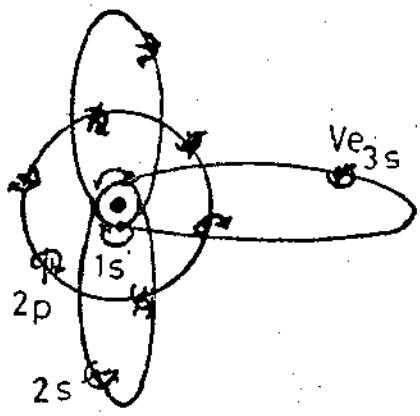


Fig. 10.3 Atomic model of Sodium showing the spinning of all electrons
Z = 11, Ve = Valence electron

Uhlenbeck in 1925 to explain the double-line structure of the alkali metals. Each electron, whether it is bound to an atom or a crystal or is alone in free space, had an angular momentum. This spin angular momentum is given by

$$p_s = \frac{sh}{2\pi} \qquad 10.2$$

Where s is the spin quantum number and has the value 1/2.

When hydrogen atoms are located in magnetic field the quantum theory requires that their electrons take on certain specified orientations. Hence we come across with two more quantum numbers; one due to the orbital motion of the electron around the nucleus and other spinning of the electrons in the magnetic field.

The component of orbital angular momentum in the field direction is given by

$$p_{m,l} = m_l \frac{h}{2\pi} \qquad 10.3$$

Where m_l is called the magnetic quantum number (or orbital projection quantum number). It takes (2/ +1) integer values between -1 and +1, i.e.

$$m_l = -l, -l+1 \ldots 1, 0, 1, (l-1), +l$$

Similiarly the projected component of spin angular momentum in the field direction is given by

$$p_{m,s} = m_s \frac{h}{2\pi} \qquad 10.4$$

Where m_s is the spin project in quantum number.

It takes (2s + 1) values. That is $\left(2 \times \frac{1}{2} + 1\right) = 2$ values are possible. They are $+\frac{1}{2}$ or $-\frac{1}{2}$.

The four quantum numbers n, l, j = $(l \pm s)$, m or n, l, m_l, m_s can be regarded as labels which specify completely the state of an electron in an atom, and hence determine its energy and angular momentum. For example, the level with n = 2 and $l = 0$ has $m_l = 0$, so that there is a maximum of two electrons, one with $m_s = +1/2$ and the other with $m_s = -1/2$ which can go into this level. The level with n = 2 and $l = 1$ has $m_s = 0$ $m_l = 1$ so that there are six possible combinations of m_l and m_s. This discussion holds good if we neglect the spin-orbit interaction and label the electrons with n, l, m_l and m_s. Detailed discussion on this is given in unit 9.

10.5 PAULI'S EXCLUSION PRINCIPLE

It follows from the discussion presented earlier that the state of an electron in an itom is completely specified by the four quantum numbers as follows :

n - *Principal quantum number* which takes integer values from unity to infinity (i.e., n =1, 2, 3 .... ∞); determines the size of electron orbit; arises from quantization of energy.

l - *Orbital quantum number* which takes values from zero to (n-1); determines the shape of the electron orbit; arises from the quantization of the magnitude of the orbital angular momentum.

m_l - *Magnetic quantum number* which takes values from $-l$ to $+l$, determines the orientation of the orbit in space arises from the quantization of the direction of the orbital angular momentum.

m_s - *Spin projection quantum number* which takes values of $\pm$ 1/2 only; determines the spin orientation, up or down; this arises from the quantization of the direction of the spin angular momentum.

To assign quantum numbers to the electrons in atoms, Pauli introduced a principle known as *exclusion principle*, According to this principle, *no two electrons can exist in an atom in the same electronic state*. This means that for each electron at least one of the four quantum numbers (n, l, m_l, m_s) mentioned above must be different from that for the other electrons.

Electrons having the same n value constitute a shell group or level; electrons having the same n but different l values constitute sub-shells, sub-groups or sub-levels of the same shell. Thus each shell consists of n sub-shells, corresponding to the n possible values of l for a given value of n. In spectroscopic notation, electrons having their orbital angular momentum quantum number l = 0, 1, 2, 3, 4 ... are called the s,p,d,f,g.... electrons respectively. But in X-ray spectra notation the shells with n = 1,2,3,4 .... are designated as the K, L, M, N .... shells respectively.

The distribution of electrons in different shells and sub-shells according to the exclusion principle is given in Table 10.1. For each value of l there are $2(2l+1)$ possible values of m_l, there are two possible values of m_s, namely, $+\frac{1}{2}$ and $-\frac{1}{2}$, giving the number of electrons which can be accommodated in a sub-shell with a given value of l as $(2l+1)$. For a given value of n there are n such sub-shells corresponding to the values 0, 1, 2,.... (n-1) of l. The number of electrons that can be accommodated in a shell with a given n value is equal to the sum of the electrons in the constituent sub-shells, that is

Table 10.1 - The distribution of electrons in different shells and sub-shells

n	l	m_l	m_s	Number of electrons in sub-shell with spectroscopic notation		Total number of electrons in shell
1	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	$1s^2$	2
2	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	$2s^2$	
2	1	-1, 0, +1	$+\frac{1}{2}, -\frac{1}{2}$	6	$2p^6$	8
3	0	0	$-\frac{1}{2}, +\frac{1}{2}$	2	$3s^2$	
3	+1	-1, 0, +1	$-\frac{1}{2}, +\frac{1}{2}$	6	$3p^6$	18
3	2	-2,-1,0,+1,+2	$-\frac{1}{2}, +\frac{1}{2}$	10	$3d^{10}$	
4	0	0	$-\frac{1}{2}, +\frac{1}{2}$	2	$4s^2$	
4	1	-1,0,+1	$-\frac{1}{2}, +\frac{1}{2}$	6	$4p^6$	32
4	2	-2,-1,0,+1,+2	$-\frac{1}{2}, +\frac{1}{2}$	10	$4d^{10}$	
4	3	-3,-2,-1,0 +1,+2,+3	$-\frac{1}{2}, +\frac{1}{2}$	14	$4f^{14}$	

$$2\left\{\left[(2\times 0)+1\right]+\left[(2\times 1)+1\right]+\left[(2\times 2)+1\right]+\ldots+\left[2(n-1)+1\right]\right\}=2n^2$$

(i.e., $N(n)=\sum_{l=0}^{n-1}2(2l+1)=2n^2$) A shell of principal quantum number n having $2n^2$ electrons is said to be completed, closed or filled; similarly, a sub-shell of orbital quantum number l is said to be completed, closed or filled with $2(2l+1)$ electrons.

Every electron is a closed sub-shell in matched by another having a m_l value of the opposite sign; similarly, the values of m_s for the electrons in a closed sub-shell are paired off. The resultant value of sum of m_l and m_s (i.e., $\Sigma(m_l+m_s)$ for a closed sub-shell is therefore, equal to zero which suggests that the contribution to the total angular momentum of the atom from the orbital and spin angular momenta of the electrons in a closed sub-shell is also zero. It then follows that a closed shell which is made up of the constituent closed sub-shells does not contribute anything to the total angular momentum of the atom.

10.6 THE PERIODIC TABLE OF ELEMENTS

With a knowledge of the way in which the electrons are distributed in the various sub-shells, one can arrange all the elements in a tabular form, known as peridic table. According to Bohr, each atom of the element of atomic number Z is built from an atom of the element of atomic number (Z-1) by adding one more electron to the latter atom; to keep the atom so formed neutral, it is necessary to increase the nuclear charge also correspondingly. In building up the atoms of the elements in this fashion it is necessary to

be guided by the chemical, magnetic and spectroscopic properties of the elements. The periodic table of elements obtained according to the above scheme is shown in Fig. 10.4. It is now possible to deduce the electronic configuration of atoms by filling by each shell in turn, starting with the most strongly bound shell. These predicted configurations and ionisation energies are given in Table 10.2. It can be noticed that when a shell or sub-shell is closed, the ionization energy arises, but when there is single electron in a shell, the ionisation energy is very low and the electron is readily detached. Fig. 10.4 shows that similarity in electronic configuration corresponds to similarity in chemical propoerties. Thus from the Fig. 10.1 and Table 10.2, we find that the 8 electrons in an oxygen atom are distributed as follows : Two's electrons in each of the shells n = 1, 2 and four p electrons in the shell n = 2. The shell n = 1 and the sub-shell l = 0 of the shell n = 2 are closed with full quota of electrons in the incomplete

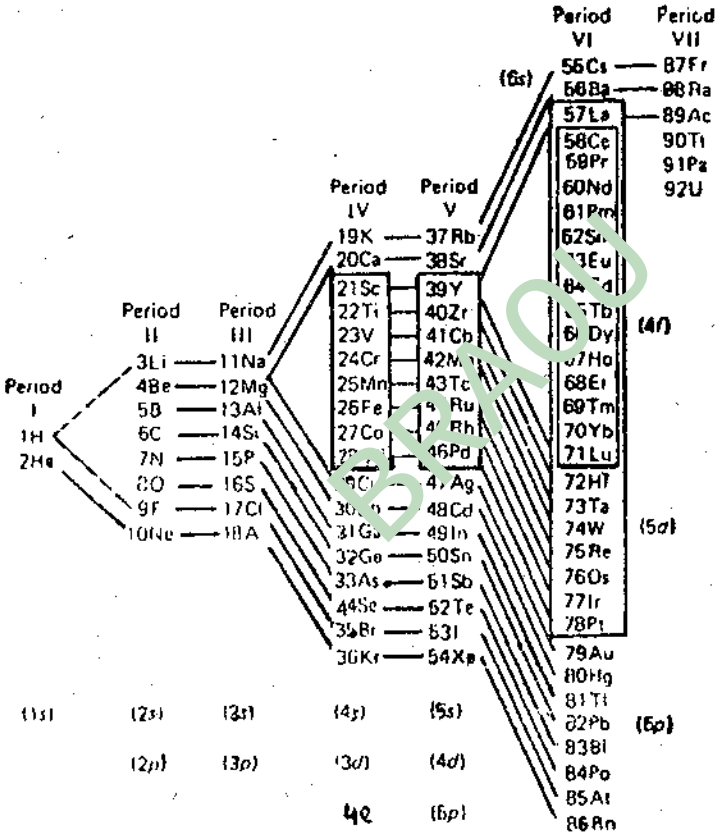


Fig. 10.4

shell l=1 of the shell n = 2. We write this electronic structures of an oxygen atom as $(1s^2 2s^2)\ 2p^4$, the parenthesis being used to denote that the orbits (shell or sub-shells) enclosed by them are closed or completed. Similarity between the same properties of oxygen and sulphur can be understood by the electronic configuration of sulphur, i.e., $(1s^2 2s^2 2p^6 3s^2) 3p^4$ The 16 electrons in sulphur atom are distributed as follows : Two s electrons in each of the shells n = 1, 2, 3 and six p electrons in shell n = 2. There are four p electrons in the shell n = 3 with incomplete configuration.

Table 10.2 - Electronic configuration and ionisation energies

Z Atomic number	Element	K-shell n=1 l=0 ls	L-shell n=2 l=0 2s	l=1 2p	/=0 3s	M-shell n=3 l=1 3p	0 3d	N-shell n=4 l=0 4s	Ionization energy (eV)
1	H	1							13.6
2	He	2							24.5
3	Li	2	1						5.4
4	Be	2	2						9.3
5	B	2	2	1					8.3
6	C	2	2	2					11.2
7	N	2	2	3					14.5
8	O	2	2	4					13.6
9	F	2	2	5					17.3
10	Ne	2	2	6					21.5
11	Na	2	2	6	1				5.1
12	Mg	2	2	6	2				7.6
13	Al	2	2	6	2	1			6.0
14	Si	2	2	6	2	2			8.1
15	P	2	2	6	2	3			10.9
16	S	2	2	6	2	4			10.3
17	Cl	2	2	6	2	5			6.1
18	Ar	2	2	6	2	6			15.7
19	K	2	2	6	2	6		1	4.3
20	Ca	2	2	6	2	5		2	6.1
21	Sc	2	2	6	2	6	1	2	6.5
22	Ti	2	2	6	2	6	2	2	6.8
23	V	2	2	6	2	6	3	2	6.7
24	Cr	2	2	6	2	6	5	1	6.8
25	Mn	2	2	6	2	6	5	2	7.4
26	Fe	2	2	6	2	6	6	2	7.9
27	Co	2	2	6	2	6	7	2	7.9
28	Ni	2	2	6	2	6	8	2	7.6
29	Cu	2	2	6	2	6	10	1	7.7
30	Zn	2	2	6	2	6	10	2	9.4

For nuclei with atomic number Z > 18 the ordering of levels becomes irregular and must be diduced from detailed study of the atomic spectra, but once the electronic configuration is established many of the chemical and physical properties of the element can be understood.

The first element in the first period in hydrogen, the second period begins with the element lithium, the third period with sodium, the fourth with potassium etc. the last elements in each of these periods are inert gases namely helium, neon, argon, krypton etc. respectively. In the fourth period (i.e. 1st long period of 18 elements) we come acorss the irregularity for the first time in assigning state to the added electron; the electron added to an atom of argon to get an atom of potassium (Z=19) is a 4s electron and not a 3d one, that is, it goes into the sub-shell l=0 of the shell n=4 and not into the sub-shell l=2 of the shell n=3, as is to be expected on Pauli's principle. This is because the 4s electron is more tightly bound

than the 3d electrons and has lower energy state. The 4s sub-shell gets filled first, as in calcium (Z=20) also, Thus Potassium and calcium exhibit spectroscopic and chemical properties similar to those of sodium and magnesium respectively. Starting with scandium (Z=21) and 3rd sub-shell starts getting filled with electrons. When we come to chromium atom (Z=24), the electron added to a vanadium (Z=23) atom to get a chromium atom does not go to into the 3rd sub-shell; instead the 6 electrons in a chromium atom outside the closed orbits containing 18 electrons in the inert gas (argon) atom in the immediately proceeding period rearrange themselves with 5 electrons in 3d sub-shell and 1 electron in 4s sub-shell (see Table 10.2); this rearrangement arises from the difference in binding energies of the electrons as in the case of potassium. Similarly other variation in the periodic table can be explained. Breaks similar to that scandium occur in the V and VI period at yttrium (Z=39) and lanthanum (Z=57) respectively. When the 4d and 5d sub-shells start getting filled electrons. The elements from cerium (Z=58) to lutetium (Z=71) which are chemically very similar, do not have counterparts in the proceding rows (or periods) and this is due to the fact that with these elements called the rare earths or lanthanides, the 4f sub-shell starts filling up with electrons. The filling of 5d sub-shell starts in a regular way with hafnium (Z=73), and in gold (Z=79) we find a single 6s electron outside closed shells and sub-shells. Radon (Z=86) has an electronic structure similar to that of inert gases. Francium (Z=87) with which VII period commences has one electron in the 7s sub-shell outside a closed core of 86 electrons (radon configuration) and comes in the first column of the periodic table. With the next element radium (Z=88) the 7-s sub-shell is closed. 6d sub-shell starts getting electrons with actinium (Z=89) and the 5f sub-shells with protactinium forming a new group called the actinide series.

10.7 MOLECULAR AND VIBRATIONAL SPECTRA

Molecular Spectra arise due to transition between allowed energy states of molecules. The energy level system is complicated than that of atoms. This complicated nature arises due to

1. Because the electronic angular momentum is not conserved to in molecules the molecular energy states cannot be specified in terms of it. In diatomic molecules the total charge distribution is symmetrical about the inter nuclear axes. The component of the angular momentum about this axes is conserved. This helps us to classify the molecular energy states.
2. The nuclear motion in molecules cannot be neglected. In a diatomic molecule the nuclear vibrate about the internuclear axes. And also the whole system rotates about its centre of mass. The energy of each of these vibrational rotational motion is quantised. Hence there are more energy level, in a molecule than in an atom. Hence the analysis of molecular spectra is difficult. But the proposed simple models of molecules can explain the molecular spectra successfully.

According to them the excited energy levels of moleculer of a fixed electronic configuration is due to

a) The excited states of the atom results in the electronic excited states in a molecule, the energy levels of which are of the order of a few electron volts are more.
b) The rotation of the molecule as a whole result in the separation of energy levels by about 0.001 ev and
c) The vibrations of atoms of molecules about their mean position result in the separation of energy levels by about 0.1 ev.

In particular if the electrons in atoms remain in the same quantum state then those atoms which vibrate about their mean position give rise to vibration energy bands. Each such vibrational level has several energy levels rising due to various ratational energy states available to the molecule.

Hence the energy levels of molecule result due to the super position of all the three types of namely electronic, rotational and vibrational.

The electronic spectra are observed both in heteronuclear and homonuclear molecules. These are observed both in emission and absorption in the visible and ultra violet regions.

Each spectrum conisiats of large number of bands. Each individual band has a sharp edge known as band head, where the intensity suddenly falls to zero and from this edge the intensity falls off gradually to the other side of the band. The lines in the bands are separated more and more as the distance between the head band increases. The band is said to be degraded towards the side opposite to the band head.

The second type is rotational spectra. These are observed only in heteronuclear diatomic molecular. As the energy involved in this type is about 0.001 ev, these spectra are observed in absoution band in the far infra red (10^6 Å to 10^7 Å) regions or in the microwave (10^7 Å to 10^8 Å) region. The spectrum is comp sed by equidistant lines.

The third is vibrational rotational spectra. These are observed in heteronuclear molecules. As the energy levels involved in such a transition are of the order of 0.1 ev, these spectra are of absorption type in the near infra red region (10^4 Å to 10^6 Å). Each spectrum consists of an intense band accompanised by a weak band of energies.

10.8 SUMMARY

The Bohr's correspondence principle states that the predictions of the quantum theory for the behavious of an atomic system must correspond to the predictions of the classical theory when the quantum theory is taken to classical limit (i.e. where n the principal quantum number is infinity).

The state of an electron in an atom is completely specified by the following quantum numbers.

- n - principal quantum number which takes the values n = 1,2,3.... ∞. This determines the size of the electron orbit.
- l - orbital quantum number which takes the values from zero to (n = l). This determines the shape of the electron orbit.
- s - spin quantum number of the electron and takes the values of 1/2 for all electrons.
- m_l - magnetic quantum number of the electron takes the values -1 to +1. This determines the orientation of the orbit in space.
- m_s - spin projection quantum number which takes the values of 1/2 only. This determines the spin orientation, up or down.

Pauli's exclusion principle stipulates the condition that no two electrons can exist in an atom in the same electronic state. This means, that for each electron at least one of the quantum numbers mentioned above must be different from that of the other electrons.

With a knowledge of the way in which the electrons are distributed in teh various sub-shells one can arrange all the elements in a tabular from know as periodic table of elements.

For atoms with atomic number $Z > 18$ the ordering of the levels become irregular and must be deduced from the detailed study of atomic spectra.

Molecular and Vibrational spectra are due to the arrangement of electrons.

10.9 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. State and explain Pauli's exclusion principle.
2. Discuss the periodic table of elements in the light of Pauli's exclusion principle.
3. Write about the Molecular and Vibrational Spectra

II. Answer the following questions in short.

1. Explain the Bohr's correspondence principle.
2. Discuss about the various quantum numbers associated with an atom.

UNIT–11 : LASER

Contents

11.1 AIMS AND OBJECTIVES

In this unit the principle and working of a laser is discussed. After studying this unit,

- You will be able to explain the principle concepts of spontaneous and stimulated emission.
- You will be able to explain the various applications of lasers in medicine, biology, weaponary, chemistry and in electronic industry.

11.2 INTRODUCTION

Man in his continuous effort for understanding the nature of light found it to be an electromagnetic wave and duality (particle nature and wave nature) during the last part of 19th Century. These discoveries opened new vistas in the construction of various source of radiation not only in the visible region but in the regions below and beyond the visible regions of the electromagnetic spectrum also. In 1960 Maiman succeeded in producing a beam of pure red light of high intensity, conherent and monochromatic. This invention which was called LASER is very much different from the generators of light used till that time. Laser is an acronym for Light Amplification by Stimulated Emission of Radiation. Light sources such as tungsten bulbs; flourescent lamps and even the so called monochromatic sources like sodium lamps produce a wide band of frequencies with no coherence, while the laser beam is highly monochromatic, coherent and intense as well. To-day the laser has become an indespensable tool in the fields of Physics, Biology, Medicine, Chemistry and in industry.

11.3 EMISSION AND ABSORPTION OF RADIATION

In an atom the negatively charged electrons revolve round the positively charged nucleus in certain specified orbits. The various electrons at an atom that revolve round the nucleus can be represented in an energy level diagram. Every electron occupies one of the energy levels as detailed in the energy level diagram scheme described in earlier lessons. According to Bohr's theory the energy can be acquired by the electron in discrete quanta.

The acquisition of energy results in a transition from one shell to the other, and then the atom is in an unstable state. When the electron is in its original level at its lowest energy state it is said to be in ground state. When once the atom is excited the electrons can go from ground level to the higher energy levels. Depending upon the occupation of higher energy levels of the electrons those levels are designated as 1st excited level, 2nd excited level, 3rd excited level etc.

When the outer most electron in an atom is in the lowest energy state, as said earlier, the atom is said to be in the ground state. In contrast when the electrons are in the excited states then the atom is supposed to be in the excited state. When sufficient energy is provided to the outermost electron in an atom, it detaches itself from the parent atom and the atom is said to be ionised. The electron in an excited state drops back to its original level, by giving out the energy in the form of electromagnetic radiation of frequency v, where E = hv, E, being the energy involved and h being Plank's constant. This phenomenon is called emission of radiation. An atom in ground state reaches the excited state by absorbing the energy from the external source. This process is called radiation absorption. This is diagramatically shown in Fig.11.1 (a,b).

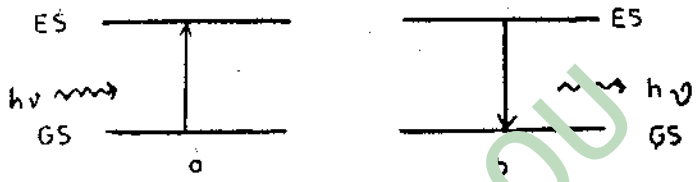


Fig. 11.1 (a) Excitation by absorption (b) de-Excitation by emission

11.4 SPONTANEOUS AND STIMULATED EMISSION OF RADIATION

If an atom from the excited state while droping to the ground state emits radiation spontaneously on its own, the phenomenon is called the spontaneous emission of radiation (Fig. 11.2). The time required for the

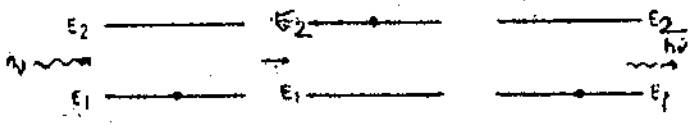


Fig. 11.2 Absorption of a photon leading to Spontaneous emission of a photon

excited atom to emit radiation spontaneously and reach the ground state is known as the life time, τ, of the excited atom and is of the order of 10^{-7} secs. If N_2 is the number of atoms in the excited state at any given instant of time, the rate at which spotaneously emitted atoms reach their ground state will be proportional to the number of excited atoms in the system.

$$\frac{dN_2}{dt} \alpha N_2$$

$$\frac{dN_2}{dt} = -A_{21}N_2 \qquad 11.1$$

Where A_{21} is called the transition probability. This is also known as Einsteins coefficient. Integration of Eqn: (11.1) yields

$$N_2 = N_o \exp(-t/A_{21}) \qquad 11.2$$

N_o being the number of atoms in the excited state at the instant when t = 0. Since most atoms spontaneously emit radiation, the resultant light waves will not be in phase with each other. The resultant photons come out incoherently, The amplitudes and phases of the waves emitted in spontaneous emission would not be the same. The resultant amplitude of vibration of these waves emitted depends upon the nature of superposition that takes place in between these waves. If there is constructive interference between any two waves as shown in Fig. 11.3, the resultant wave amplitude will be maximum.

1. Wave of amplitude A
2. Wave of amplitude B
3. Resultant Wave amplutude (A + B)

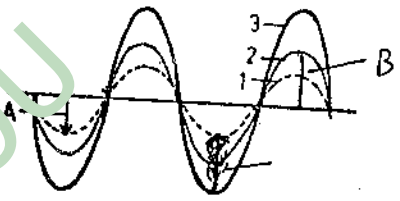
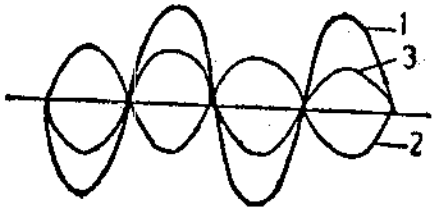


Fig. 11.3 Waves Superposition in Phase

Then the two waves are said to be in phase with each other. Eventhough the wavelength of the two photons is same the destructive interference causes the reduction in amplitude of the resultant wave as shown in Fig. 11.4.



1. First Wave
2. Second Wave
3. Resultant Wave

Fig. 11.4 Waves superposition out of phase

The incandescent lamps we use in daily life emit white light. In these lamps the passage of current through the tungsten filament causes the filament wire to be heated up thereby exciting the atoms in the filament. Through spontaneous emission they emit radiation and reach the ground state, there by resulting in the emission of an incoherent white radiation.

Fluorescence and Phosphorescence

The phenomenon of fluorescenc must be first understood before describing the operation of the laser. A fluorescent substance is one which emits light when it is exposed

to radiation. The frequency of the emitted radiation is generally found to be lower than that of exposed radiation. Visible light from a fluorescent lamp is emitted when the material is exposed to ultraviolet radiation. The coating on the inner side of the fluorescent tube is sensitive to ultraviolent light produced when an electric discharge runs through the gas inside the tube. The fluorcescent material coated on the innerside of the tube absorbs the ultraviolet light and then reradiates visible light. This phenomenon is known as flurescence. In this phenomenon as soon as the incident radiation is withdrawn the emission stops immediately. If the illumination of the emission of radiation continuous for sometimes even after the removal of incident radiation then the phenomenon is called phosphorescene. It is also possible to excite a mixture of two types of atoms, say A and B by irradiation with a suitable frequency. The incident radiation must correspond to a transition involving the ground state of the atoms of A type so that the incident photons can be absorbed to excite these atoms from their ground state to an excited state. If these excited atoms make collisions with atoms of B type, they can tansfer energy to the latter by

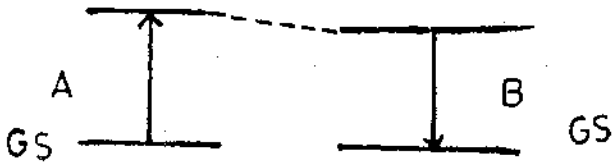


Fig. 11.5 Sensitised flurorescence
A - excitation by absorption, GS - Ground State, B - De-excitation by emission

exciting them (B type of atoms) to an appropriate excited state as shown in Fig. 11.5. The atoms of type A are left in the ground state and carry away any excess energy in the form of kinetic energy, while the atoms of B type can de-excited by emission of radiation. This process is known as sensitized flourescene.

Stimulated Emission

In conventional light sources the atoms do not emit radiation all the time as described earlier and there will not be any coherence between the radiation emitted by various individual atoms. Each atom emits radiation independently and their phases might be different. The amplitue of the resultant wave produced by the superposition of all these waves vary instantaneously. This is analoguous to the phenomena of throwing handful of stones in a water pond which is perfectly still. A series of disturbances will be produced at random. In such a case the amplitude and phase of the water waves produced will be different and the resultant wave motion will be completely incoherent. The light produced by the traditional sources of light resembles that of the water waves described above. This type of light sources are called incoherent light sources. Instead, if each stone is dropped systematically one after the other in regular intervals a series of waves of equal amplitude are produced. The light sources of this type of characteristics are called coherent source possessing of light. In traditional and conventional light sources, the low intensity is another drawback, this is because of the non-cooperative effect produced by the superposition of the various light waves (photons) generated by different atoms.

The efficiency of the light sources can be improved by seeing that all the atoms emit radiation of same frequency at the same time. That is, the atoms in a light source should be excited to a excited level and then they are made to emit radiation in phase. Einstein proposed that it is possible to bring the excited atoms to the ground state using radiation of suitable frequency. Let an atom be excited from the ground state E_o to the excited state

E_1. Then the excited atom by emitting radiation spontaneously reaches another lower excited state E_2. If a photon with energy difference $E_2 - E_o$ collides with one of the atoms in the excited state E_2, stimulated emission takes place and two photons are produced as shown in Fig. 11.6, which are in phase with each other. This phenomena is known as stimulated emission of radiation. The two photons thus

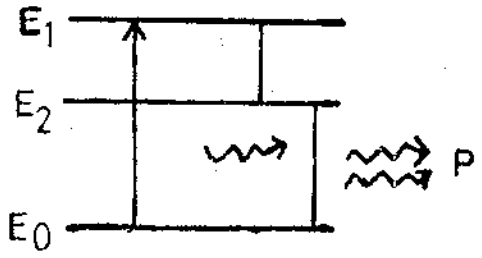


Fig. 11.6 Stimulated emission
P - Photons, E_o, E_1, E_2 - Energy Levels.

produced in turn may collide with two more excited atoms in the excited state E_2 to produce four photons which are in phase with each other. This process multiplies and a parallel beam of light emerges out of the source. The process is thus responsible for the production of highly intense, parallel, coherent light beam in laser by stimulated emission.

11.5 BASIC REQUIREMENTS OF LASING ACTION AND EINSTEIN COEFFICIENTS

When a body is in equilibrium with an incident electro-magnetic radiation field, there will be always a proportion of atoms in the excited state. The rate of emmission of energy by the excited atoms is the same as the rate at which the energy is absorbed from the radiation field by the atoms in the ground state. In 1917 Einstein suggested that the excited atoms not only emit spontaneous radiation but also emit stimulated radiation.

$$B_{12}\rho(v)N_1dv = A_{21}N_2dv + B_{21}\rho(v)N_2dv$$

where

B_{12} - is the transition probability of an atom going from the ground state to the excited state by the absorption of radiation.

A_{21} - transition probability of an exited atom to emit spontaneously and reach its ground state.

B_{21} - transition probability of an atom to emit radiation by stimualted emission and come back to its ground state density.

ρ - density of radiation.

N_1, N_2 - number of atoms in ground state and in the excited state respectively.

A_{21}, B_{12} and B_{21} are also known as Einstein coefficients. It is found in the visible part of the spectrum, the number of atoms returning to the ground state in a given period, as a

result of stimulation, is insignificant of the total number of atoms returning to the ground state by spontaneous emission. At longer wavelengths, the proportion of stimulated emission increases. The rates of spontaneous to stimulated emissions are the same for a wavelength of 60 μ

Einstein Coefficients

Let us consider the relations between the absorption, spontaneous and stimulated emission of radiations constants. These constants are known as Einstein Coefficients.

Let us consider two states of energies are in the ground state (GS) and the other in the Excited State (ES). Let an energy of radiation of frequency ν is aborbed by the GS. the probable rate of transition from GS to ES is proportional to the energy density $\rho(\nu)$ and is written as

$$P_{12} = B_{12}\ \rho(\nu)$$

where B_{12} is the constant of proportionality and is known as Einstein coefficient for the absorption of radiation.

The probability of spontaneous emission from ES to GS is

$$P_{21} = A_{21}$$

where (spontaneous) A_{21} is the Einstein Coefficient of spontaneous emission of radiation.

The probability of stimulated emission from ES to GS depends on the energy density of incident radiation and is written as

$$P_{21} = B_{21}\,\rho(\nu)$$

where (stimulated) B_{21} is another proportionality constant know as Einstein Coefficient for the stimulated emission of radiation.

Hence the total probability of emission transition from ES to GS is

$$P_{21} = A_{21} + B_{21}\,\rho(\nu)$$

11.1

now let us derive a relation for the Einstein relations.

Let N_1 and N_2 be the atoms present in the GS and ES respectively at any instant of time. The number of atoms that absorb photons frequency ν in GS is given by

$$N_1P_{12} = N_1B_{12}\ \rho(\nu) \qquad 11.2$$

similarly the number of photons in ES that can cause emission process is N_2P_{21}. From equ.11.1

$$N_2P_{21} = N_2[A_{21}+B_{21}\ \rho(\nu)] \qquad 11.3$$

At equilibrim the rates of absorption & emission should be equal.

$$\therefore \quad N_1P_{12} = N_2P_{21}$$

$$\therefore \quad N_1B_{12}\ \rho(\nu) = N_2[A_{21} + B_{21}\ \rho(\nu)]$$

$$= N_2A_{21} + N_2B_{21}\ \rho(\nu)$$

$$\therefore \quad \rho(\nu)\ [N_1B_{12} - N_2B_{21}] = N_2A_{21}$$

$$\rho(\nu) = \frac{N_2A_{21}}{N_1B_{12} - N_2B_{21}}$$

Deviding both numerator and denominator with N_2B_{21}...

$$\rho(\nu)\frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}} - 1\right)}\right] \qquad 11.4$$

According to Boltzmann distribution law, at thermal equilibirium at temp. T

$$N_1 = N_o e^{-E_1/KT} \quad \text{and} \quad N_2 = N_o e^{-E_2/KT}$$

where N_o is the total number of atoms present and K is the Boltzman constant.

$$\frac{N_2}{N_1} = e^{\frac{-(E_2 - E_1)}{KT}}$$

$$= e^{\frac{-h\nu}{KT}} \quad \text{since } E_2 - E_1 = h\nu$$

Substituting for $\frac{N_1}{N_2}$ in equation 11.4

$$\rho(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\exp(h\nu/kT)\left(\frac{B_{12}}{B_{21}} - 1\right)}\right] \qquad 11.5$$

Comparing this equation with Plank's radiation law

$$\rho(v) = \frac{8\pi h v^3}{C^3} \cdot \frac{1}{e^{\frac{hv}{KT}} - 1}$$

we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{C^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1 \qquad 11.6$$

From eqn. 11.6, we see that

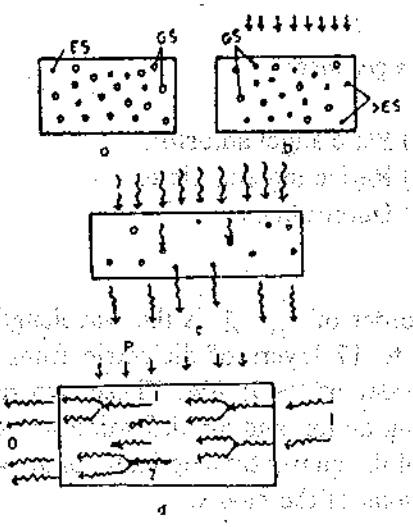
$$B_{12} = B_{21}$$

and $$\frac{A_{21}}{B_{21}} \alpha v^3$$

That is the ratio of spontaneous and stimulation emission is proportional to v^3. Also the probability of stimulated emission is the same as that of absorption.

The rate of absorption of energy depends upon the number of atoms present in the ground state and the rate of emission depends on the number of atoms present in the excited state. If the proportion of atoms present in the ground state is less compared to those present in the excited state, the rate of absorption will be less than the rate of emission.

For stimulated emission to take place the population of atoms in the excited state should be more compared to the groundstate and more over they should stay for considerably longer interval of time in the excited state. Population inversion is the name given to the condition in which there are more number of atoms in the excited state than in the ground state as shown in Fig. 11.7.



(a) Sample in equilibrium [Notice here that more GS atoms are present than excited state atoms]

(b) Population inversion, The pumping field is absorbed by the atoms of the material and there are far more ES atoms than GS atoms.

(c) Sample in equilibrium with surrounding radiation field. Some photons are absorbed by ground state atoms: but an equal number are emitted spontaneously by Es State atoms. Thus there is as much radiation leaving the sample as entering it

(d) An electro magnetic wave passing through a sample in which there is a population inversion gets amplified.

Fig.11.7 Population inversion and amplification

The process in which the population inversion is achieved is called 'pumping'. Depending upon the nature of the active medium in a laser (ruby, semiconducting materials, gases, chemicals etc.), different types of pumping methods are broadly (a) optical pumping (b) elctrical pumping and (c) chemical pumping etc. In a laser for light amplification to take place the basic requirements are :

1. Population inversion,
2. stimulated emission along the direction of the laser beam and
3. amplification.

Let us study how laser action has been achieved in the working of the helium-neon laser.

11.6 HE-NE-LASER

The He-Ne-Laser designed by Ali Javan and his coworkers consists of a quartz tube filled with neon gas at a pressure of 0.1 mm Hg and helium at a pressure of 1 mm Hg, in the ratio 1:5. The length of the quartz tube is 80 cms long having a diamerter of 1.5 cms. They have employed a radio frequency field to pump the system to the metastable state and achieve population invertion. Two highly reflecting mirrors parallel to each other are attached at the ends of the quartz tube. The schematic diagram is shown in Fig.11.8 indicating the various parts of the He-Ne-Laser. The optical

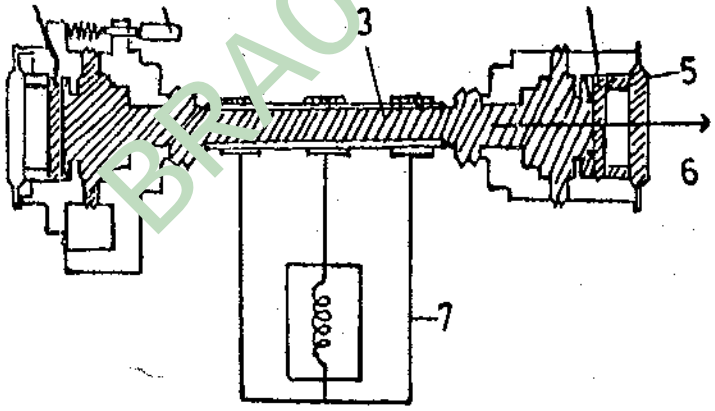


Fig.11.8 The He-Ne gas laser

1) Reflecting plate,
2) Plate angel adjustor,
3) He-Ne-Mixture,
4) Reflecting end plate,
5) Window,
6) Output beam,
7) Rf exciter.

planes of the parallel mirrors should be of the order of $\frac{\lambda}{100}$, λ is the wavelength of the laser beam. These mirrors are made up of 15 to 17 layers of dielectric films of high reflectivity. The reflectivity should be of the order of 99-99.8%. These two reflecting mirrors serve as *Fabry-Perrot Cavity*. The lasing action and amplification are achieved because of these reflecting mirrors. At the ends of the quartz tube optically plane windows are arranged making Brewster's angle with the axis of the cavity.

11.7.2 Military Applications

It is used in military due to its great quantity of energy that can be concentrated, laser has been prominently mentioned as a potential death ray' tube of incendiary weapon for use against enemy missiles. However the technology require to develop such a system is not clearly known. It has been estimated, for example, that in order to destroy a missile from earth, a power output of 10 billion joules would be required. Such a system would require great banks of capacitors to store as well as supply this much of energy. The presently available. Laser system can put out energy of the order of 1500 joules, This intensity if concentrated by proper lens systems, could set on fire easily inflamable material at a distance of 4 km; and could be used to set-off ammunition at that range. Much research is being done in the field of processing and other fields of laser weaponary. It is used for communication purposes also in material applications.

11.7.3 Scientific Applications

Laser has been used to repeat one of the most important scientific experiments-Michelson-Morely experiment, which showed that the velocity of light was constant and thus paved the way for Einstein's theory of relativity. The laser beam is employed for three dimensional photography called holography. Laser Raman Spectroscopy is a potential field for delenating the structure of matter.

The use of lasers in computers is being investigated and several potential applications are there. One of them is to transmit an entire memory bank from one computer to the other. This system would make use of the enormous potential information carrying capability of a beam of light.

11.8 SUMMARY

An atom from the excited state while dropping to the groundstate emits radiation spontaneously. This phenomenon is known as spontaneous emission.

Conventional light sources produce light radiation which is incoherent in nature.

An atom excited from the ground state reaches an excited state. From there it will reach another lower state by spontaneous emission. If the photon with a suitable energy collides with an atom in the 2nd state two photons will be emitted. This is called stimulated emission.

In He-Ne Laser the Ne atoms are active atoms. It can be excited to produce a laser beam both in the infrared region and visible region by varying the cavity length of the Fabry-Perrot resonator.

Lasers find lot of potential applications in medicine, biology, weaponry, chemistry and in electronic industry.

11.9 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Describe the lasing action in the He-Ne laser.

2. Discuss some of the application of lasers.

II. Answer the following questions briefly.

1. Discuss the different between the spontaneous and stimualted emission of radiation.

2. Discuss about the population inversion in lasers.

11.10 GLOSSARY

Black body	:	A body which absorbs all the radiations energy incident upon it.
Emissive power	:	Energy radiated per unit time from unit area in the form of radiation with frequencies lying between ν and $\nu + d\nu$.
Photon	:	A quantum of electromagnetic energy.
Photolelectron	:	Electron emitted from a substance through photoelectric effect.
Recoil electron	:	An electron recoiling from a collision with a photon in a Compton collision.
Threshold frequency	:	The minimum radiation frequency required to realease photoelectron from a given material.
Work function	:	Energy in electron volts required to detach an electron from the surface of a body.
Electron volt	:	The energy required to raise the potential of an electron by one volt.
Compoton shift	:	The shift in wavelength of radiation undergoing compton scattering
Stationary state	:	A Permitted atomic state in which no energy is radiated and which is associated with a definite discrete energy.
Coulomb force	:	The force acting on an electrically charged particle in an electric field.
Rest mass energy	:	The energy associated with a particle of mass m_o when its velocity in zero and is given by $m_o C^2$ according to Einstein's mass-energy formula.
Fabry-Perrot resonator	:	A quartz or a glass tube fixed with parallel semisilvered quartz plates at its ends. It depends

In the He-Ne laser the active atoms are Ne atoms. When He-Ne mixture is excited by means of rf field the He atoms will go to the excited state from the ground state whose ionisation energy is 19.8 eV. This state being a metastable state a large number of He atoms will get excited to this state. In this excited state the He atoms will not radiate energy. These atoms will loose energy when they collide state (as described in Sec. 11.2). When the excited He atom collides with a Ne atom which is in the ground state, the energy is in turn transfered to the Ne atom and it gets excited. The transitions responsible for the emission of infrared radiation in the He-Ne laser are shown in Fig.11.9.

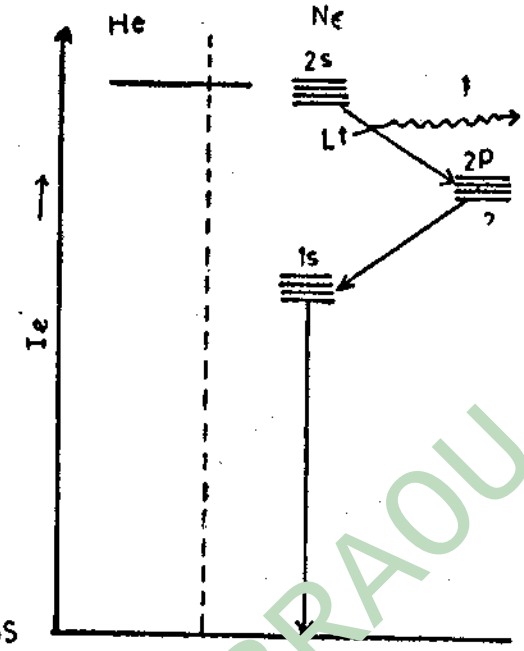


Fig.11.9 The energy levels of helium-neon laser operating in the infra-red part of the Spectrum

When the excited He atoms collide with Ne atoms, the Ne atoms get excited to the 2s state. When the Ne atoms fall from 2s state to 2p state infrared radiation is emitted. The atoms from 2p state will reach the 1s state rapidly resulting in a population inversion condition between 2p and 2s states. Most of the Ne atoms drop to one of the ten closely spaced levels, called 2p levels emitting in the infrared region of the electromagnetic spectrum. By varying the distance between the mirrors (i.e. changing the cavity length) one can obtain radiation of wavelengths ranging from 10,600 to 15,000 $\overset{o}{A}$. The intensity of the radiation of wavelength 11532 $\overset{o}{A}^{o}$ is of most intense. The Ne atoms fall from 2p state to 1s state, as described earlier, by spontaneous emission. The pink colour glow of the laser tube is because of this reason only.

It is also possible to excite the He atoms to an excited state of energy 20.7 eV. This state is also a metastable state. He atoms existing in the state will collide with the Ne atoms in the ground state and will transfer their energy to the Ne atoms, in turn these Ne atoms will go to 3s excited state as shown in Fig.11.10. When these atoms come to 2p

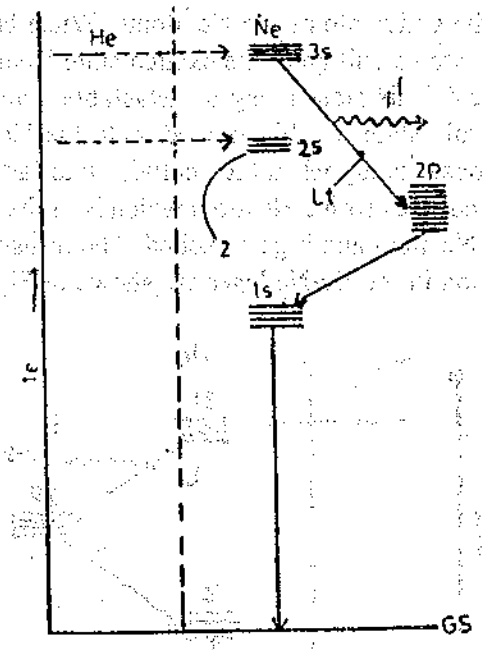


Fig.11.10 The energy levels of a He-Ne laser Operating in the visible point of the Spectrum

state the resulting radiation required for laser beam will be emitted whose wavelength lies in the visible region and at a wavelength of 6328 $\overset{o}{A}$. Generally He-Ne laser of output power 0.5 to 10mw will be constructed. He-Ne laser with an output power of 100 mW have been constructed. In this laser the length of the quartz tube maintained at 2 meters with a diameter ranging from 0.7 to 1.0 cm. The following table illustrates the wavelength and the power output of He-Ne lasers.

Table 11.1 - Output power of He-Ne Lasers

Wavelength in $\overset{o}{A}$	Output power mW
6118	2
6328	25
10,840	10
11,520	12
33,810	2

11.7 APPLICATION OF LASERS

The lasers have such remarkable properties which have provided a number of applications in various fields of pure science, medicine, biology, range finding and detection, communication, weaponary and microwelding etc.

11.7.1 Medical Applications

Lasers have been successfully used in the treatment of detached retina and show great promise in the treatment of cancer. Surgeons are seeing the possibility of using the laser as an improved tool for microsurgery. Experiments are now under way to use laser to effect chromosomal changes.

upon the interference effects produced by multiple reflections between the fixed parallel half silvered quartz plates.

Brewster's window : A glass or quartz plate arranged at Brewster's angle to the axis of the resonator cavity to produce a polarised beam of light.

11.11 RECOMMENDED BOOKS

1.	M.N. Saha and B.N. Srivastava	A Treatise on Heat	The Indian Press Pvt. Ltd. Allahabad
2.	J.B. Rajam	A Text Book of Heat	S.Chand & Co., New Delhi
3.	V.V. Krishna Rao	Book of Heat	V.D. Murthy & Co., Nellor
4.	C.L. Hemenway R.W. Henry and M. Cantoon	Physical Electronics	John Wiley & Sons Inc., New York
5.	R.M. Eisberg	Fundamentals of Modern Physics	John Wiley & Sons Inc., New York
6.	S.E. Liverhant	Outlines of Atomic Physics	Regents Publishing Co. Inc., New York.
7.	H.E. White	Modern College Physics	Affiliated East-West Press Pvt. Ltd. New Delhi.
8.	S.B. Borowitz and	A contemporary view of Elementary Physics	McGraw-Hill Book Company, New York
9.	F.K. Richtmeyer E.H. Kennard and T. Lauritsen	Introduction to Modern Physics	McGraw Hill Book Company, Inc., New York.
10.	A.H. Compton and S.K. Allison	X-rays in Theory and Experiment	Affiliated East West Press Pvt. Ltd. New Delhi
11.	D.F. Jackson	Concepts of Atomic Physics	McGraw-Hill Book Company, England

12.	G. Herzberg	Atomic Spectra and Atomic Structure	Dover Publications England
13.	F.K. Ritchmeyer, E.H. Kennard and T. Lauritsen	Introduction to Modern Physics	McGraw Hill Book Company, Inc., New York
14.	R.M. Eisberg	Fundamentals of Modern Physics	Wiley & Sons, United States
15.	A.L. Schawlow	Scientific American	W.H. Freemann Company, San Fransisco
16.	R. Brown	Lasers-A survey of their performance and applications	Business Book Ltd. London,
17.	N. Sabolev	Lasers and their Prospects	Mir Publications, Moscow.
18.	K. Gopalakrishna	Atomic and Nuclear Physics	Macmillan India Ltd., Delhi.

BLOCK - 3 : NUCLEAR STRUCTURE AND PROPERTIES OF NUCLEI

UNIT–12 : NATURAL RADIO ACTIVITY

Contents

12.1 AIMS AND OBJECTIVES

In this unit the phenomenon of radio activity and the laws of radio active disintegration are discussed.

After going through this unit

- You can evaluate the disintegration constant (λ) half life. ($T_{1/2}$) and mean life (T) of a radio active element.
- You will be able to estimate the Age of the earth with the help of radio active measurements.
- You will be able to describe the Radio Active series.

12.2 INTRODUCTION

The phenomenon of radioactivity was discovered by Henry Becquerel in 1896. While studying the luminiscense of uranium salts excited by ordinary light, Becquerel observed that the shadow casting radiation from uranium persisted, even when the exciting light was removed. Becquerel also showed that the radiation was found with all uranium compounds and the intensity of the radiation was proportional to their uranium content. The spontaneous emission of radiation, called radioactivity, is a property of the uranium itself. The actual radiations studied by Becquerel were fast electrons in the β-decay of the daughter products of U^{238}.

Pierre Curie and Marie Curie surveyed many other elements for radiactive emission and discovered thorium to exhibit radiactivity. They also discovered by chemical extraction of the uranium from pitchblende, a hightly active element, namely Polonium, named after the native country of Maire, Curie, that is, Poland. The activity of Polonium was found to decrease slowly and Curies continued their work and found another highly active element which behaved chemically like barium and was named by its discoverers as radium.

The experiements of Curie and of Rutherford showed that the radiation emanating from radioactive substances contained components of different penetrating power as

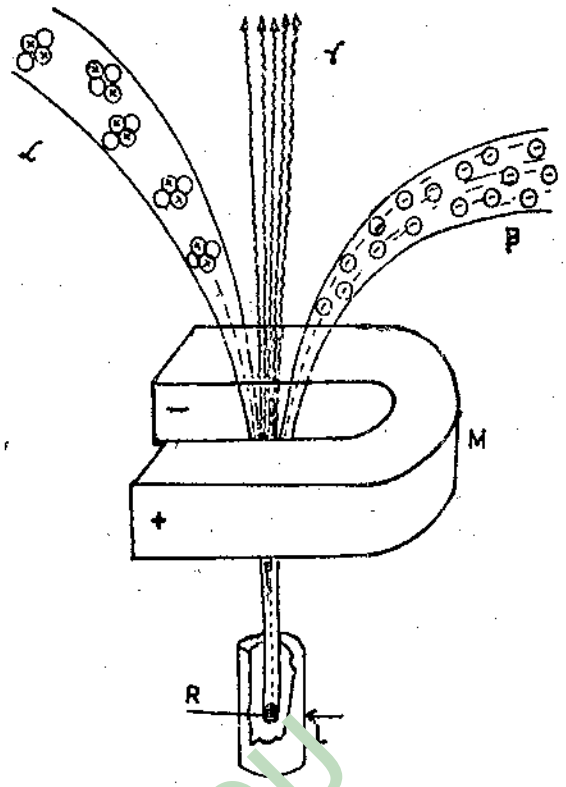


Fig. 12.1 Seperation of radioactive radiations into its components under the action of a magnetic field. L - Lead Cylinder, R - Radioactive source, M - Magnet.

assessed by their absorption in matter. The less penetrating rays which were completely aborbed by a few centimeters of air were called alpha (α-rays). The more penetrating radiation which were absorbed by about 1 mm of lead were named beta rays (β-rays). In 1990 Villard identified a third component which is a more penetrating type of ionizing radiation capable of traversing as much as 10 cm of lead. These rays could not be deviated by a magnetic field and were described as gamma rays (γ-rays). These rays are known to be electromagnetic in nature. (α-rays) and β-rays are deflected by magnetic field. Fig. 12.1 illustrates the difference between these rays.

The left-side deflected radiation which has large radius of curvature are α-rays which have been identified as ionized helium nuclei, consisting of two protons and two neutrons. The right side deflected radiation which has small radius of curvature and β-rays identified as electrons. The centre radiation which is not affected by the magnetic field is gamma radiation.

In this lesson we shall study in detail the laws of radioactive disintegration, radioactive series, radioactive equilibrium and properties of radioactive radiations and how the radioactive decay processes helps one to evaluate the age of the earth.

12.3 NATURAL RADIOACTIVITY - LAW OF RADIOACTIVE DISINTEGRATION

The phenomenon of radioactivity exhibited by naturally occuring elements is termed as natural radioactivity. All atoms of atomic number greater than 83 are radioactive. These are transformed into other atoms by the spontaneous emission of alpha or beta particles. The mode of decay and the readiness with which the transformations occurs is a

characteristic of the particular species of atom. Sometimes gamma rays accompany α-rays or beta rays. The phenomenon of radioactivity is unaffected by electric and magnetic fields or any external agency.

A quantitative description of radioactive growth and decay was provided by the experiments conducted by Crookes, Becquerel and Rutherford and Soddy. Crookes found that if a uranium salt was precipitated from solution by the addition of ammonium carbonate and then redissolved in excess of the reagent, a small residue was left. This residue when removed from the solution, was found to be highly radiactive. The product obtained by evaporating the solution which contained uranium had very little activity. This indicated that the active substance contained in the residue is different from uranium and given the name uranium X, (UX) to distinguish it from uranium. Becqaerel observed that if UX and uranium fractions were allowed to stand seperately for sometime, the activity of UX decreased while that of uranium fraction increased. Rutherford and Soddy obtained similar results with thorium salts. When the active material thorium X was seperated from thorium, thorium was found to be inactive. After a few days it was noticed that thorium X was losing its activity while thorium which had been freed from thorium X was recovering its activity.

Rutherford and Soddy studied quantitatively the rate of decay of Th X activity and the rate of growth of the activity and obtained the curves as shown in Fig. 12.2. The experimental dacay curve for Th X is exponential in nature

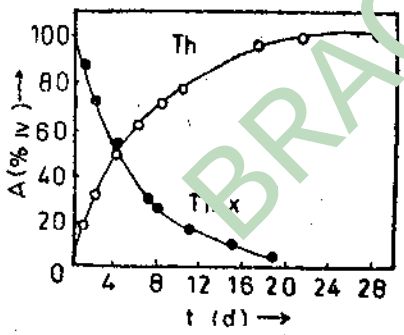


Fig. 12.2 The decay of Th X activity (A) and growth of the activity with time t; N - % of Normal activity.

Hence the activity could be expressed as a function of time (t) by the following equation

$$A_x(t) = A_x(o)e^{-\lambda t} \qquad 12.1$$

where $A_x(o)$ is the initial activity of Th X, $A_x(t)$ is the activity after a time t and λ is a constant called the disintegration constant. The rate of growth of the activity can be expressed as

$$A(t) = A_o\left(1 - e^{\lambda t}\right) \qquad 12.2$$

where $A_{(t)}$ represents the activity of Th at any time t and A_o represents the initial activity of Th at t = o which will be zero. The decay constant (λ has) the same value as in Eq.12.1. The decay and growth curves are symmetrical.

Rutherford and Soddy formulated a theory of radioactive change. Accordingly the atoms of radioactive elements undergo spontaneous disintegration with the emission of α or β particles leading to the formation of atoms of new element. The intensity of radioactivity, called the activity, is proportional to the number of radioactive nuclei present at that instant of time. To express this statement in a mathematical form let us consider that $N_{(t)}$ represents the number of radioactive atoms present at time t. Let $dN_{(t)}$ represent the number of atoms that disintegrate in a samll interval of time dt. Then the rate of decay $\frac{dN_{(t)}}{dt}$ can be represented by

$$-\frac{dN_{(t)}}{dt} \alpha N(t) \qquad 12.3$$

The negative sign in the above equation indicates that the process is a decay process. We can write Eq. 12.3 as

$$-\frac{dN_{(t)}}{dt} = \lambda N(t) \qquad 12.4$$

where the constant of proportionality is called the disintegration constant of the particular radioactive isotope. Rewriting Eq. 12.4 as

$$\frac{dN_{(t)}}{N_{(t)}} = -\lambda dt \qquad 12.5$$

Integrating Eq. 12.5

$$\int \frac{dN_{(t)}}{N_{(t)}} = -\lambda \int dt$$

$$\log_e N_{(t)} = -\lambda t + C \qquad 12.6$$

when t - 0 let $N_{(t)} = N_o$.

(The no. of atoms present at t = 0 i.e at the starting of the decay process).

Then

$$\log_e N_{(o)} = C$$

Substituting for C in equ. 12.6

$$\log_e N_{(t)} = -\lambda t + \log_e N_{(o)}$$

or $$\log_e N_{(t)} - \log_e N_{(o)} = -\lambda t$$

or $$\log_e \frac{N_{(t)}}{N_{(o)}} = -\lambda t$$

or $$N_{(t)} = N_{(o)} e^{-\lambda t} \qquad 12.7$$

Eq. 12.7 which is similar to Eq. 12.1 and represents the decay process of a radioactive element.

Worked Example - 1

The disintegration constant of radium D is 1.13 x $10^{-9}s^{-1}$. How much would radium D remain after 5 years from an initial amount of 10g?

The relation governing radioactive decay is

$$N_{(t)} = N_o e^{-\lambda t}$$

Since N is proportional to the mass m we can write the decay equation in terms of mass as

$$m = m_o e^{-\lambda t}$$

As per the problem

$$m_o = 10g$$

t = 5yrs = 5 x 365 x 24 x 60 x 60 seconds

λ = 1.13 x $10^{-9}s^{-1}$

$$m = (10g)e^{-1.13\times10^{-9}(5\times365\times24\times60\times60)}$$

$$m = (10g)e^{-0.178} = (10g)(0.84)$$

$$m = 8.4g$$

The amount of radium D present after 5 years from an initial amount of 10 g is 8.4 g.

12.4 DISINTEGRATION CONSTANT, HALF LIFE AND MEAN LIFE OF A RADIOACTIVE SUBSTANCE

A radioactive nuclide is characterized by the rate at which it disintegrates. It is generally characterized by any one of the three quantities namely the disintegration constant, the half-life or the mean life.

The half-life of a radioactive substance is defined as the time interval during which the activity reduces to one half of its value at the beginning of the time interval. Since the activity is proportional to the number of radioactive nuclides present we have

$$N_{(t)} = N_{(o)}e^{-\lambda t}$$

$N_{(t)}$ becomes $\frac{N_o}{2}$. When t = $T_{1/2}$ the half life.

$$\frac{N_{(o)}}{2} = N_o e^{-\lambda T_{1/2}}$$

or $e^{-\lambda T_{1/2}} = \frac{1}{2}$

or $e^{\lambda T_{1/2}} = 2$

Taking logarithms on either side we get

$$\lambda T = \log_e 2 \Rightarrow 2.303 \log_{10} 2 \Rightarrow 0.693$$

$$\therefore \quad T_{\frac{1}{2}} = \frac{0.693}{\lambda} \qquad 12.8$$

The relation between the activity and half-life of a radioactive substance is illustrated in Fig.12.3. After n half lives, that is t = $nT_{1/2}$ the fraction of the activity remaining is $\left(\frac{1}{2}\right)^n$ This fraction never becomes zero but becomes very small as t increases.

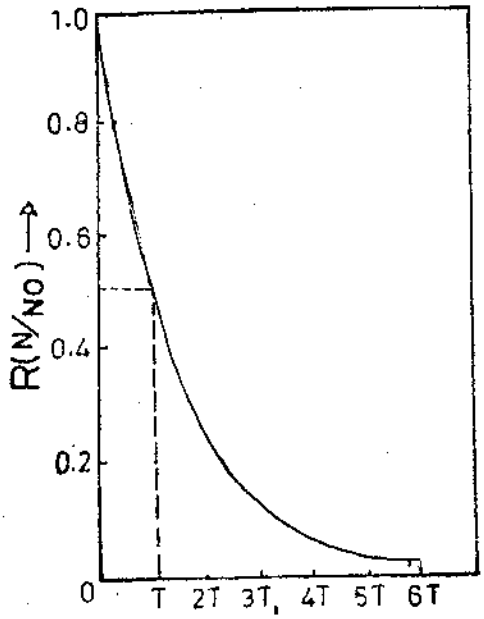


Fig. 12.3 Activity versus integral multiplies of half-life of radioactive substance. R - Relative activity.

After 10 half-lives interval of time the activity falls to $\left(\frac{1}{2}\right)^{10}$ of original amount, which is negligible in comparison with he initial value.

The mean life or average life τ repersents the average life time of all atoms of a radioactive material. It is usually given by the sum of the times of existence of all the atoms, divided by the initial number. The number of atoms which decay in the time interval between t and t+dt is given by

$$dN = \lambda N dt$$

The number of atoms still existing at time t is given by

$$\therefore \quad N = \lambda N_o e^{-\lambda t}$$

$$\therefore \quad dN = \lambda N_o e^{-\lambda t} dt = N dt \qquad 12.9$$

Since the decay process is a statistical one, the life time of any single atom may have any value ranging from 0 to ∞. Hence the mean life τ can be given by

$$\tau = \frac{1}{N_o} \int_o^\infty N_o e^{-\lambda t} t dt \qquad 12.10$$

$$\tau = \lambda \int_o^\infty t e^{-\lambda t} dt \qquad 12.11$$

$$\tau = -\left\{\frac{(\lambda t + 1)}{\lambda} e^{-\lambda t}\right\}_o^{\infty}$$

$$\tau = -\left\{0 - \frac{1}{\lambda}\right\} = \frac{1}{\lambda} \qquad 12.12$$

The mean life is simply the reciprocal of the disintegration constant. From Eq. 12.8 we have

$$T_{1/2} = 0.693\tau \qquad \because \; T = \frac{1}{\lambda}$$

If we know λ, τ can be easily evaluated.

The disintegration constant λ can be determined experimentally using the relation

$$A_{(t)} = A_{(o)} e^{-\lambda t} \qquad 12.13$$

where, $A_{(t)}$ and $A_{(o)}$ represent the activity of the radioactive substance at any time t=t and at time t=o. Eq. 12.13 can be rewritten as

$$\log_{10} A_{(t)} = \log_{10} A_{(o)} - \frac{\lambda t}{2.303}$$

or $$\log_{10} A_{(t)} = \log_{10} A_{(o)} - 0.4343\lambda t$$

A Plot of logarithm of the measured activity against time yields a straight line, the slope of which is equal to -0.4343 λ. Fig.12.4 a plot of activity versus time on a semi-log paper. For the data given in the figure, the slope is -0.00808 with the time expressed in minutes. The value of λ is 3.1 x $10^{-4}s^{-1}$.

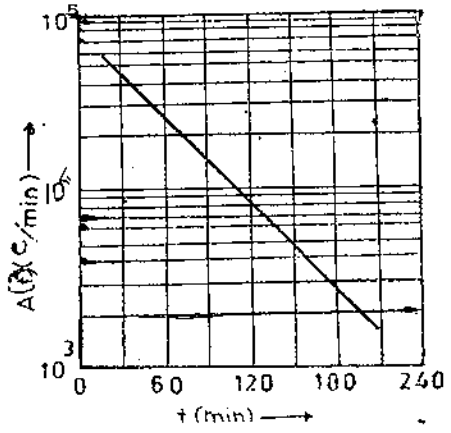


Fig. 12.4 Activity versus time Evaluation of λ A(t) Activity (counts/minute)

Worked Example - 2

The activity of a radioactive substance after 5 minutes is $\frac{1}{3}$ of its original activity. Determine the disintegration constant, half-life and mean life of the radioactive substance.

We have

$$A_{(t)} = A_{(o)}e^{-\lambda t}$$

when t = 5 minutes = 300s $A_{(t)} = \frac{A_o}{3}$

$$\therefore \quad \frac{A_o}{3} = A_{(o)}e^{-\lambda 300}$$

$$\therefore \quad e^{\lambda 300} = 3$$

$$\lambda 300 = 2.303\log_{10} 3$$

$$\therefore \quad \lambda = \frac{2.303\log_{10} 3}{300} = \frac{2.303(0.4771)}{300}$$

$$\lambda = 3.7 \times 10^{-3} s^{-1}$$

Half-life $T = \frac{0.693}{\lambda} = \frac{0.693}{3.7\times10^{-3}s^{-1}} = 1.89\times10^{2}s$

Mean life $\tau = \frac{1}{\lambda} = \frac{1}{3.7\times10^{-3}s^{-1}} = 2.73\times10^{2}s$

12.5 UNITS OF RADIOACTIVITY

The activity of a radioactive substance is generally expressed in terms of the number of atoms which disintegrate per unit time. The standard unit of radioactivity is Curie. It is defined as that quantity of radioactive material giving 3.7×10^{10} disintegrations per second. The milli-Curie and micro Curie corresponding to 3.7×10^{7} and 3.7×10^{4} disintegrations per second respectively and are also used as units, representing the activity of a substance.

Worked Example - 3

Determine the amount Pb-214 which has an activity of 1 Curie. The disintegration constant of Pb-214 is $4.31 \times 10^{-4} s^{-1}$.

Let W be the weight of Pb-214 that has an activity of 1 Curie. As per the disintegration law

$$-\frac{dN}{dt} = \lambda N$$

The number of atoms of N is W gram of Pb-214 are given by

$$N = \frac{6.02 \times 10^{25} \times W}{214}$$ where 6.02 x 10^{23} represents the Avagadro number.

Substituting for λ and N

$$-\frac{dN}{dt} = 4.33 \times 10^{-4} s^{-1} \left(\frac{6.02 \times 10^{25}}{214} W \right) = 1.21 \times 10^{18} W$$

disintegrations per sec.

If $-\frac{dN}{dt} = 1 Curie = 3.7 \times 10^{10}$ disintegration/sec.

Then

$3.7 \times 10^{10} = 1.21 \times 10^{18} W$

$$\therefore \; W = \frac{3.7 \times 10^{10}}{1.21 \times 10^{18}} = 3.1 \times 10^{-8} g$$

12.6 RADIOACTIVE SERIES

In the year 1913, Soddy, Russel and Fajans formulated displacement law, obeyed by all radioactive nuclides. According to this law (i) the loss of an α -particle an element two places to the left in the periodic table and towers its mass by four units and (ii) the loss of a β -particle displaces an element one place to the right in the periodic table but does not alter the atomic mass. All atoms whose atomic number is greater than 83 decay following the displacement law. All the naturally occuring radioactive elements were found to belong to any one of the three long chains called radioactive series. These series are named as the uranium, actinium and thorium series respectively after the elements at or near the head of the series. In the uranium series, the mass number of each member of the series can be expressed as (4n+2), where n is an integer. Hence uranium series is also called as (4n+2) series. In actinium series, the mass number of any member is given by (4n+3). In thorium series the mass number of any member of the series is given by 4n. There is no naturally occurring radioactive nuclide whose mass number can be given by (4n+1), where n is an integer. Artificially produced radioactive nuclides belong to this (4n+1) series and go by

the name neptunium series. The naturally occurring series end in lead isotopes.

Check Your Progress

The loss of a _____ particle displaces an element one place to the _____ in the periodic table.

12.7 SUCCESSIVE RADIOACTIVE DISINTEGRATION

In all the radioactive series, the parent nuclide decays into a daughter nuclide, which decays in turn and so on until finally a stable end product is reached. In the study of radioactive series it is essential to know the number of atoms of each member of the series as a function of time. This involves solving of a system of differential equations. To start with, let us consider a radioactive nuclide, denoted by A, decays into another radioactive nuclide, denoted by B, which in turn decays into a stable end product, denoted by C. The number of atoms of A, B and C at any instant of time t be given by N_1, N_2 and N_3 respectively. Let the disintegration constants of A, B, and C be λ_1, λ_2 and λ_3 respectively. This system can be described by the following equations

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad 12.14$$

$$\frac{dN_2}{dt} = -\lambda_1 N_1 - \lambda_2 N_2 \qquad 12.15$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 \qquad 12.16$$

The above equations indicate that the parent nuclide decays according to the basic exponential decay law. Atoms of the second kind are formed at the rate of $\lambda_1 N_1$ because of the decay of parent atom and disappear at the rate $\lambda_2 N_2$. Atoms of the stable end product appear at the rate of $\lambda_2 N_2$ as a result of the decay of the atoms of the second kind.

We can write the number of atoms N_1 of radioactive element A as

$$N_1 = N_1^{o} e^{-\lambda_1 t} \qquad 12.17$$

where N_1^{o} represents the number of atoms of A at t=0.

Inserting Eq. 12.17 in Eq. 12.15, we get

$$\frac{dN_2}{dt} = \lambda_1 N_1^{o} e^{-\lambda_1 t} - \lambda_2 N_2$$

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1^{\,o} e^{-\lambda_1 t}$$

Multiplying both sides with $e^{\lambda_2 t}$ we get

$$e^{\lambda_2 t}\frac{dN_2}{dt} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_1^{\,o} e^{(\lambda_2 - \lambda_1)t}$$

or $$\frac{d}{dt}\left\{N_2 e^{\lambda_2 t}\right\} = \lambda_1 N_1^{\,o} e^{(\lambda_2 - \lambda_1)t}$$

Integrating on both sides we get

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o} e^{(\lambda_2 - \lambda_1)t} + C_1 \qquad 12.18$$

where C_1 represents the constant of integration. Eq. 12.18 can be rewritten as

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o} e^{-\lambda_1 t} + C_1 e^{-\lambda_2 t} \qquad 12.19$$

C_1 can be determined by imposing the condition that

at t = 0 let $N_2 = N_2^{\,o}$. Then

$$N_2^{\,o} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o} + C_1$$

$$\therefore \quad C_1 = N_2^{\,o} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o} \qquad 12.20$$

Inserting Eq. 12.20 in Eq. 12.19 and simplifying we get,

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_2^{\,o} e^{-\lambda_2 t} \qquad 12.21$$

The number of atoms of C can be found by inserting Eq. 12.21 in Eq. 12.16 and integrating, it gives

$$N_3 = \left[\frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^{\,o} - N_2^{\,o}\right] e^{-\lambda_2 t} - \frac{\lambda_2}{(\lambda_2 - \lambda_1)} N_1^{\,o} e^{-\lambda_1 t} + C_2 \qquad 12.22$$

where C_2 represents the constant of integration which can be determind imposing the condition that at t = 0 let $N_3 = N_3^{\,o}$. Then

$$N_3^{\,o} = \frac{\lambda_1}{(\lambda_2 - \lambda_1)} N_1^{\,o} - N_2^{\,o} - \frac{\lambda_2}{(\lambda_2 - \lambda_1)} N_1^{\,o} + C_2$$

$$N_3^{\,o} = -N_2^{\,o} - \frac{N_1^{\,O}}{(\lambda_2 - \lambda_1)}[\lambda_2 - \lambda_1] + C_2$$

$$\therefore \quad C_2 = N_3^{\,o} + N_2^{\,o} + N_1^{\,o} \qquad 12.23$$

Substituting for C_2 from Eq. 12.23 in 12.22 we get

$$N_3 = N_3^{\,o} + N_2^{\,o}\left(1 - e^{\lambda_2 t}\right) + N_1^{\,o}\left(1 + \frac{\lambda_1}{(\lambda_2 - \lambda_1)} e^{-\lambda_2 t} - \frac{\lambda_2}{(\lambda_2 - \lambda_1)} e^{-\lambda_1 t}\right)$$

generally $N_1^{\,o}$ will be present initially with $N_2^{\,o} = N_3^{\,o} = 0$. Then

$$N_2 = \frac{\lambda_1}{(\lambda_2 - \lambda_1)} N_1^{\,o}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_3 = N_1^{\,o}\left\{1 + \frac{\lambda_1}{(\lambda_2 - \lambda_1)} e^{-\lambda_2 t} - \frac{\lambda_2}{(\lambda_2 - \lambda_1)} e^{-\lambda_1 t}\right\}$$

The mathematical treatment just discussed above can be extended to a chain consisting of any number of radioactive products. For this system the differential equations are

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3$$

$$\therefore \quad \frac{dN_n}{dt} = \left(\lambda_{n-1} N_{n-1} - \lambda_n N_n\right)$$

The solution for this system of equations under the assumption at t = 0, $N_1 = N_1^o$ and $N_2^o = N_3^o = \ldots N_n^o = 0$ was derived by Bateman.

The number of atoms of the nth member of the chain is given by

$$N_n(t) = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + C_3 e^{-\lambda_3 t} + \ldots C_n e^{-\lambda_n t}$$

where

$$C_1 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\ldots(\lambda_n - \lambda_1)} N_1^o$$

$$C_2 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\ldots(\lambda_n - \lambda_2)} N_1^o$$

$$C_n = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_1 - \lambda_n)(\lambda_3 - \lambda_n)\ldots(\lambda_{n-1} - \lambda_n)} N_1^o$$

The application of Bateman's equations to the decay of Radium A is illustrated in Fig. 12.5

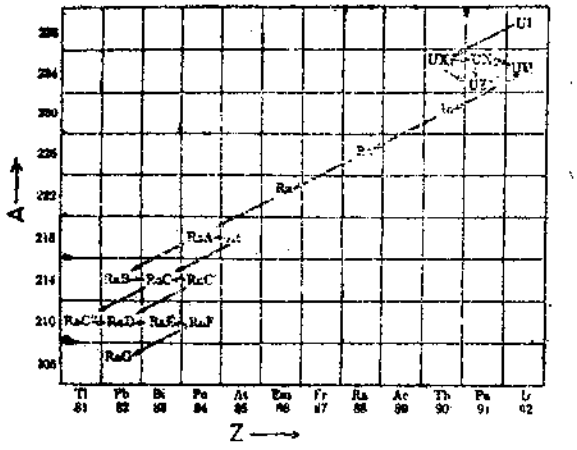


Fig. 12.5 Successive radioactive disintegration, K- Relative number of atoms

RaA (Po^{218}) decays into RaB (Pb^{214}) with a half life of 3.05 min. RaB decays into RaC (Bi^{214}) with a half life of 26.8 min. Radium C decays into Radium D (Pb^{210}) with a half-life of 19.7 min. The end product RaD has a half-time of 22 years. The half-life of

RaD is sufficiently long so that the number of radium D atoms which disintegrate may be neglected. The number of RaA atoms decreases exponentially. The number of RaB atoms which is initially zero passes through a maximum in 10 minutes and later decreases with time. The number of RaC atoms passes through a maximum after 35 min. The number of RaD atoms increases reaching a maximum when RaA and RaB have disappeared. At the end RaD would decay exponentially with a half-life of 22 years. The sum of all atoms presented at any time is N_o, the initial number of atoms of RaA.

12.8 RADIOACTIVE EQUILIBRIUM

A system is said to be in equilibrium if the derivative of the function describing the system with respect to time is equal to zero. When this condition is applied to radioactive chain, we have

$$\frac{dN_1}{dt} = -\lambda_1 N_1 = 0$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 = 0$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3 = 0$$

$$\frac{dN_n}{dt} = \lambda_{n-1} N_{n-1} - \lambda_n N_n = 0$$

The above conditions cannot be satisfied rigorously if the parent is a radioactive substance since $\lambda_1 \neq 0$. But it is possible to achieve a state very close to equilibrium if the parent decays very slowely in comparision with other members of the chain. This means the half-life of the parent must be long compared with any of its decay products. This condition is satisfied by the naturally occuring radioactive chains. Uranium I has a half-life of 4.5 x 10^9 yr. In such a case , N_1 can be taken to be constant and λ_1 is very much smaller than that of any of the λ's in the chain. The first Eq. 12.24 is then a very good approximation and the rest of the conditions are regorously valid. This type of equilibrium is called secular equilibrium and satisfies the conditions.

$$\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = ... \lambda_{n-1} N_{n-1} = \lambda_n N_n$$

Secular equilibrium can also be attained when a radioactive substance is produced at a steady rate. Then $\lambda_1 N_1$ will be a constant.

Let us now study the conditions under which secular equilibrium can be achieved. Consider the case of a long-lived parent ($T \simeq \infty$) and a short-lived daughter. Let the parent be initially pure. The expressions for the number of atoms of parent and daughter are given by,

$$N_1 = N_1^o e^{-\lambda_1 t} \simeq N_1^o \qquad \text{since } \lambda_1 \simeq 0.$$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^o \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$\simeq \frac{\lambda_1}{\lambda_2} N_1^o \left(1 - e^{-\lambda_2 t}\right)$$

$$\text{or} \quad \lambda_2 N_2 \simeq \lambda_1 N_1^o \left(1 - e^{-\lambda_2 t}\right) \qquad 12.25$$

Eq. 12.25 describes the acivity of the daughter as a function of time in terms of the activity of the parent. The total activity can be given by

$$\text{A total} = \lambda_1 N_1^o + \lambda_2 N_2 = 2\lambda_1 N_1^o - \lambda_1 N_1^o e^{-\lambda_2 t} \qquad 12.26$$

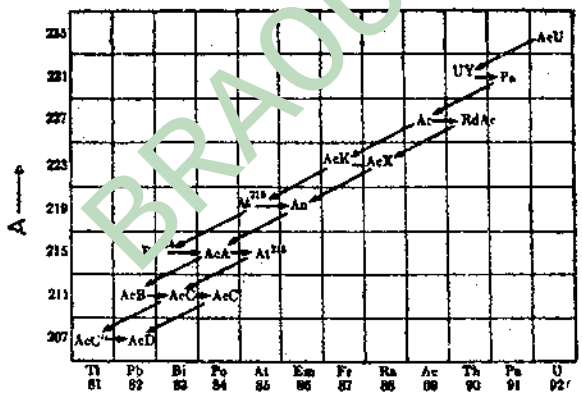


Fig.12.6 Secular equilibrium
a - Daughter Activity growing in freshly purified parent fraction. b - Activity of parent, ($T_{1/2} = \infty$). c - Total activity of initially pure parent fraction. d - Decay freshly isolated daughter fraction. N - Number of atoms.

Eq. 12.26 indicates that as t increases, the activity of the daughter increases and after several half-lives $\lambda_2 N_2$ approaches $\lambda_1 N_1^o$ satisfying the condition for secular equilibrium. These aspects are illustrated in Fig. 12.6.

A different type of equilibrium called transient equilibrium results if the parent lives longer than the daughter ($\lambda_1 < \lambda_2$) but the half-life of the parent is not very long. In this case $\lambda_1 = 0$ cannot be assumed. If the parent is initially pure, then

$$N_1 = N_1^o e^{-\lambda_1 t}$$

$$N_2 = \frac{\lambda_1}{(\lambda_2 - \lambda_1)} N_1^{\,o}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

When t is large, $e^{-\lambda_2 t}$ becomes negligible compared with $e^{-\lambda_1 t}$ and hence

$$N_2 \simeq \frac{\lambda_1}{(\lambda_2 - \lambda_1)} N_1^{\,o} e^{-\lambda_1 t} \qquad 12.27$$

Eq. 12.27 indicates that the daughter decays with the same as the half-life as the parent. Since $N_1 = N_1^{\,o} e^{-\lambda_1 t}$, we get

$$\frac{N_1}{N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1}$$

The ration of the measured activities at equilibrium is

$$\frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1}$$

The daughter's activity is greater than that of the parent by the factor $\frac{\lambda_1}{\lambda_2 - \lambda_1}$. These results are illustrated in Fig.12.7.

When the parent has a shorter half-life than the daughter i.e., $\lambda_1 > \lambda_2$, no state of equilibrium can be obtained. When the parent and daughter are seperated initially, as the parent decays, the number of daughter atoms increases, pass through a maximum and eventually decay with the half-life of the daughter.

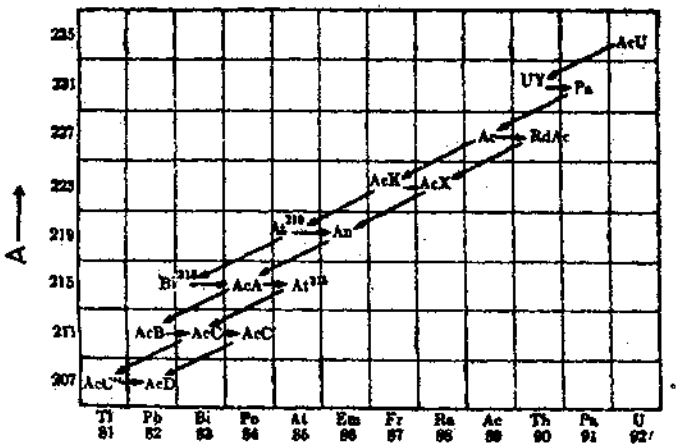


Fig.12.7 Transient equilibrium

a - Daughter Activity growing in freshly purified parent fraction. b - Activity of parent, ($T_{1/2}$ =8.0 hour). c - Total activity of initially pure parent fraction. d - Decay freshly isolated daughter fraction $T_{1/2}$ =0.8 hrs. e - Total daughter activity in - parent - plus - daughter fractions. A - Activity in Logscale.

12.9 PROPERTIES OF $\alpha-,\beta-$, AND $\gamma-$ RADIATIONS

Alpha particles are nuclei of helium atom. They are positively charged, which is twice the charge of the electron. The range of $\alpha-$ particles in solid materials is extremely small. In materials of density 1 g/cm^3, the range of $\alpha-$ particles would be 0.005 cm, $\alpha-$ particles cause strong ionization. They cause luminiscence and react on contact with photographic plates. $\alpha-$ particles are deflected by the electric and magnetic fields. Alpha rays cannot penetrate cloth and are not dangerous as external radiation. When the source of $\alpha-$ rays is within the organism, the results are disastrous. The dense ionization caused will lead to the total destruction of the surrounding tissue. The energy of $\alpha-$ particles emitted by a radioactive nuclide ranges from 4.5 MeV to 9.6 MeV.

Beta particles are electrons or positrons. Their speed will be almost equal to that of light. The energy of $\beta-$ particles ranges from zero to a high value of the order of 1 MeV. The ionizing power of $\beta-$ particles is less. Positive $\beta-$ decay is rare compared to negative $\beta-$ decay. Neutrinos are emitted during beta-decay. Beta-decay is caused by the transformation, within the nucleus, of a neutron to a proton or vice versa with the emission of $\beta-$ particle and neutrino. Beta-rays can ionize gases, cause luminiscence and make an impression on a photographic plate. Beta-rays are deflected by electric and magnetic fields. Beta-rays are harmful to the skin and dangerous for the eyes.

Gamma-rays are electromagnetic in nature. Gamma-rays are intranuclear in orgin. Gamma-rays are identical to X-rays except X-rays being extranuclear in origin. Compared to $\alpha-$ and $\beta-$ rays, $\gamma-$ rays have a much greater power of penetration. But they have very less ionizing power. The energy of gamma-rays ranges from a few KeV to MeV. Gamma-rays accompany the emission of $\alpha-$ and $\beta-$ rays.

12.10 AGE OF THE EARTH

Radioactivity studies of terrestrial and meteorite samples can be used to estimate the age of the earth. An examination of the radioactive chains of natural radioactive series shows that the decay of the parent body and its products is accompanied (a) by the evolution of helium from the emitted $\alpha-$ particles and (b) by the formation of lead of a particular atomic weight for each of the naturally occuring series. When an uranium, bearing Mineral, was formed, if there was no presence of helium or lead of atomic weight 206 and no alteration of decay rate or loss of products occured, the determination either total helium or total radiogenic lead in the mineral as a ratio to uranium contest can permit the estimation of age of the mineral. Hence if the decay of uranium is assumed to take palce at a constant rate the age of the mineral is given by

$$\lambda_U N_U t = N_{Pb}$$

Where N_U and N_{Pb} represent the present number of atoms of U238 Pb206 in the mineral, Substituting for λ_U

$$\text{Age of mineral} = \frac{\text{Weight of } Pb^{206}}{\text{Weight of } U^{238}} \times 7.5 \times 10^9 \text{ yr.}$$

Many estimates of the age of the geological strata have been made in this way of using refined radiochemical and mass spectrometric techniques and introducing corrections for the decay of the parent uranium activity. Results for pitchblends range from 640 to 1.38 x 10^9 years. This indicates that the age of earth must be greater than 1.38 x 10^9 years.

Radioactive data can also indicate the much longer time which has elapsed since the formation of the atoms as distinct from the consideration of the minerals on earth. A simple estimate is by comparing the present relative abundance of U^{238} and U^{235} (140/1) and their half-lives assuming that the two isotopes were created in comparable amounts. This estimate gives a passage of 6.6 x 10^9 yrs since formation. More recent methods of dating are based on the radio isotopes namely K^{40}, Rb^{87}, In^{115} etc. The age of the earth as estimated comes out between 4 to 4.5 x 10^9 years.

The noble gas technique for measuring time was first applied in 1951 by E.K. Gerling and his co-workers in USSR. Argon is the daughter of K^{40}. The half-life of K^{40} is 1.3 x 10^9 years. 89% of K^{40} disintegrates yielding Ar^{40}. Hence any mineral containing Potassium gradually accumulates Ar^{40} in its lattice. If the system is undisturbed the rate of Ar^{40} accumulation is definite and immutable, governed by the potassium content and by the rate of decay. By measuring K^{40} and Ar^{40} content in such an undisturbed sample, it is possible to estimate the date on which the sample crystallized. The pottasium - argon ages of the most common stone meteorites tend to have values in the range 4 to 4.5 billion years. Some of the ages fall below this range, but none are higher. These findings are consistent with the age of 4.6 x 10^9 yrs indicated by lead-to-uranium ratios.

An age of 4.6 billion years in now generally accepted for both meteorites and the crust of the earth. All evidence indicates that the stable, cold objects in the solar system took their present form at about 4.6 billion years ago.

Worked Example - 4

$_{92}U^{238}$ and $_{92}U^{235}$ occur in nature in an atomic ratio of 140:1. Assuming that at the time of the earth's formation they were present in equal amounts, estimate the age of the earth. The half-life of U^{238} is 4.5 x 10^9 years and that of U^{238} is 7.13 x 10^8 years.

Let No represents the number of atoms U^{238} and U^{235} initially when the earth was formed. If N^{238} and N^{235} represent the number of atoms of U^{238} and U^{235} to-day then

$$N_{238} = N_o e^{-\lambda_{238} t}$$

$$N_{235} = N_o e^{-\lambda_{235} t}$$

Where t represents the age of the earth

$$N_{238} / N_{235} = e^{-\lambda_{238} - \lambda_{235} t}$$

As per the data given in the problem

$$e^{-(\lambda_{238}-\lambda_{235})t} = 140$$

$$\therefore \quad t = \frac{\log_e 140}{\lambda_{235} - \lambda_{238}}$$

But

$$\lambda_{235} = \frac{0.693}{7.13\times10^{8} Yrs} = 0.972\times10^{-9} Yrs^{-1}$$

$$\lambda_{238} = \frac{0.693}{4.5\times10^{-9} Yrs} = 0.154\times10^{-9} Yrs^{-1}$$

$$t = \frac{2.303\log_{10}140}{\left(0.972\times10^{-9} - 0.154\times10^{-9}\right)} Yrs$$

$$= \frac{2.303(2.146)}{0.818}\times10^{9} Yrs$$

$$\therefore \quad t = 6.04\times10^{9} Yrs$$

12.11 SUMMARY

The spontaneous emission of radiation by heavy elements is called radioactivity. The radiation consists of α or β or γ radiation. γ radiation follows α or β emission.

α-particles are helium nuclei. β-particles are electrons. γ-particles are electromagnetic radiation.

Radioactive elements decay obey two laws namely exponential law and displacement law.

According to exponential law the rate of decay or growth of radioactive element is exponential in nature. The number of a radioactive element at anytime t is given by

$$N(t) = N_{(o)}e^{-\lambda t}$$

where $N_{(o)}$ represents the number of atoms at t = 0 and λ represents the disintegration constant.

According to displacement law if a radioactive element emits an α-particle, its

position in the periodictable will be displaced to the left by two units. If the radioactive element emits a β-particle, its position in the periodic table will be shifted to the right by one unit.

A radioactive element is characterised by its disintegration constant (λ), half-life (T) or mean life τ. These three are related as

$$T = 0.693/\lambda, \tau = 1/\lambda = T/0.693$$

The unit of radioacivity is Curie. It is defined as that quantity of radioactive material giving 3.7×10^{10} disintegration per sec.

All naturally occuring radioacivity elements can form three distinct radioactive series. They are called uranium series, Actinium series and thorium series. The mass number of each member of the series can be expressed as (4n+2) for uranium series, (4n+3) for actinium series and (4n) for thorium series. Here n represents an integer.

In successive radioactie disintegration processes two types of equilibria can be obtained depending upon the half-life of the parent and the daughter nuclides. If the parent is long-lived and the daughter is short-lived after several half-lives of the daughter we get the situation where $\lambda_1 N_1 = \lambda_2 N_2$. This type of equilibrium is called Secular equilibrium.

A different type of equilibrium called transient equilibrium results if the parent lives longer than the daughter i.e., $\lambda_1 < \lambda_2$ but the half-life of the parent is not very long. After several half-lives of the daughter, the daughter decays with the same half-life as the parent.

The age of the earth as determined by radioactive dating is 4.6×10^9 years.

12.12 MODEL ANSWERS

Check Your Progress

The loss of a β particle displaces an element one place to the right in the periodic table.

12.13 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Derive the expression for (1) half-life (2) mean-life and (3) disintegration constant of a radioactive substance.
2. Write about the radioactive equilibrium in detail.

II. Answer the following questions briefly.

1. State and explain the laws of radioactivity
2. Write a note on radioactive series.
3. Explain how the age of the earth can be determined by radioactive dating.
4. Compare the properties of α, β, γ radiations.

III. Solve the following problems

1. The decay constant of $_{11}Na^{24}$ is 1.3×10^{-5} s^{-1}. Determine its half-life. How many atoms would still be present after 14.8 hrs. of time from an initial activity of 2 mCi.
(Ans. $T_1/2 = 14.8$ hrs., $N(T_1/2) = 2.846 \times 10^{12}$)

2. The half-life of Po^{218} is 3.05 min. Its daughter atom Pb^{214} has a half life of 26.8 min. Determine the time at which teh activity of Pb^{214} reaches maximum starting from the pure sample of Po^{218}.
(Ans. 10.8 min.)

3. A piece of wood from an ancient boat found to show C^{14} activity to 5 counts/min. An equal amount of wood of recent date gave a count rate of 20 counts/min. If the half life of C^{14} is 5568 Yr. Calculate the age of the boat.
(Ans. 11146 yrs.)

UNIT–13 : TRANSMUTATION OF ELEMENTS

Contents

13.1 AIMS AND OBJECTIVES

In this unit transmutation reactions by various particles are discussed.

After going through this unit you will be able to explain the endoergic and exoergic reactions depending on the value of "Q".

13.2 INTRODUCTION

The discovery of radioactivity and spontaneous disintegration of heavy nuclei led nuclear physicists to think about the possibility of disintegration of light and intermediate nuclei by bombarding them with energetic particles. The easily available projectiles for this purpose are radiations coming from radioactive nuclides. Among alpha beta and gamma radiations, α-particles seemed to be most apt projectiles for the study of transmutation of nuclei. The experiments conducted by Rutherford, Chadwick, Cock Craft, Walton and that others, at the beginning of this century, established clearly that stable nuclides may be transmuted into new nuclides when bombarded by energetic particles. In this unit, we shall study the experiments conducted by Rutherford which established the occurrence of transmutation reactions and the experiments of Chadwick leading to the discovery of neutron and also different types of transmutation reactions.

13.3 TRANSMUTATION OF ELEMENTS - RUTHERFORD'S EXPERIMENTS

When an atom is bombarded by a highly energetic particle, the particle may enter deep inside the nucleus and cause disruption of the nucleus.

α-particles from natural radioactive nuclides are chosen as projectiles and light nuclei are chosen as targets. The artificial transmutation was first demonstrated by Rutherford in 1919. He demonstrated experimentally that the nuclei of nitrogen emit protons when bombarded by alpha particles from RaC. The experimental set up employed by Rutherford is illustrated in Fig. 13.1.

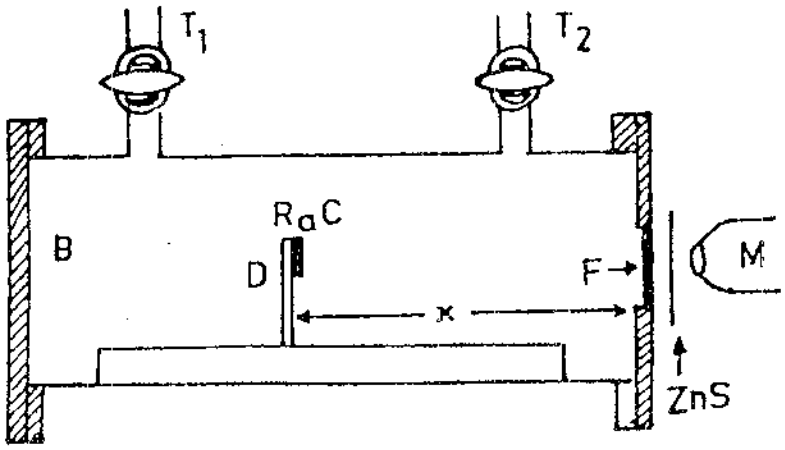


Fig. 13.1 Experimental set up of Rutherford's apparatus to study disintegration of nitrogen by α -particles. F - Agfoil, M - Microscope, ZnS - Zinc sulphide screen..

The apparatus consists of a box B, one end of which has, an opening covered by silver foil F. A zinc Sulphide screen was placed at S just outside the opening. The source of α -particles, RaC, is placed on a small disc D whose distance from the ZnS screen could be varied. The silver foil F was thick enough to absoprb the α -particles coming from the source. Different gases could be introduced into the box and also exhausted using the values T_1 and T_2. The scintillation, if any, caused on the zinc sulphide screen were observed using the microscope M.

When the box was filled with oxygen or carbon-dioxide at atomospheric pressure, no scintillations were observed on the screen when the source was about 7cm way from the screen. This air thickness is sufficient enough to absorb all α -particles. When nitrogen gas was introduced into the box B, scintillations were observed on the screen even when the source was as much as 40 cm away from the screen. Rutherford concluded that the scintillations were caused by particles ejected from nitrogen nuclei due to bombardment by α -particles. By magnetic diflection experiment Rutherford showed that these particles were protons.

By careful experimental study, Rutherford ruled out the possibility that the protons responsible for scintillations on the screen came from hydrogen present in the nitrogen gas as an imputity. He concluded that protons resulted as a consequence of artificial disintegration of nitrogen atoms.

Rutherford and Chadwick extended the investigations to all element from boron to potassium except carbon and oxygen. In some cases, certain ejected protons were found to be more energetic than that the of the bombarding alpha-particle. This result provided additional evidence that the protons were emitted as a result of disintegration process, the extra energy being acquired due to nuclear rearrangement.

Two different hypotheses were suggested to explain the disintegration process leading to the emission of protons. According to one hypothesis, the nucleus of the bombarded atom simply loses a proton as a result of collision with swift alpha-particle. As per this proposition the α -particle still should exist after the collision process. According to the second hyphothesis, the alpha-particle is captured by the atom it hits, leading to the formation of a new nucleus called compound nucleus which emits a proton. As per this propostion

α-particle should disappear after the collision process.

The dilemma was settled in 1925 when Blackett studied the tracks produced by α-particles passing through nitrogen in a cloud chamber. The tracks which could be seen are that of (1) incident alpha-particle, (2) a proton and (3) a recoil nucleus. The absence of the track corresponding to an alpha-particle after collision proved that alpha-particle disappeared completely. This result established the validity of the second hypothesis. The disintegration process can be written as

$$_7N^{14} +_2 He^4 \rightarrow \left[_9F^{18}\right] \rightarrow_8 O^{17} +_1 H^1$$

The transmulation process can also be represented by

$$_7N^{14}(\alpha, p)_8O^{17}$$

The transmulation of other nuclides by α-particles from radioactive substances may be given as

$$_5B^{10} +_2 He^4 \rightarrow \left[_7N^{14}\right] \rightarrow_6 C^{13} +_1 H^1$$

$$_{13}Al^{27} +_2 He^4 \rightarrow \left[_{15}P^{31}\right] \rightarrow_{14} Si^{30} +_1 H^1$$

$$_{19}K^{39} +_2 He^4 \rightarrow \left[_{21}Se^{43}\right] \rightarrow_{20} Ca^{42} +_1 H^1$$

In general the (α, p) reaction can be written as

$$_zX^A +_2 He^4 \rightarrow \left[_{Z+2}C_n^{A+4}\right] \rightarrow_{Z+1} Y^{A+3} +_1 H^1 \qquad 13.1$$

Where X, C_n and Y represent the target, compound and product nuclei respectively.

13.4 ANALYSIS OF NUCLEAR TRANSMUTATION REACTIONS

The nuclear transmutation reaction, represented by Eq. 13.1 can be analyssed quantitatively in terms of the masses and energies of the nuclei and particles involved. Let the nuclear reaction be represented by

$$x + X \rightarrow Y + y \qquad 13.2$$

where x, X, Y and y represent the bombarding particle, target nucleus, product nucleus and product particle respectively. Let us assume that the target nucleus is at rest initially, that is, its kinetic energy is zero, Since the total energy of a particle or atom is the sum of

the rest energy and kinetic energy, the statement that the total energy must be conserved in a nuclear reaction yields to the distintegration process given by Eq. 13.2 as

$$\left(E_x + m_x C^2\right) + M_x C^2 = \left(E_Y + M_Y C^2\right) + \left(E_y + m_y C^2\right) \qquad 13.3$$

In the above equation m_x, M_x, m_y, and M_Y represented the masses of the incident particle, target nucles, product particle and product nucleus respectively, E_x, E_Y and E_y represent the kinetic energies of incident particle, product nucleus and product particle respectively, C represents the velocity of light.

Let us define a quantity Q which represents the difference between the kinetic energies of the products of the reaction and that of the incident particle.

$$\therefore \quad Q = E_Y + E_y - E_x$$

From Eq. 13.3

$$E_Y + E_y - E_x = \left(M_X + m_x - M_Y - m_y\right)C^2$$

$$\therefore \quad Q = \left(M_X + m_x - M_Y - m_y\right)C^2 = E_Y + E_y - E_x \qquad 13.4$$

The quantity Q is called the energy balance of the reaction. If Q is positive the kinetic energy of the products is greater than the kinetic energy of the reactants and the reaction is called exothermic or exoergic. The total mass of the reactants is greater than the total mass of the products. If Q is negative, the reaction is said to be endoergic or endothermic Nuclear reaction yields information about nuclear masses, particle energies or about Q-values; depending on the type of information available and quatities that can be measured experimentally.

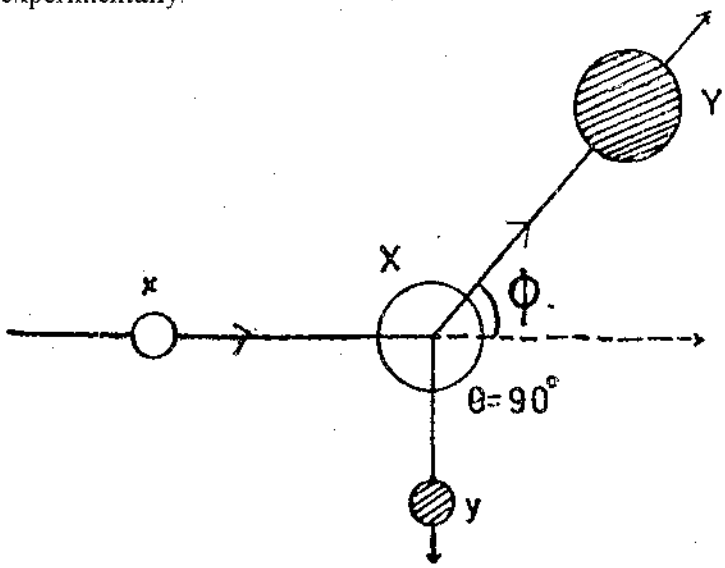


Fig. 13.2 Collision between incident particle and target nucleus leading to nuclear reaction y and Y represent the product particle and product nucleus.

In Eq. 13.4 E_Y represents the recoil energy of the product nucleus which is usually small and difficult to measure. It can be eliminated from Eq. 13.4 by applying the principle of conservation of momentum to the reaction process. For simplicity, consider a special case in which the product particle is observed at angle of 90° with respect to the direction of the incident particle, taken as X-axis as shown in Fig. 13.2.

Before collision the momentum vector is directed along the X-axis: Hence the resultant momentum after collision must also be directed along X-axis. Hence

$$m_x V_x = M_Y V_Y Cos\phi \qquad 13.5$$

$$m_y V_y = M_Y V_Y Sin\phi \qquad 13.6$$

where V_x, V_y, and V_Y represent the velocities of the incident particle, the product particle and product nucleus respectively ϕ represent the angle between the X-axis and the direction of recoil of the product nucleus.

Squaring Eqns. 13.5 and 13.6 and adding we get

$$m_x^2 V_x^2 + m_y^2 V_y^2 = M_Y^2 V_Y^2 \left(Cos^2\phi + Sin^2\phi\right) = M_Y^2 V_Y^2 \qquad 13.7$$

Since $E_x = (1/2) m_x V_x^2 E_y = (1/2) m_y V_y^2$ and $E_Y = (1/2) m_Y V_Y^2$

Eq. 13.7 can be rewritten as

$$m_x 2E_x + m_y 2E_y = M_Y 2E_Y$$

or $$E_Y = \frac{m_x}{M_Y} E_x + \frac{m_y}{M_Y} E_y \qquad 13.8$$

Inserting Eq. 13.8 in 13.4, we get

$$Q = E_y\left(1 + \frac{m_y}{M_Y}\right) - E_x\left(1 - \frac{m_x}{M_Y}\right)$$

The Q value of a reaction can be determined by measuring the energies of incident and product particle and determining the mass number of the product nucleus. If the outgoing particle is ejected at any angle θ instead of 90° then

$$Q = E_y\left(1 + \frac{m_y}{M_Y}\right) - E_x\left(1 - \frac{m_x}{M_Y}\right) - \frac{2}{M_Y}\left(E_x E_y m_x m_y\right)^{\frac{1}{2}} Cos\theta \qquad 13.9$$

The value of the last term in Eq. 13.9 decreases as the mass of the target nucleus increases.

Check Your Progress

If the kinetic energy of the products is greater than the Kinetic energy of the reactents the reaction is called _____ reaction.

Worked Example - 1

In the transmutation reaction $_5B^{11}$ (d, α) $_4Be^9$ the energy of deutron beam is 1.51 Mev. The α-particles come of at an angle of 90° with the direction of deutron beam and have an energy of 6.4 MeV. Determine the Q-value of the reaction.

The transmutation reaction

$$X + x \rightarrow Y + y$$

$$_5B^{11} +_1 H^2 \rightarrow_4 Be^9 +_2 He^4$$

The Q-value of the reaction is given by

$$Q = E_y\left(1 + \frac{m_y}{M_Y}\right) - E_x\left(1 - \frac{m_x}{M_Y}\right)$$

$$Q = 6.4(1 + 4/9) - 1.51(1 - 2/9)$$

$$Q = 6.4(1.444) - 1.51(0.7778)$$

$$Q = 9.24 - 1.17 = 8.07\,MeV.$$

In an endoergic reaction, the energy - Q is necessary to excite the compound nucleus sufficiently so that it will break up. This energy must be supplied in the from of kinetic energy of the projectile. But not all of the kinetic energy is available for excitation because some of the energy is utilized in imparting momentum to the compound nucleus. This momentum is later distributed among the products of the reaction. Hence to see that - Q is available for excitation of the compound nucleus we have to supply some more energy in addition to -Q. The amount of energy required for an endoergic reaction to take place is called threshold energy of the reaction.

In M_C and V_C represent the mass and velocity of compound necleus the principle of conservation of momentum requires that

$$m_x V_x = M_C V_C$$

$$\therefore \quad V_C = \frac{m_x}{M_C} V_x$$

The part of the kinetic energy of the incident particle needed for excitation of the compound nucleus is

$$-Q = \frac{1}{2} m_x V_x^2 - \frac{1}{2} M_C V^2 C$$

$$-Q = \frac{1}{2} m_x V_x^2 - \frac{1}{2} M_C \frac{m^2 x}{M^2 x} V_x^2$$

$$-Q = \frac{1}{2} m_x V^2 x - \frac{1}{2} M_C \frac{m^2 x V^2 x}{M_C}$$

$$-Q = \frac{1}{2} m_x V^2 x \left[1 - \frac{m_x}{M_C}\right]$$

Since $M_C - m_x + M_X$

$$-Q = \frac{1}{2} m_x V^2 x \left[1 - \frac{m_x}{M_y + m_x}\right]$$

$$-Q = \frac{1}{2} m_x V^2 x \left[\frac{M_X}{M_X + m_x}\right]$$

The threshold energy $E_{th} = \frac{1}{2} m_x V_x^2$ is given by

$$E_{th} = -Q\left[1 + \frac{m_x}{M_X}\right] \qquad 13.10$$

In Eqns. 13.4 and 13.10, the masses involved are nuclear masses. In actual calculation these masses are replaced by atomic masses. This will not alter the results since the number of electrons added to the reactants to form neutral atoms are equal to the number

of electrons added to the products of the reaction to form neutral atoms. The general equation representing a nuclear reaction can be given by

$$X + x \rightarrow [C_n] \rightarrow Y + y + Q$$

Worked Example - 2

Determine the Q-value of the reaction

$${}_{11}Na^{23} + {}_{2}He^{4} \rightarrow [{}_{13}A^{27}] \rightarrow 12^{Mg^{26}} + 1^{H1}$$

given the atomic masses Na^{23} = 22.997047 a.m.u. He^{4} = 4.0038727 a.m.u., Mg^{26} = 25.990796 a.m.u., H^{1} = 1.0081473 a.m.u. As per Eq. 13.4 the Q-value of the reaction is given by

$$Q = \left(M_X + m_x - M_Y - m_y\right)C^2 = \text{(change in mass in a.m.u.)}$$

$$Q = \left[\left(22.997047 + 4.0038727\right) - \left(25.990796 + 1.0081473\right)\right]$$

$$Q = 0.0019764 \text{ a.m.u.}$$

In terms of energy 1 a.m.u. = 931.48 MeV.

$$Q = (0.0019764)(931.48) = 1.841 MeV$$

13.5 CHADWICK'S DISCOVERY OF NEUTRON

A logical sequence of experiments following the discovery of artificial disintegration by Rutherford and the poineering work of Chadwick led to the discovery of neutron.

In the experiments conducted by Rutherford and Chadwick, the transmutation of light elements by alpha-particles yielded protons as product particles. Rutherford was of the view that in all cases protons alone need not be ejected and deep insight into this aspect is necessary. He was having the idea that an electron may closely combine with a proton leading to a neutral particle and initiated experiments to detect such a particle in a hydrogen discharge. He was of strong opinion that such a neutral particle might exist independently and need to be discovered. The sequence of transmutation experiments that led to the discovery of neutron are discussed below.

During 1930, Bothe and Becker observed the production of what appeared to be electromagnetic radiation in the bombardment of elements such as Li, Be, B, Mg and Al with α-particles. They also observed that the radiation was highly intense in the case of Be target. It was suggested that the radiation might be due to deexcitation of the excited states of the residual nucleus produced in (α, p) reactions. Further experimental work by Boethe, by Webster and by Mme Curie-Joliot indicated that the radiation emitted, with

Be as target for the α-particle projectiles, possessed a penetrating power considerably greater than any of the known gamma radiation. Mme Curie-Joliot and M.Joliot found that the radiation coming from the transmutation reaction

$$ {}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{13} + \gamma \qquad 13.11 $$

was able to eject high speed protons from hydrogenous material such as paraffin. These protons were found to have energies about 5 MeV and a range about 26 cm in air. Assuming that the protons are emitted by elastic collision with gamma-radiation calculations show that the photons must have energies about 50 MeV. The energy attainable by gamma-rays can be calculated by evaluating the Q-value of the reaction represented by Eq.13.11. The Q-value is given by

$$ Q = \{[M_X + m_x - M_Y]a.m.u.\}931.48 MeV $$

Using the values M_X = 9.01504a.m.u., m_x = 4.00388a.m.u. and M_Y = 13.00748s.m.u. we get Q = 10.5 MeV. If the kinetic energy of α-particles is assumed to be 5 MeV the total energy available for the transmutation reaction is 15.5 MeV. This energy is utilized in imparting recoil energy of the product nucleus and the energy of the product particle. Hence, the maximum energy that the gamma-rays may possess would be 15.5 MeV which is much less than 50 MeV. Chadwick solved the riddle of this penetrating radiation by conducting a series of direct and compelling experiments at Cavendish laboratory.

Chandwick conducted experiments on ($Be + \alpha$) reaction. Chadwick found that the penetrating radiation ejects ionizing particles not only from hydrogen but from all other light elements. The essential features of Chadwicks apparatus is shown in Fig. 13.3.

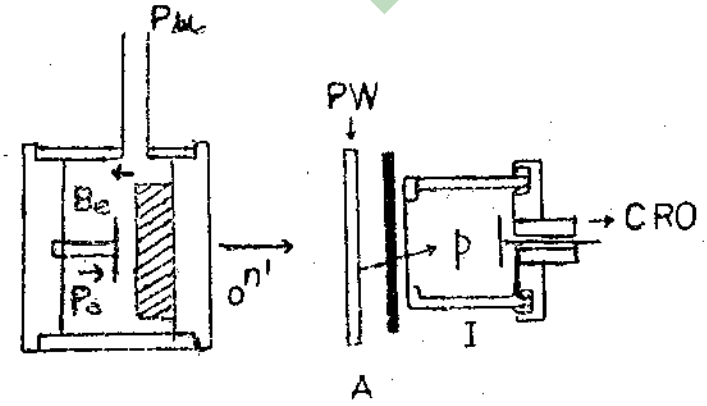


Fig. 13.3 Chadwick's apparatus for identifying neutrons
Po - Polonium source; Be - Beryllium absorber $_0n^1$ neutron beam; PW - paraffin wax; A - Aluminium absorber; I - Ionization chamber; P - Proton beam.

The source of α-particles, Po^{212} and the target material Be^9 are kept inside an evacuated chamber as shown in the figure. The energy of α-particles is 5.3 MeV. These rays strike the target. The nuclear tranmutation reaction gives out penetrating radiation in all possible directions. Some of this radiation travels toward the ionization chamber. The ionization pulse from the ionization chamber is amplified electronically and given to an oscilloscope. The height of each pulse would then be a measure of the ionization produced by single recoil atom in the ionization chamber.

When there was no absorbing material kept between the source vessel and the ionization chamber, pulses were observed which were attributed to recoil nitrogen atoms produced in the gas with which the ionization chamber was filled. The counting rate remained unchanged when 2 cm thick lead sheet was interposed between the source vessel and ionization chamber. This result indicated the highly penetrating power of the radiation . When 2 mm thick sheet of paraffin is introduced in place of lead sheet, a substantial increase in counting rate was observed. The pulse heights were identified as characteristic of protons in the ionization chamber. The range of these recoil protons wad determined using aluminium foils between the paraffin absorber and the detector. The results yielded a range in aluminium equivalent to 40 cm of air at 15°C and 760 mm pressure. Using the relationship between range and velocity the protons were estimated to have a velocity of 3.3×10^7 ms^{-1} and hence an energy of about 5.7 MeV. As detailed earlier, if the radiation coming from Be target were to be gamma radiation then to yeild protons of 5.7 MeV energy, gamma-rays should have an energy of around 55 MeV. Chadwick studied energy of protons emitted by replacing paraffin by Li, Be, B, C and N and also by filling the ionization chamber successively by H_2, He, N_2 O_2 and A. If the energy of the recoil nuclei was assumed to be produced by elastic collisions with photons, it turned out that the energy that had to be attributed to the photon increased with the mass of the recoil atom. This requirement is contrary to the principles of conservation of energy and momentum in elastic collisions. Chadwick showed that the observed energy of recoil niclei can be explained by assuming that the penetrating radiation coming from Be source is a stream of particles having mass equal to that of protons and possessing no charge. The nuclear reaction proposed accordingly is

$$ {}_4Be^9 + {}_2He^4 \rightarrow \left[{}_6C^{13} \right] \rightarrow {}_6C^{12} + {}_0n^1 $$

Here ${}_0n^1$ represents the neutron. Chadwick also calculated the mass of neutron and found it to be equal to that of proton.

Since neutron is a neutral particle its mass can not be determined by deflection experiments under the influence of electric and magnetic fields. Its mass could be determined only by indirect experiments. Chadwick calculated the mass of neutron by estimating the velocities of recoil nucleus in head on collison with neutron. The velocity imparted to a nucleus is maximum when a moving neutron makes a headon collison with it, when it is at rest, makes it to move along the direction of neutron. Accordingly consider a particle of mass m_1 moving with a velocity v colliding with a stationary particle of mass m_2. After collision let v_2 be the velocity of m_2 and v_1 be the velocity of m_1. Applying the principle of conservation of energy to the interaction process, we have

$$ \frac{1}{2}m_1V^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \qquad 13.12 $$

Applying the conservation of linear momentum, we have

$$ m_1V = m_1V_1 + m_2V_2 \qquad 13.13 $$

Eliminating V_1 from Eqs. 13.12 and 13.13, we get

$$V_2 = \left\{\frac{2m_1}{m_1 + m_2}\right\}v \qquad 13.14$$

If neutron having a velocity v collides with another particle of mass m_3 and imparts to it a velocity v_3 then

$$V_3 = \left\{\frac{2m_1}{m_1 + m_3}\right\}v \qquad 13.15$$

Dividing Eq. 13.14 by Eq. 13.15, we get

$$\frac{V_2}{V_3} = \frac{m_1 + m_3}{m_1 + m_2}$$

knowing m_2, m_3, v_2 and v_3; m1 can be estimated. The maximum velocity imparted to hydrogen nuclei ejected from paraffin when hit by neutrons coming from Be target after bombardment by α -particles from polonium source was found to be 3.3 x 10^7 ms^{-1}. The recoil of nitrogen nuclei after being struck by the same neutrons were observed in cloud chamber experiments and their naximum velocity was found to be 4.7 x $10^6 ms^{-1}$. Taking the masses of proton and nitrogen to be 1 and 14 a.m.u. respectively,

$$\frac{m_1 + 14}{m_1 + 1} = \frac{3.3 \times 10^7}{4.7 \times 10^8}$$

we get m_1 = 1.15 a.m.u. The uncertainty involved in the estimation m_1 is about 10% due to errors involved in the determination of velocities of the recoil nuclei.

Chadwick obtained a better value of the neutron mass by considering the reaction

$$_5B^{11} +_2 He^4 \rightarrow \left[_7N^{15}\right] \rightarrow_7 N^{14} +_o n^1$$

The Q-value of the above reaction was found from the kinetic energies of the incident α -particles and nitrogen nuclei measured in cloud chamber experiments and of the neutron estimated from the maximum recoil energy of a proton after being struck by a neutron and using the expression the Q-value and knowing the atomic masses of B^{11}, He^4 and N^{14} the mass of neutron had been estimated using the Eq.13.4 In this way Chadwick found a value between 1.005 and 1.008 a.m.u. as the mass of the neutron. The best methods now available for the estimation of the mass of neutron give a value of 1.008983 a.m.u.

The other types of nuclear reactions in which neutron is produced are

$$_3Li^7 +_2 He^4 \rightarrow \left[_5B^{14}\right] \rightarrow_5 B^{10} +_o n^1$$

$$_7N^{14} +_2 He^4 \rightarrow \left[_9F^{18}\right] \rightarrow_9 F^{17} +_o n^1$$

$$_{11}Na^{23} +_2 He^4 \rightarrow \left[_{13}Al^{27}\right] \rightarrow_{13} Al^{26} +_o n^1$$

$$_{13}Al^{27} +_2 He^4 \rightarrow \left[_{15}P^{31}\right] \rightarrow_{15} P^{30} +_o n^1$$

$$_{18}A^{40} +_2 He^4 \rightarrow \left[_{20}Ca^{44}\right] \rightarrow_{20} Ca^{43} +_o n^1$$

13.6 ARTIFICIAL TRANSUMATION BY DIFFERENT PROJECTILES

After the discovery of artificial transumation by α-particles emitted from natural radioactive sources, experiments were conducted to study the transmutation of elements by neutrons coming from (α, n) reactions. Artificial transmutation was found to be possible with neutron. Further, it was thought that other particles like, protons, deuterons and γ-rays might induce disintegration of nuclei. This was made possible with development of particle accelarators. Cockcroft and Walton were the first perople to achieve artifical transmutation of Li^7 by protons which have been accelerated using voltage multiplier. The development of other accelerators like cyclotron, linear accelerators, synchrocyclotron, proton synchotron etc. made possible to obtain the particles with very high energies and many transmutation reactions with projectiles as protons, deuterons and γ-rays are discussed below.

13.6.1 Transumutation by protons

(a) (p,α) reactions.

Cockcroft and Walton bombarded lithium with protons accelerated to 0.1 to 0.7 MeV. Scintilations caused by particles ejected from lithium were observed on a zinc sulphide screen. These particles were proved to be α-particles by photographing their tracks in a cloud chamber. The reaction is given by

$$_3Li^7 +_1 H^1 \rightarrow \left[_4Be^8\right] \rightarrow_2 He^4 +_2 He^4$$

This reaction is very important since it gave a quantitative proof of the validity of Einstein's mass energy relation. The value of Q obtained using the relation

$$Q_M = \left[M\left(Li^7\right) +_m \left(1H^1\right) - 2M\left(_2He^4\right)\right]C^2$$

is given by

$$Q_M = \left[7.01822 + 1.008144 - 2(4.003873)\right]931.48$$

- 17.34 MeV.

The value of Q obtained from the energies of incident protons and the emergent alpha-particles determined experimentally is 17.33 MeV. This agreement indicates the validity of Einstein's mass energy relation where energy is released at the expense of mass.

The general type of p, α reaction is given by

$$_ZX^A + _1H^1 \rightarrow \left[_{Z+1}Cn^{A+1}\right] \rightarrow _{Z-1}Y^{A-3} + _2He^4$$

Other examples are

$$_3Li^6 + _1H^1 \rightarrow \left[_4Be^7\right] \rightarrow _2He^3 + _2He^4$$

$$_4Be^9 + _1H^1 \rightarrow \left[_5Be^{10}\right] \rightarrow _3Li^6 + _2He^4$$

$$_{13}Al^{27} + _1H^1 \rightarrow \left[_{14}Si^{28}\right] \rightarrow _{12}Mg^{24} + _2He^4$$

An interesting reaction occurs when the targ[illegible]

$$_5B^{11} + _1H^1 \rightarrow \left[_6C^{12}\right] \rightarrow _4Be^{\cdot} + _2He^4$$

$_4Be^8$ is highly unstable and breaks up into two more alpha-particles. Hence.

$$_5B^{11} + _1H^1 \rightarrow \left[_6C^{12}\right] \rightarrow _2He^4 + _2He^4 + _2He^4$$

We get three alpha-particles. This reaction is an example for multiple particle production reaction.

(b) (p,n) reactions

Proton bomabardment may also lead to the ejection of neutrons. The general reaction can be given by

$$_ZX^A + _1H^1 \rightarrow \left[_{Z+1}Cn^{A+1}\right] \rightarrow _{Z+1}Y^A + _on^1$$

Examples are

$$_5B^{11} +_1 H^1 \rightarrow \left[_6C^{12}\right] \rightarrow_6 C^{11} +_o n^1$$

$$_8O^{18} +_1 H^1 \rightarrow \left[_9F^{19}\right] \rightarrow_9 F^{18} +_o n^1$$

$$_{28}Ni^{58} +_1 H^1 \rightarrow \left[_{29}Cu^{59}\right] \rightarrow_{29} Cu^{58} +_o n^1$$

In (p,n) reactions the mass change is usually negative and hence the reactions are usually endothermic.

(c) (p,γ) reactions

In certain reactions, the proton is simply captured by the target nucleus. The compound nucleus formed will be highly unstable and becomes less unstable with the emission of gamma-rays rather than emission of α-particle or neutron. The general reaction is given by

$$_ZX^A +_1 H^1 \rightarrow \left[_{Z+1}Cn^{A+1}\right] \rightarrow_{Z+1} Y^{A+1} + \gamma$$

Examples of the above type of reactions are

$$_3Li^7 +_1 H^1 \rightarrow \left[_4Be^8\right] \rightarrow_4 Be^8 + \gamma$$

$$_6C^{12} +_1 H^1 \rightarrow \left[_7N^{13}\right] \rightarrow_7 N^{13} + \gamma$$

$$_{13}Al^{27} +_1 H^1 \rightarrow \left[_{14}Si^{28}\right] \rightarrow_{14} Si^{28} + \gamma$$

The gamma-ray photons emitted in these reactions are often very energetic and in turn may be used to transmute elements. The bobardment of lithium by protons yields photons with energy 17.2 MeV far more energetic than the most energetic photons of 2.6 MeV available from natural radioactive elements.

(d) (p-d) reactions

When nuclei are bombarded by protons some times deuterons are produced. The general reaction is given by

$$_ZX^A +_1 H^1 \rightarrow \left[_{Z+1}Cn^{A+1}\right] \rightarrow_Z Y^{A-1} +_1 H^2$$

Some examples of (p,d) reactions are

$$_4Be^9 +_1 H^1 \rightarrow \left[_5B^{10}\right] \rightarrow_4 Be^8 +_1 H^2$$

$$_3Li^7 +_1 H^1 \rightarrow \left[_4Be^8\right] \rightarrow_3 Li^6 +_1 H^2$$

13.6.2 Transmutation by deuterons

Many nucleur reactions were observed with high energy deutons as projecticles. The deuterons could be accelerated to several MeV energy in a cyclotron or in an electrostatic generator. The general reaction is given by

a) (d-α) reactions

$$_ZX^A +_1 H^2 \rightarrow \left[_{Z+1}Cn^{A+2}\right] \rightarrow_{Z-1} Y^{A-2} +_2 He^4$$

Deuteron-induced reaction was first observed in lithium atom. The reaction is given by

$$_3Li^6 +_1 H^2 \rightarrow \left[_4Be^8\right] \rightarrow_2 He^4 +_2 He^4$$

Other possible reactions are

$$_8O^{16} +_1 H^2 \rightarrow \left[_9F^{18}\right] \rightarrow_7 N^{14} +_2 He^4$$

$$_{13}Al^{27} +_1 H^2 \rightarrow \left[_{14}Si^{29}\right] \rightarrow_{12} Mg^{25} +_2 He^4$$

In the above type of reactions, the mass defect is usually positive leading to positive Q-values. Hence these reactions are exoergic.

(b) (d,p) reactions

In deuteron-induced nuclear reactions, sometimes protons are produced.

The general reaction is given by

$$_ZX^A +_1 H^2 \rightarrow \left[_{Z+1}Cn^{A+2}\right] \rightarrow_Z Y^{A+1} +_1 H^1$$

$$_6C^{12} +_1 H^2 \rightarrow \left[_7N^{14}\right] \rightarrow_6 C^{13} +_1 H^1$$

$$_{11}Na^{23} +_1 H^2 \rightarrow \left[_{12}Mg^{25}\right] \rightarrow_{11} Na^{24} +_1 H^1$$

$$_{15}P^{31} +_1 H^2 \rightarrow \left[_{16}S^{33}\right] \rightarrow_{15} P^{32} +_1 H^1$$

These reactions are exoergic.

(c) (d,n) reactions

The general reaction is given by

$$_Z X^A +_1 H^2 \rightarrow \left[_{Z+1}Cn^{A+2}\right] \rightarrow_{Z+1} Y^{A+1} +_o n^1$$

Neutrons are often produced as a result of deuteron bombardment X Reactions of this type are

$$_6C^{12} +_1 H^2 \rightarrow \left[_7N^{14}\right] \rightarrow_7 N^{13} +_o n^1$$

$$_3Li^7 +_1 H^2 \rightarrow \left[_4Be^9\right] \rightarrow_4 Be^8 +_o n^1$$

$$_4Be^9 +_1 H^2 \rightarrow \left[_5B^{11}\right] \rightarrow_5 B^{10} +_o n^1$$

If the target is deuteron, both (d,p) and (d,n) reactions are possible.

$$_1H^2 +_1 H^2 \rightarrow \left[_2He^4\right] \rightarrow_1 H^3 +_1 H^1$$

$$_1H^2 +_1 H^2 \rightarrow \left[_2He^4\right] \rightarrow_2 He^3 +_o n^1$$

$_2He^3$ is stable, $_1H^3$ called tritium is unstable and has a half-life of 12 years.

13.6.3 Transmutation by neutrons

Since neutrons do not have charge they can easily penetrate the nucleus unlike protons, deuterons and alpha-particles. Both high energetic and low energetic neutrons are capable of transmutation of nucleui. Before the development of nuclear reactors, the neutron sources are nuclear reactions like $_1H^2(d,n)_2He^3$, $_4Be^9(d,n)_5B^{10}$, $_4Be^9(\alpha,n)_6C^{12}$. The fast neutrons are slowed down by passing them through the hydrogenous material like water or paraffin. Some of the neutron induced reactions are discussed below.

(a) (n, α) reactions

The general reaction is represented by

reactions are

$$_1H^2 + \gamma \rightarrow \left[_1H^2\right] \rightarrow _1H^1 + _on^1$$

$$_4Be^9 + \gamma \rightarrow \left[_4Be^9\right] \rightarrow _4Be^8 + _on^1$$

In the reaction given by $\left[_1H^2(\gamma,n)_1H^1\right]$ the binding energy of deuteron is 2.2 MeV. In the reaction given by $\left[_4Be^9(\gamma,n)_4Be^8\right]$ the disintegration is possible because one neutron is loosely bound. The Q-value of the reaction is -1.67 MeV.

13.7 SUMMARY

Artificial transmutation was first demonstrated by Rutherford. When α-particles emanating from RaC hit nitrogen nuclei, a compound nucleus $_9F^{18}$ is formed which immediately disintegrates with the ejection of a product nucleus $_8O^{17}$

The Q-value of a nuclear reaction is given by the difference between the kinetic energies of the products the reaction and that of the incident particle. It is also given by

$$Q = \left[M_X + m_x - M_Y - m_y\right]C^2 a.m.u.$$

To express it in MeV. it should be multiplied with 931.48 where M_X, m_x, M_Y and m_y represent the masses of target nucleus, incident particle, product nucleus and product particle respectively.

If the Q-value of a nuclear reaction is positive, the reaction is said to be exoergic. If the Q-value is negative, the reaction is said to be endoergic.

For an endoergic reaction to take palce, energy greater than-Q is to be supplied. This energy required for the reaction to take palce is called the threshold energy, E_{th}.E_{th} can be calculated using the relation.

$$E_{th} = -\left(1 + \frac{m_x}{M_X}\right)Q$$

where m_x and M_X represent the masses of incident particle and target nucleus respectively.

When $_4Be^9$ was bombarded by α-particles, a highly penetrating radiation was observed. This radiation was first thought of as gamma radiation. Mme Curie Joliot found that this radiation when passed through paraffin gave protons. Chadwick analysed this radiation using different absorbers in place of paraffin and confirmed the penetrating radiation coming from $_4Be^9$ was neutrons. He estimated the mass of neutron to lie between

1.005 to 1.008 a.m.u.

Other elements like ${}_5B^{11}, {}_3Li^7, {}_7N^{14}, {}_{11}Na^{23}, {}_{18}A^{40}$ were found to yield neutrons when bombarded by α-particles.

With the advent of particle accelarators, transmutation reactions were observed with protons and deuterons. Gamma-rays were also observed to cause disintegration of nuclei called photodisintegration reactions. The various transmuation reactions observed are (p, α), (p, n), (p, γ), (p, d); (d, α), (d, p), (d, n), (n, α), (n, p), (n, 2n), (n, γ) and (γ, n). In these brackets, the first particle represents the incident particle and the second particle represents the product particle.

13.8 MODEL ANSWERS

Check Your Progress

If the kinetic energy of the products is greater than the kinetic energy of reactents the reaction is called exothermic reaction.

13.9 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Discuss the experimental studies carried out by Rutherford which established the transmutation of elements.
2. Discuss the various experimental investigation that led to the discovery of neutron.
3. Derive an expression for the Q-value of a nuclear reaction.
4. What are the different types of projectiles that may be used in transmutation reactions? Discuss by giving examples the different types of transmutation reactions.

III. Solve the following problems

1. The transmutation reaction ${}_5B^{11}(d,p)_5B^{12}$ is caused by deutrons of energy 1.5 Mev. The protons come out at an angle of 90° with the direction of deutron beam. If the Q-value of the reaction is 1.5 MeV determine the energy of protons.
(Ans. 2.54 MeV)
2. The transmutation reaction ${}_4Be^9(p,n)_5Be^9$, ${}_6C^{13}(p,n)_7N^{13}$ and ${}_8O^{18}(p,n)_9F^{18}$ were found to have threshold energies of 2.059 MeV, 3.236 MeV and 2.590 MeV respectively. Evaluate the Q-values of the above reactions.
(Ans. Q_1 = -1.853 MeV, Q_2 = -3.003 MeV and Q_3 = -2.453 MeV.)

$$_{Z}X^{A} +_{o} n^{1} \rightarrow \left[_{Z}Cn^{A+1}\right] \rightarrow_{Z-2} Y^{A-3} +_{2} He^{4}$$

some of the interesting examples of (n, α) reactions are

$$_{3}Li^{6} +_{o} n^{1} \rightarrow \left[_{3}Li^{7}\right] \rightarrow_{1} H^{3} +_{2} He^{4}$$

$$_{5}B^{10} +_{o} n^{1} \rightarrow \left[_{5}B^{11}\right] \rightarrow_{3} Li^{7} +_{2} He^{4}$$

$$_{13}Al^{27} +_{o} n^{1} \rightarrow \left[_{13}Al^{28}\right] \rightarrow_{11} Na^{24} +_{2} He^{4}$$

Reactions $_{3}Li^{6}$ (n, α) $_{1}H^{3}$ and $_{5}B^{10}$(n, α) $_{3}Li^{7}$ are used to detect the presence of neutrons.

(b) (n,p) reactions

The compound nucleus formed by the capture of neutrons emits a proton. The general equation is represented by

$$_{Z}X^{A} +_{o} n^{1} \rightarrow \left[_{Z}Cn^{A+1}\right] \rightarrow_{Z-1} Y^{A} +_{1} H^{1}$$

A few examples of the above reaction are

$$_{7}N^{14} +_{o} n^{1} \rightarrow \left[_{7}N^{15}\right] \rightarrow_{6} C^{14} +_{1} H^{1}$$

$$_{13}Al^{27} +_{o} n^{1} \rightarrow \left[_{13}Al^{28}\right] \rightarrow_{12} Mg^{27} +_{1} H^{1}$$

$$_{30}Zn^{64} +_{o} n^{1} \rightarrow \left[_{30}Zn^{65}\right] \rightarrow_{29} Cu^{64} +_{1} H^{1}$$

(c) (n,2n) reactions

Sometimes neutron-iduced reactions yield two neutrons. The general reaction is given by

$$_{Z}X^{A} +_{o} n^{1} \rightarrow \left[_{Z}Cn^{A+1}\right] \rightarrow_{Z} X^{A-1} +_{o} n^{1} +_{o} n^{1}$$

An example of the above reaction are

$$_{13}Al^{27} +_{o} n^{1} \rightarrow \left[_{13}Al^{28}\right] \rightarrow_{13} Al^{26} +_{o} n^{1} +_{o} n^{1}$$

Fast neutrons are required for effecting this reaction. It is an endoergic reaction.

(d) (n, γ) reactions

The capture of neutron by the target nucleus yields emission of photons. This type of reaction is called radioactive capture reactions. The general reaction is given by

$$_{Z}X^{A} +_{o} n^{1} \rightarrow \left[_{Z}Cn^{A+1}\right] \rightarrow_{Z} Y^{A+1} + \gamma$$

Radioactive capture reactions are possible with almost all elements. The Q-value is positive. The excess energy is carried away by the photons. Some radioactive capture reactions are given below.

$$_{1}H^{1} +_{o} n^{1} \rightarrow \left[_{1}H^{2}\right] \rightarrow_{1} H^{2} + \gamma$$

$$_{1}H^{2} +_{o} n^{1} \rightarrow \left[_{1}H^{3}\right] \rightarrow_{1} H^{3} + \gamma$$

$$_{13}Al^{27} +_{o} n^{1} \rightarrow \left[_{13}Al^{28}\right] \rightarrow_{13} Al^{28} + \gamma$$

$$_{92}U^{238} +_{o} n^{1} \rightarrow \left[_{92}U^{239}\right] \rightarrow_{92} U^{239} + \gamma$$

Radioactive capture reactions sometimes yield a product nucleus which is radioactive. This type of reaction is an important source of artificial radioactive nuclides.

13.6.4 Transmutation by Photons

Atomic nuclei can be disintegrated using high energetic photons. This process is called photodisintegration. It is the kinetic energy associated with the photon that is responsible for the reaction. This kinetic energy must be greater than the binding energy of the particle which is emitted in the reaction. Hence photodisintegration reactions are endoergic.

The general reaction is given by

$$_{Z}X^{A} + \gamma \rightarrow \left[_{Z}Cn^{A}\right] \rightarrow_{Z} Y^{A-1} +_{o} n^{1}$$

The threshold energies are usually of the order of 10 MeV. In the reaction

$$_{15}P^{31} + \gamma \rightarrow \left[_{15}P^{31}\right] \rightarrow_{15} P^{30} +_{o} n^{1}$$

In the above reaction photons of energy about 17 MeV are used. Photo disintegration does not occur with gamma-rays from radioactive nuclides except in $_{1}H^{2}$ and $_{4}Be^{9}$. These

UNIT–14 : RADIATION DETECTORS

Contents

14.1 AIMS AND OBJECTIVES

In this unit the design working and application of radiation detectors are discussed.

14.2 INTRODUCTION

In the study of nuclear physics we come across various particles which may be categorised into charged particles, uncharged particles and electromagnetic radiation. α-radiation, β-radiation, proton electron and mesons are taken as charged particles, Neutrons, light particles in Cosmic radiation are taken as uncharged particles. X-rays & γ-rays are taken as electromagnetic radiation. The Properties such as magnitude of charges, mass, energy, momentum, velocity and likewise of these particles are very important in the study of particle interactions. The advent of nuclear physics and cosmic ray research is mainly due to the development of particle detectors. Particle detectors have been constructed, based on the nature of interaction and effect of interaction of these particles with matter in the three forms, namely, solid, liquid and gas. These detectors can be used to detect the presence of the particles or to determine any of the parameters which characterise the particle under consideration.

Particle detector may be broadly classified as (1) electrical pulse detectors and (2) particletrack detectors. In electrical pulse detectors the nuclear particle when it enters the sensitive volume of the detector gives rise to an electrical pulse which can be detected using suitable electronic circuit. Gas-filled detectors such as ionization chambers, proportional counters and Geiger-Muller counters, scintillation counters, Cerenkov counters and Semi-conductor particle detectors, come under the category of electrical pulse detectors. In the particle-track detectors, the track of the particle is recorded. To this category belong the bubble chambers, spark chambers, cloud chambers and photographic emulsion techniques. These instruments are highly useful in the study of nuclear reactions, particle interactions and in cosmic ray research. These instruments we get a permanent record of the nuclear event which can be analysed to get particle, pulse.

In this Unit we shall study, in detail, the construction, working and application of GM detectors, cloud chamber, bubble chamber and scintillation detector.

14.3 GEIGER MULLER COUNTER

Gas-filled detectors namely ionization chambers, proportional counters and Geiger Muller counters are based on the collection of ions, produced in the sensitive volume of the

detector due to the passage of ionizing radiation. Before we go to study, in detail, the working and constructional features of Geiger Muller counter, let us understand the machanism of ionization caused by the ionizing particle in the sensitive volume of the detector and the effect of applied voltage on the output pulse.

When a charged particle passes through the sensitive volume of the ionization chamber, a large number of ion pairs will be created in its path. For typical field conditions the electrons drift at velocities of the order of 10^4 m s^{-1} and ions at 10. The nature of migration of these ion pairs, the recombination and secondary processes caused by these primary electrons and ions depend in a complex way upon (1) the physical design of the chamber (2) the applied voltage, (3) the gaseous filling and (4) the auxiliary circuits. To understand this, let us consider a cylindrical ionization chamber containing a simple gas such as nitrogen or hydrogen or argon as shown in Fig. 14.1. Let us also assume the (a) the incident charged particle creates

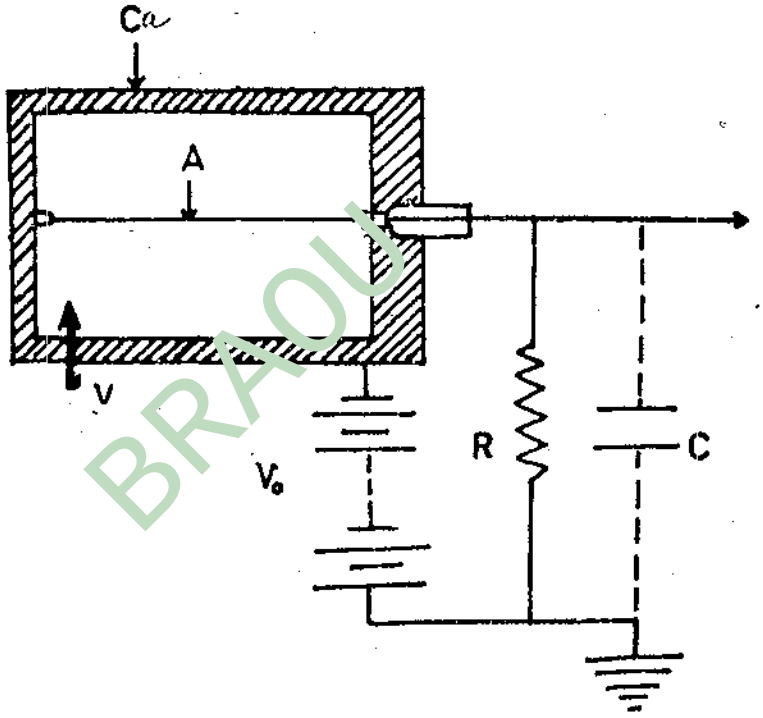


Fig. 14.1 Cylindrical ionization chamber.
V - Sensitive volume of the chamber. A - Anode. Ca - Cathode.

'n' ion pairs in the chamber, (b) the time constant RC, where C is the capacitance of the Chamber, is greater than the migration time of electrons and positive ions and (c) the voltage V_o is sufficient enough that no appreciable recombination takes palce and also no appreciable secondary ionization occurs. Under these conditions, the magnitude of the output voltage, $[V_a(t)]$ will look like the one shown in Fig.14.2. The sharp rise of $[V_a(t)]$ is due to rapid migration of electrons to the anode. The slow continuous rise is due to the migration of positive ions to the outer cathode.

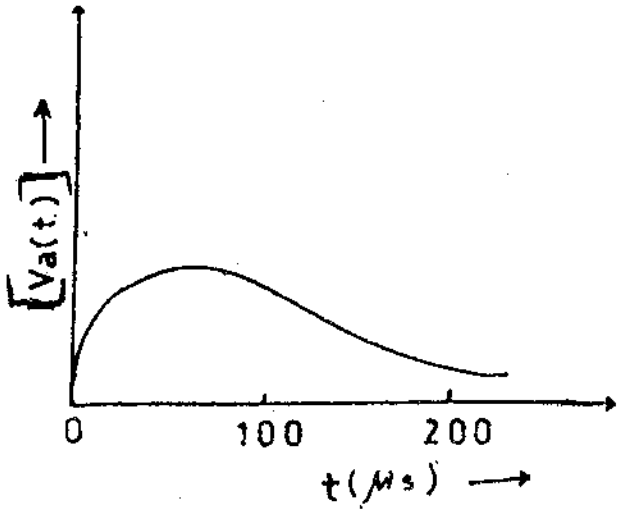


Fig.14.2. Voltage output versus time for a cylindrical chamber

When RC is much greater than the migration time of positive ions, the peak voltage is approximately equal to ne/C. The decay of $[V_a(t)]$ is due to the discharge of the chamber through the external resistor R.

To understand the more general characteristics of the ionization process, let us consider that three incident charged particles passing through the chamber produce instantly 'n'ion pairs, '2n'ion pairs and '3n'ion pairs respectively. Then the variation of pulse height with applied voltage V_o will look like the one shown in Fig. 14.3.

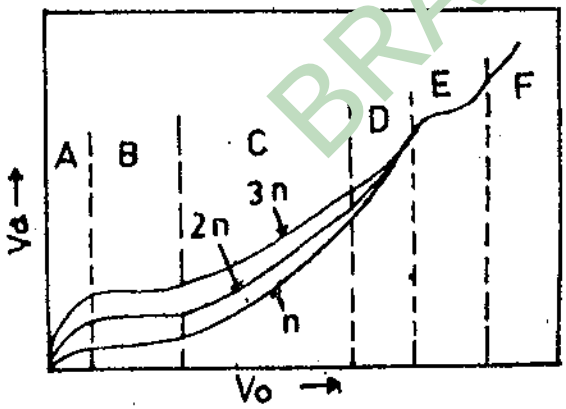


Fig.14.3 Variation of pulse height with applied voltage for three initial conditions.

the curve of pulse height versus voltage may be divided into several characteristic regions as indicated in Fig.14.3.

Region A : This is a very low voltage region. The charge collected varies markedly with the applied voltage and is less than ne, 2ne and 3ne respectively. In this region significant recombination of electrons and positive ions takes place before the ions are collected.

Region B : The field strength is of the order of 20 to 50 V cm^{-1}. All ions initially formed are collected and neither recombination nor secondary ionization is appreciable in this region. This region of operation is used in ionization chambers in steady operation to measure the flux of the parties.

Region C : This region is called proportional region. Here more ions are collected than those initially formed. The additional ions are formed due to gas multiplication which occurs due to intense electric fields existing in the near vicinity of the anode and get accelarated. This accellarated electron ionizes the gas molecules. Even the secondary electrons produce further ionization. Thus, gas multiplication takes place. If V_0 is very high, an avalanche occurs in the immediate neighbour-hood of the anode such that each primary electron finally reaches the anode accompained by a large number of additional electrons. In this region the pulse height is proportional to the initial number of ion pairs. Proportional countes are operated in this region of ionization.

At the transition of region C and in the region D, proportionality between the pulse height and the initial number of ion pairs is lost.

Region E : In this region, the pulse height is very large and is independent of initial number of ion pairs. This region is called Geiger Muller region of operation.

Region F : This region is associated with a continuous glow or arc discharge which is indifferent to the presence of incident charged particles.

Geiger-Muller counter is designed utilizing the property of Geiger Muller region in which the charge collected is independent of initial ionization. In the Geiger region the avalanche associated with the drift of a single electron to one point on the anode initiates avalanches, over the entire anode length. Photons emitted during the course of one avalanche, act to spread the discharge over the entire tube. The rapid collection of electrons by the anode in a time of the order of $1\ \mu s$, momentarily leaves a sheath of positive ions surrounding the anode which tends to cancel the field and finally limit the avalanching of electrons to the anode. This positive ion sheath then spreads out radially and reaches the cathode in a time of the order of $100\ \mu s$. The rest of the discharge process depends upon the gaseous filling and the external circuit. The discharge in the Geiger region associated with an initial particle can be regenerated if a gas ion upon striking the cathode releases a new electron which is then accelarated to the anode initiating a second avalanche. This type of regenerative process will continue unless the applied voltage is reduced in some way below the Geiger threshold. A number of methods have been used to quench the self-perpetuating avalanches in GM counters. External circuits were used with older GM counting systems. These circuits were designed to reduce the voltage across the detector below the value required to maintain the discharge. The voltage is reduced only mometarily, but this is sufficient to terminate the discharge of the counter. External quenching results in long resolving times and for this reason 'Self-quenching' GM Counters are commonly used.

Self-quenching GM counters contain a quenching gas which is added as an impurity to the major gas filling the tube usually argon. In self quenched GM counters, as positive ions move towards the cathode there is transfer of charge to the molecules of the quenching gas. When these charged molecules reach the cathode they dislodge electrons from the chamber wall, but this process results in the dissociation of the quenching gas molecules rather than production of additional electrons.

Two types of self-quenchign tubes are available, those quenched by the addition of a halogen and those which are orginally quenched. Typical organic quenching gases are polyatomic gases including ethyl alcohol, ethyl formate and amyl acetate. For organic quenching gases the dissociation is irreversible and hence the life of an organically quenched

GM counter is limited. The useful life time is typically 10^8 to 10^{10} counts. Bromine and chlorine gases are commonly used in halogen quenched GM counters. In this type of counters the quench gas molecules recombine after a finite period and hence have unlimited useful life time. But all gas filled counters have a practical limitation on their life-time set by the leakage of counter seals.

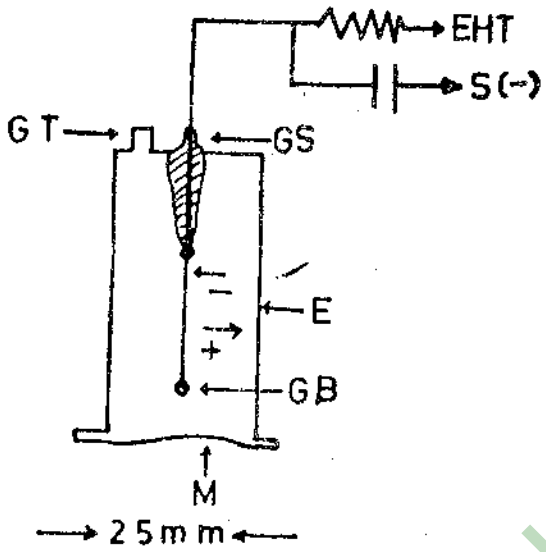


Fig. 14.4 Geiger Mulle Counter
S - Signal; E - Earth; GS - glass metal seal; GB - glass bead; GT - gas filling tube; M - Mica window;

Geiger Muller counters are typically cylindrical in shape, as illustrated in Fig. 14.4, and filled with a mixture of argon and quenching gas (<1%). Argonis filled at a pressure of 100 mm of Hg and quenching gas, say, ethyl alcohol vapour is filled at a pressure of 10 mm/Hg. The collecting electrode is usually a tungsten wire of 0.001 inch diameter. The outer electrode is thin stiffened steel or aluminium shell coated with graphite to ensure uniform field distribution. For alpha or beta detection, counters are designed with thin mica windows.

The main features of a GM counter are

1. Constant output pulse size independent of initial ionization.
2. sensivity for the production of a single ion pair and
3. a fairy long insensitive time following the entry of each particle as shown in Fig. 14.5

This time is made up of a dead time during which the counter voltage drops below counting threshold and a recovery time during which pulses of reduced size are produced. The insensitive time is usually made definite by suitable design of the external electrical circuit. It is then called the paralysis time. The paralysis time may be usually a few hundred micro seconds.

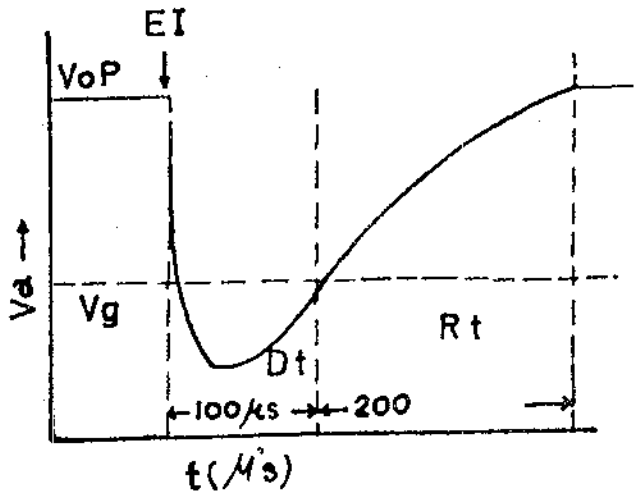


Fig. 14.5 Voltage on wire immediately following entry of ionizing particle versus time. V_α - Voltage on wire. EI - Entry of ionizing particle Dt - Dead time. Rt - Recovery time.

The operating characteristics of a GM counter exposed to a source of ionizing radiation as is clear from Fig. 14.6 are

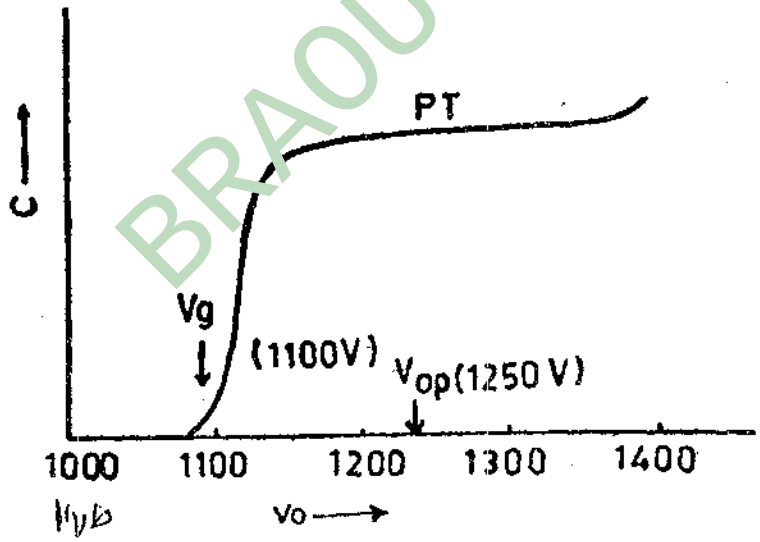


Fig. 14.6 Counting rate (c) versus anode voltage (V_o) for a constant source of radiation. PT - Plateau

1. a threhold voltage Vg
2. a plateau of small slope, over which the counting rate increases slightly with the operating voltage.
3. a background counting rate due to contamination of the materials, to cosmic radiation and to spurious discharges.

GM counters are extensively used in the detection of β and γ rays.

Worked example 1

A beta particle of energy 1.5 MeV enters a thin window detector tube filled with methane and argon initiating an avalanche of 5 x 10^8 ion pairs. Assuming that the energy

loss suffered by the beta particle in passing through the tube window is negligible and the average energy loss per ion pair creation in the tube gas is 30 ev, calculate the multiplication factor M.

The number of ion pairs created in the saturation region of the tube by 1.5 MeV electron is given by

$$n = \frac{1.5 \times 10^6 eV}{30 eV} = 5 \times 10^4 \text{ ion pairs}$$

The multiplication factor

$$M = \frac{\text{Number of ion pairs in the avalanche}}{n}$$

$$M = \frac{5x10^8}{5 \times 10^4} = 10^4$$

Worked Example - 2

5000 alpha particles each of energy 6 MeV enter an ionization chamber in one second. If the energy required for ion pair creation in the chamber is 35 eV. Calculate the ionization current.

Energy of each alpha particle = 6 x 10^6 eV
Energy required for ion pair creation = 35 eV

Number of ion pairs created $n = \frac{6 \times 10^6}{35} = 1.71 \times 10^5$ ion pairs.

The ionization charge per alpha particle

q = 1.71 x 10^5 (ion pair) x 1.60 x 10^{-19} c/ion pair

q = 2.736 x 10^{-14} C.

Since 5000 alpha particles enter the chamber per second, the ionization current IqN=5000 x 2.736 x 10^{-14} C/s

= 1.368 x 10^{-10} A.

14.4 CLOUD CHAMBER

Cloud chamber is the first successful particle track detector invented by C.T.R. Wilson in 1911. The basic principle involved in the condensation of super-saturated vapour into droplets around free ions. There are two types of cloud chambers called expansion

cloud chamber and diffusion cloud chamber signifying the way how super-saturation of the vapour is achieved in the sensitive volume of the chamber. In expansion cloud chamber, rapid expansion of a volume of the gas containing the vapour leads to super saturation. In diffusion cloud chamber, the diffusion of vapour from a warm region where it is not super saturated to a cold region brings in super-saturation of the vapour near the cold region.

The essential features of an expansion cloud chamber are illustrated in Fig.14.7. Initially the sensitive volume of the chamber will be filled with a gas saturated with vapour. The usual gas-vapour mixture is air with water or argon with ethyl alcohol at atmospheric pressure. The gas-vapour mixture is made to expand by quick motion of the piston. The expansion is adiabatic and lowers the temperature. The cooling is more and more the air becomes super-saturated with water vapour. If an ionizing particle enters the chamber, the ions formed along the track of the particle act as nuclei for the condensation of the vapour and the path of the particle appears as a thin track of fog. The track can be illuminated by light from the side and photographed through the window. The ions formed can be removed by means of an electric field applied between the piston P and the metal ring R. The piston can be brought back to its original position and the chamber is ready for the next event to be recorded.

Liquid droplets may form in a super-saturated vapour on nuclei present in the gas. The nuclei include ions produced by the ionizing particle.

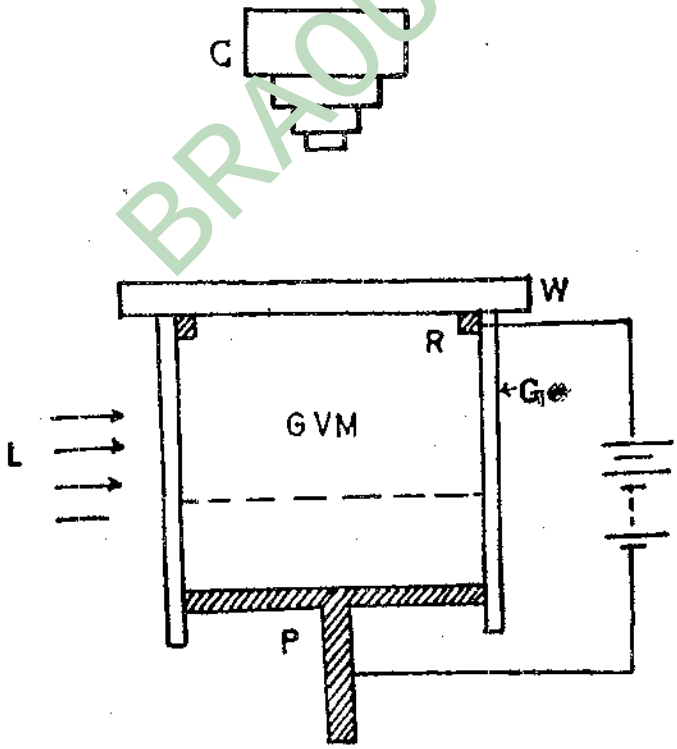


Fig. 14.7 Expansion cloud chamber
W - glass window; R - metalring; C - Camera; G - glass Cylinder; L - powerful flash light; P - piston; GVW - gas vapors mixtures.

passing through the chamber and others formed in the chamber due to foreign suspended particles such as dust, chemical compounds etc. Liquid droplets may also form spontaneously on microscopic fluctuations in density of the vapour if the super-saturation is sufficiently high. This process determines the upper limit of super-saturation desirable. Of all these

condensation nuclei the only ones required exist in the chamber at the time of production and for subsequent photography are those ions produced by the ionizing particle. The unwanted ions are removed by an electronstatic clearing field. Dust particles etc are removed by producing super-saturation successively until the nuclei are carried to the bottom of the chamber where they adhere to the wall.

The value of super-saturation necessary for drop formation on ions, depends on the nature of the vapour and the sign of the ion. For water vapour the degree of super-saturation required is small for negative ions and high for positive ions. If the vapour is ethyl alcohol vapour the degree of super-saturation should be high for negative ions and low for positive ions.

The rate of growth of drops from a super-saturated vapour determines the length of the time required for a drop to reach visible size. Rapid growth makes possible short photographic delay times thereby minimizing distorsion effects due to motion of the gas. If the droplet is large it falls quickly to the bottom of the chamber.

In the expansion cloud chamber, super-saturation is achieved by a rapid, nearly adiabatic expansion of the gas and vapour. If T_1 represents the temperature before expansion and T_2 after expansion then.

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad \text{(Volume defined)} \qquad 14.1$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{(Pressure defined)} \qquad 14.2$$

where V_1, V_2, and P_1, P_2 represent volume and pressure of the gas and vapour before and after expansion respectively. From Eq. 14.1 and 14.2 we see that the change in temperature is dependent on γ the ratio of specific heats ($\gamma = \frac{C_P}{C_V}$) Hence monoatomic gasses are used in cloud chambers becuase of large γ. Most of the expansion cloud chambers are volume defined. But in cloud chambers containing metal plates, thin rubber diaphragm separates pressure vessels and hence are pressure defined.

Condensation centres produced by ionization when the particle passes through the chamber probably have a radius of the order of 3×10^{-10} m in the first instance. If sufficient degree of supersaturation is existing at the time of ionization the drops grow to a diameter of 3×10^{-5} m about 0.5s and during this time of growth the track may be photographed with a suitably phased flash of fight. The interval after production of super-saturation during which ionization leads to track formation is known as the sensitive time. The sensitive time is primarily determined by the warming up of the chamber from temperature T_2 with subsequent reduction of supersaturation.

Many modifications of the original cloud chamber have been made to study in detail the properties of ionizing particles and nuclear reactions. To obtain a large number of photographs automatic arrangement have been made so that the expansion can be repeated

rapidly and photographs taken continuously on motion picture film. By taking two stereoscopic pictures simultaneously the path of the particle in space can be constructed.

Cloud chambers have been designed for use with magnetic field. The size of the chamber is suitably reduced so as to keep it in between the pole pieces of an electromagnet. With this type of arrangement it is possible to distinguish between positive and negative charged particles and to determine the monentum of the particle.

To study highly energetic particles, since the stopping power of gas is low, it is desirable to use plates of heaviour metals in the chamber leaving gas spaces in between, in which the tracks may be seen and photographed. Cloud chambers have been designed incorporating the plates inside the chamber. The minimum distance between the plates is generally kept at 3/4 of an inch and faces of the plates are coated with reflecting materials to increase the light, scatted by droplets. This type of chambers are used to study nuclear reactions, and shortlived and unstable particles produced in these reactions.

Counter controlled cloud chambers have also been developed. Here a counter which senses the arrival of a particle triggers of the positia, pistion that expands the gas and vapour inside the chamber. This makes the vapour super-satured and fog droplets rapidly grow along the ionization trail left by the passage of the particle. The droplets form clear tracks which are photographed stereoscopically for analysis. Blacket and Occhialini developed this technique in 1930s which led to many detailed studies of the cosmic radiation and to the discovery of strange particles by Rochester and Dutter.

The expansion cloud chamber has the limitation of a short sensitive time which has been eliminated in the diffusion cloud chamber. It operates by the diffusion of a condensable vapour from a warm region where it is not saturated, to a cold region where it becomes supersaturated. The vapour is methyl alcohol or ethyl alcohol. Air is satisfactory for the gas at pressures from 20 cm Hg to 4 atomospheres and hydrogen from 10 to 20 atomospheres. A schematic diagram of the diffusion cloud chamber is shown in Fig.14.8. The vapour is introduced at the top of the chamber which is the warm end.

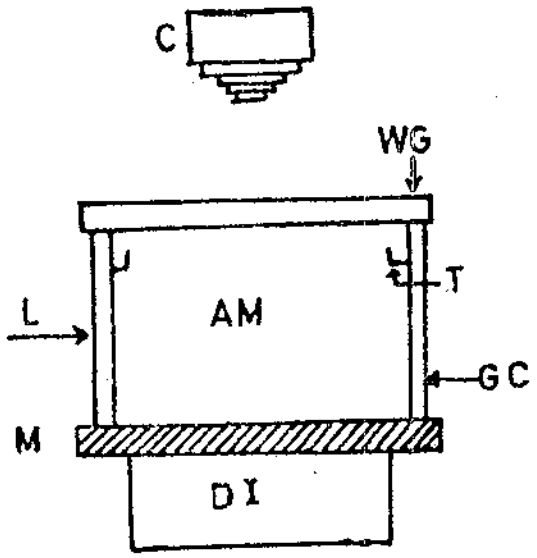


Fig.14.8 Diffusion cloud chamber
AMV - Air + methyl alcohol vapour; L - Light; C - Camera; WG - Warm glass plate; T - Trough filled with alcohol; GC - Glass cylinder; DI - Dryice; M - Metal plate.

It diffuses downward continously through the region in which a steady vertical temperature gradient is maintained by cooling the bottom of the chamber with dry ice. The diffusion cloud chamber is continuously sensitive.

A low pressure chamber constructed by Choyke and Nielsen, uses helium as the vapour at pressure ranges of 75 cm of Hg to 15 cm of Hg. The bottom of the chamber is cooled with liquid air. The temperature of the top of the chamber is maintained at - 20°C. This type of chamber is useful for observation of low energy electrons.

Many historic discoveries have been made by cloud chamber photography namely identification of neutron, discovery of positron, confirmation of the production of positron electron pairs by gamma rays and nuclear disintegration.

14.5 BUBBLE CHAMBER

In the year 1953 Donald A. Glaser working at the University of Michigan developed bubble chamber for which he received Nobel Prize in 1960. Glaser conceived the idea that when a fast moving charged particle passes through a super-heated liquid, it triggers the formation of microscopic bubbles and starts the boiling process. The bubbles grow along the tract left by the charged particle to a size at which they may be photographed. Fig.14.9 illustrates the construction and operation of bubble chamber.

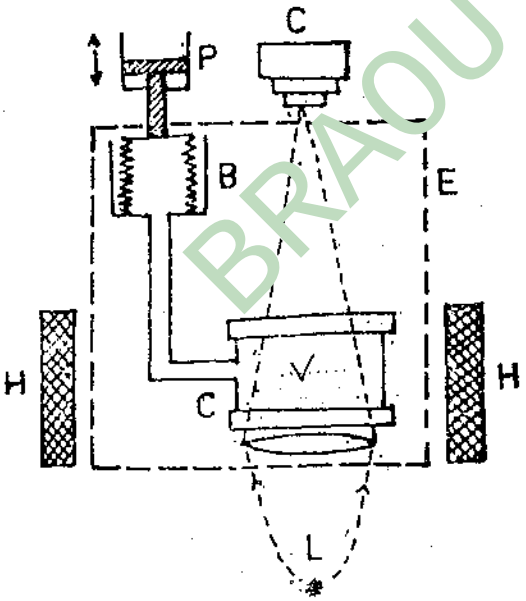


Fig.14.9 Bubble chamber
V - sensitive volume of bubble chamber. P - Expansion Piston. H - Magnetic coils. L - light source. E - Enclouser for temperature control camera. B - Bellows.

Here the sensitive volume of the chamber contains a suitable liquid heated above its normal boiling point and maintained in liquid phase by the application of a pressure above the saturation vapour pressure at the operating temperature. The pressure is at the operating temperature. The pressure is then reduced by about 10 percent by the movement of the piston and the liquid becomes super-heated and sensitive to the presence of ionizing particle. After the track forms and the photography is taken, the pressure is increased to its initial value at which the bubbles collapse and the chamber is ready for another expansion. The sensitive time of the bubble chamber after expansion is only a few milliseconds and entry of particles and photography must take place during the time.

The liquid that should be used in a bubble chamber should satisfy the following conditions.

1. The liquid should be non-conducting so that the ions retain their charge.
2. The liquid should have iow surface tension so that the force tending to collapse a bubble is weaker.
3. The liquid should have high vapour pressure which would tend to enlarge the bubble formed in the liquid.

Liquid hydrogen and diethylether were the two liquids used in the early stages. Deuterium, helium, propane, pentane, freon and xenon may also be used. Liquid hydrogen chambers are extensively used because it presents a target for the incoming particle, electrons and protons and the data can be most readily interpreted.

A chamber described by Glasser and Rahn contained isopentane as the liquid. It becomes sensitive to ionizing particles 3.5 ms after the expansion was initiated and remained sensitive for about 10ms. In liquid hydrogen bubble chamber the bubbles grow much more slowly and the sensitive time is of the order of 50 ms.

Bubble chamber tracks are generaly photographed under the influence of a magnetic filed to determine the sign of the charge and momentum of the ionizing particle. In liquid hydrogen bubble chambers, super-conducting magnets are used. The successful analysis of bubble chamber information is critically dependent on track quality and momentum definition. The measurement of bubble chamber tracks in now a highly specialised activity based on automatic machines under computer control. The operating conditions of bubble chamber fluids are detailed on Table 14.1

TABLE 14.1 Operating conditions for bubble chamber fluids.

Fluid	Temp oC	Pressure (atmospheres)	Density (Kg m-3)
Hydrogen	-246	5	60
Deuterium	-241	7	130
Helium	-269	1	130
Propane	58	21	430
Pentane	157	23	500
Xenon	-20	26	2300

While bubble chamber has the decided advantages of short sensitive time and the posibility to study particle interactions, there is one serious disadvantage that it can not be triggered like counter controled cloud chambers. Since there is no way to select rare events, one has to take the photograph of the chamber at every expansion cycle, develop the films and examine hundreds or thousands of exposures looking for events of interest. Hence bubble chambers are used with large accelators where a timing sequence first expands the chamber and then sends a burst of particles to be analyzed.

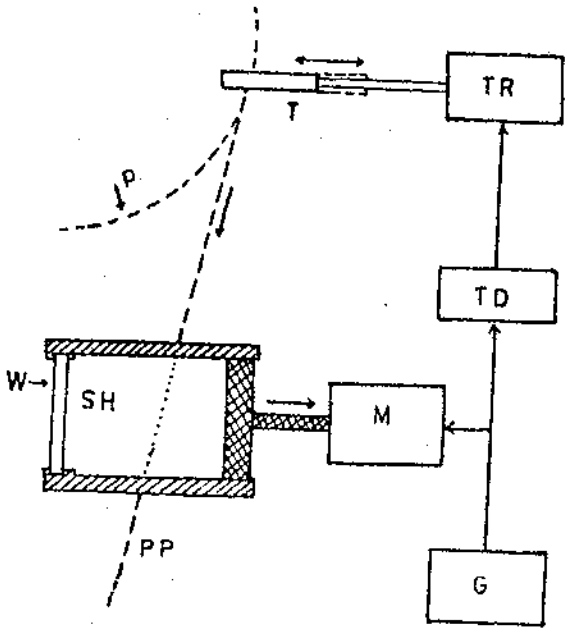


Fig.14.10 Bubble chamber operated with particle accelarator. A timing mechanism moves a target into the beam of circulating Protons in an accelarator, there by inducting particles into the chamber at this instant it is most sensitive to bubble growth
P - Circulating proton beam in an accelerator, G - Time controlled ascillations generation, T - Target, M - Mechanical Driving System, TR - Target RAM, TD - Time delay. W - glass window, PP - particle track, SH - Super Heated Liquid

This is illustrated is Fig.14.10. The chamber recovers back for next cycle in about 1s time.

14.6 SCINTILLATION DETECTOR

Scintillation detectors are generally used to detect any elementary particle either by direct or indirect process. The incident particle induces the detector to emit a light pulse which serves as the signal for a count.

Scintillation detection was one of the earliest methods employed to detect nuclear radiations. Rutherford and his collaborators observed the emission of alpha particle using spinthariscope. The spinthariscope consisted of a zinc sulphide screen, which could flouresce or scintillalte when struck by alpha-particle. The number of light flashes that are caused on a limited area can be visually observed or counted using a microscope. Modern scintillation counting involves the use of a photomultiplier tube where light pulse is converted into an electrical pulse.

Scintillation counting basically depends on the nature of interaction of the incident particle with the scintillator also called as phosphor. On absorbing energy from the incident particle the phosphor undergoes citation to higher electronic state. The excited electronic states undergo deexcitation to the ground state either immediatelly leading to flouroscence or after some time leading to phosphorscence The wavelength of the radiation emitted depends on the difference in energy of the excited and ground states of the system. If the material is transparent to light of this wavelength, this light may be observed outside the phosphor. For most of the materials the emitting transistion also corresponds to an absorption transition so that the material is opaque to the electromagnetic radiation. To eliminate this, impurities are introduced into the scintillators to which the exciton can be transferred. The

impurity gives rise to trapping levels from which transfer to the ground state can occur at a wavelength significantly different from the absorption wavelength so that light is emitted in a region for which the material is transparent. These impurities are called activators.

Sometimes impurities are added to the phosphor to serve as wavelength shifters. Especially organic scintillators flouresce readily which excit but the wavelength of the light emitted falls in the near or far ultraviolet region. Detection of this light is relatively inefficient. By introducting small concentrations of impurities into the scintillator, one can get enough concentration of trapping centres leading to high proportion of fluorescence emission from these lower energy levels. The net effect is the emission of light in the blue or green regions which can be detected by photomultiplier tube.

Scintillators are available in several forms, namely. crystals, liquids, plastics and gases. Among the materials found suitable in crystal form are inorganic and organic substances. The inorganic crystals are NaI (Thallium activated), CsI, (Thallum activated), Cadmium tungstate, LiI (europium activated) and KI (Thillium activated). Among organic crystals anthracene transstilbene, phenanthrene and napthalene are used as scintillators 5gm per litre of terphenyl in toluene is used as liquid scintillator. Terphenyl in polysterene and terphenyl in polyvinyl toluene serve as plastic scintillators.

Noble gases such as helium, argon, krypton and xenon are used as gaseous scintillators. In plastic, liquid and gaseous scintillators suitable materials are used as wavelength shifters.

Crystal scintillators are generally kept in metal containers, Particularly in the case of hygroscopic alkali halides, crystals must be sealed from moisture. Containers for liquid scintillators are generally made up of aluminium with thin transparent windows made up of alite material. When photomultiplier can not be put in direct contact with the scintillator lucite light pipes are used. These serve as light guides.

The essential parts of a scintillation detector are shown in Fig.14.11.

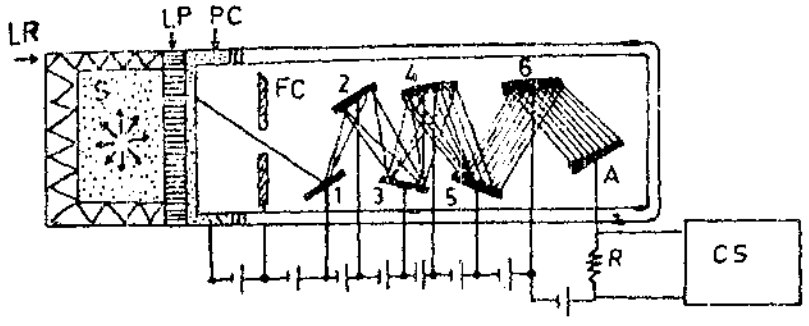


Fig.14.11 Scintillation detector system
S - Phosphor (scintillator), LR - Light Reflector, LP - Light pipe, PC - Photo cathode of photo multipliertube, FC - Focussing electrode. 1,2,3,4,5,6 Dynodes. A - Anode CS - Counting system.

The fluorescent light emitted by the scintillator, when an incident particle passes through the scintillator, is guided through on to the photo cathode of the photomultiplier tube. Each incident light photon is converted to one or more electrons which are accelarated by an electric field and hit a low work function electrode called a dynode. Here each electron hitting the dynode leads to the emission of two or more secondary electrons. By

repeating this secondary electron emission process at successive more positively charged dynodes one can get a large swarm of electrons at the last dynode from which the resulting charge pulse can be collected and further amplified. In general, the magnitude of the electronic pulse will be proportional to the number of photons reaching the photocathode and hence to the energy of the incident radiation. The number of pulses received per unit time will be proportional to the intensity of the incident radiation.

Scintillation counters are highly efficient. They have short rise and recovery time. They also have long and indefinite usuable life. Scintillation counters are used to detect almost all types of particles.

Inorganic scintillators are used to detect heavy charged particles. For gamma ray counting NaI (TI) and CsI (TI) are well suited. For beta counting organic scintillator are used. Scintillation counters find wide application in neutron counting. They are also used in the measurement of high energy particles in the investigation of short-lived nuclear isomers.

14.7 SUMMARY

Geiger-Muller counter operates in the Geiger-Muller region in which the charge collected is independent of initial ionization. The sensitive volume of the chamber consists of an inert gas usually argon and a quenching gas (alcohol or halogen). GM Counters are characterized by a plateau region where the counting rate is independent of the operting voltage. GM counters are generally used for detection of beta and gamma particles.

C.T.R Wilson developed cloud chamber. It is a particle track detector. When the ionizing radiation passes through sensitive volume of the chamber consisting of super-saturated vapour condensation of vapour on the ions takes place. When illuminated the track of the particle becomes visible and can be photographed. Depending on the way how super-saturation of the vapour is achieved the cloud chambers are divided as expansion cloud chambers and diffusion cloud chambers.

Cloud chambers are extensively used in cosmic ray research. Identification of neutron, discovery of positron, production and nuclear disintegration are some of the nuclear phonomena studied using cloud chambers.

Donald A. Glaser developed bubble chamber. When ionizing radiation passes through a super-heated liquid it triggers off the formation of microscopic bubbles which grow along the track of the particle, which can be photographed. This is the basic principle on which bubble chamber works.

Bubble chamber is used to study nuclear reactions and particle interactions.

Scintiallation counter is to detect any particle both charged and uncharged. The incident particles the detector (scintillator) to emit a light pulse which serves as the signal for count. Scintillation counter has high detection efficiency, high speed reaction to the passage of radiation, minimum dead time and response proportional to the energy of the radiation.

Scintillators exists in crystal, liquid, plastic and gaseous forms. NaI (TI activated) is the extensively used inorganic crystal scintillator. It is used for gamma-ray detection. For alpha-particle detection ZnS crystal is used. Anthrancene, an organic crystal scintillator, is

used for beta particle detection. Noble gases like Xenon are used as gaseous scintillators. A mixture of zinc sulphide and boron is used for detection of neutrons.

14.8 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Discuss the characteristics of the Ionization process in an ionization chambers when the applied voltage is increased gradually discuss the usefulness of various regions of ionization.

2. Discuss the constructional details and working of a Geiger Muller counter.

3. Discuss the construction and working of a expansion Cloud Chamber, Give some of its important application.

4. Discuss the construction and working of diffusion Cloud Chamber, Give some of its important application.

5. Describe the construction and working of Bubble Chamber.

6. Describe the working of a scintillation counter and mention its uses.

II. Answer the following question briefly.

1. Give an account of different type of Scintillators used in Scintillation Counter.

2. Explain the difference between the self quncling and externally quenched GM Counters.

UNIT–15 : ARTIFICIAL RADIO-ACTIVITY

Contents

15.1 AIMS AND OBJECTIVES

In this unit formulae for nuclear binding energy was deduced. The importance of nuclear binding energy in accounting for the stability of nucleus was discussed.

After going through this unit

- you will be able to explain the stability of the nucleus depending of the basis of nuclear binding energy.
- you will be able to describe the nature of different types of nuclear reactions.

15.2 INTRODUCTION

In this unit we shall study the artificial radio activity and transuranic elements.

15.3 NUCLEAR BINDING ENERGY

A clear picture regarding the structure of the nucleus emerged after the discovery of neutron by Chadwick. Accordingly, if A represents the mass number of a nucleus, it consists of Z number of protons and A-Z number of neturons. Around this nucleus, Z number of electrons revolve in definite orbits. It is generally expected that the mass of an atom must be equal to the sum of the masses of its constituents. A survey of the atomic masses of various nuclides determined using mass spectrographs atomic masses less than the sum of its constituent particles in the free state. To account for this mass deficit, the principle of equivalence of mass and energy, due to Albert Einstein, is made use of. According to this principle if Δm in the decrease in mass when a certain number of protons, neutrons and electrons combine to form an atom, an amount of energy equal to

$$\Delta E = (\Delta M)C^2$$

is released in the process. In the above equation C represents the velocity of light. ΔE is called the binding energy of the nucleus and ΔM is called the mass defect.

Atomic masses can be represented either in (1) absolute scale related to the kilograms or in (2) the atomic mass scale. Usually atomic masses are given in atomic mass scale. The atomic masses are now-a-days given, taking C^{12} as standard. The mass of one atom of the nuclide C^{12} is equal to 12.000 atomic mass units (a.m.u.). To calculate the binding

energy it is essential to represent the value of 1 a.m.u in terms of absolute scale and its equivalence in terms of energy based on Einstein's mass energy relation. We know that 1 mole of C^{12} consists of Avagadro number of atoms and weighs 0.012 kg. Hence weight of one atom of C^{12} is given by

$$\text{Weight of 1 atom of } C^{12} = \frac{0.012kg}{6.02252 \times 10^{23}}$$

Since 1 atom of C^{12} is equal to 12 a.m.u. weight of 1 a.m.u. is given by

$$1a.m.u = \frac{0.012kg}{12 \times 6.0225 \times 10^{23}} = 1.661 \times 10^{-27} Kg$$

In terms of energy

$$1a.m.u = 1.661 \times 10^{-27} Kg \times \left(2.998 \times 10^{8} ms^{-1}\right)^2$$

$$1a.m.u = 14.949 \times 10^{-11} J$$

Since 1 J is equal to 0.6242 x 10^{13} MeV

1a.m.u = 14.949 x 10^{-11} x 0.6247 x 10^{13} MeV

1 a.m.u = 931.48 MeV

Let us now calculate the binding energy of a nuclide represented by ${}_{Z}X^{A}$. This nuclide consists of Z number of protons and (A-Z) number of neutrons inside the nucleus and Z number of electrons outside the nucleus. if ${}^{M}Z$, A represents the actual mass of the nuclide, the mass defect ΔM is given by

$$\Delta M = ZM_p + ZM_e + (A - Z)M_n - M_{Z,A}$$

where M_p, M_e and M_n represent the masses of proton, electron and neutron respectively. We can write $M_p + M_e = M_H$ the mass of the hydrogen atom, since the mass defect in the formation of hydrogen atom is neglible.

Therefore

$$\therefore \quad \Delta M = ZM_H + (A - Z)M_n - M_{Z,A} \qquad 15.1$$

Since the mass of the hydrogen atom is equal to 1.0078252 a.m.u. and the mass of neutron is equal to 1.0086654 a.m.u as referred to C^{12} standard we can rewrite Eq.15.1 as

$$\Delta M = \left[1.0078252Z + 1.0086654(A - Z) - Mz, A\right] a.m.u$$

Since 1 a.m.u = 931.48 MeV, the binding energy of the nucleus is given by

$$\Delta M = \left[1.0078252Z + 1.0086654(A - Z) - Mz, A\right]931.48\,MeV \qquad 15.2$$

The average binding energy per nucleon $(\Delta E/A)$ can be obtained by dividing the total binding energy with the mass number A.

Worked Example 1

Claculate the binding energy per nucleon of ${}_7N^{14}$ given its atomic mass is 14.0075179 a.m.u.

Using Eq. 15.2

$$\Delta E = \left[1.0078252 \times 7 + 1.0086654 \times 7 - 14.0075179\right] \times 931.48$$

$$\Delta E = (14.1198867 - 14.0075179)931.48 = 104.33\,MeV$$

$$\text{Binding energy per nucleon} = \frac{104.33\,MeV}{14} = 7.45\,MeV$$

The binding energy of a nuclide represents the amount of energy required to break the consituents of the atom. It is the amount of energy released when the constituents of the atom are brought from free state to the bound state. Hence binding energy is a measure of the stability of the nucleus. The more the binding energy, the more stable the nuclide is. This aspect can be understood by studying the variation of binding energy per nucleon $(\Delta E/A)$ as a function of mass number which is schematically shown in Fig.15.1.

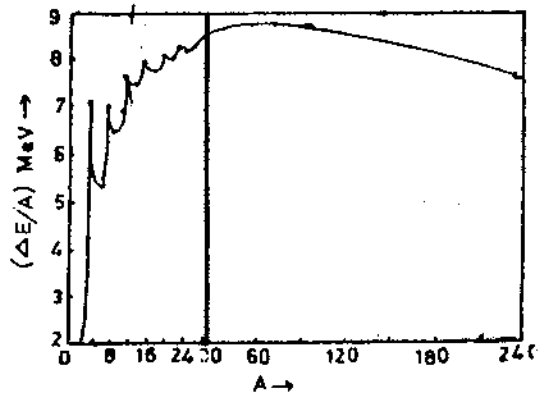


Fig. 15.1 Binding energy pey nucleon versus mass number. A - for different nuclides

The main features that can be inferred from the diagram are as follows :

I. A positive binding energy for all nuclei indicates that any nucleus is more stable than an unconnected assembly of its constituent neutrons and protons. This reflects the

attractive forces acting between nucleons inside the nucleus.

2. The binding energy per nucleon increases rapidly for light nuclei with a notable peak at A = 4. 4(He^4) and further peaks at A = 4n (Be^8, C^{12}, O^{16}, Ne^{20}, Mg^{25}). This reflects the peculair stability of the α-particle structure.
3. The binding energy per nucleon is nearly the same around 7.3 to 8.7 MeV for all nuclei with A greater than 16. This indicates that except for light nuclei, the binding energy of any nucleus is proportional to A. If each particle interacts with every other particle in the nucleus then the binding energy must be proportional to A^2. But, the above experimental fact indicates that inside the nucleus each nucleon interacts with only a limited number of other nucleons.
4. The peak at A = 4 indicates that neutron interacts strongly with the other neutron and two protons. The fifth, sixth and seventh particlese in Li^5, Li^6 and Li^7 are less strongly bound.
5. The binding energy per nucleon is maximum (8.7 MeV) for A = 60 (Fe, Ni, Co).
6. The binding energy per nucleon decreases from 8.7 MeV at A = 60 to 7.3 MeV at A = 238. This is associated with the disruptive effect of the nuclear charge which ultimately sets a limit to the number of elements which can be formed.

15.4 ARTIFICIAL RADIOACTIVITY

The phenomenon of artificial radioactivity was discovered in 1934 by Curie and Joliot while they were studying the transmutation of light elements by alpha-particles. The term artificial radio activity mainly denotes the nuclear decay spontaneously according to the same laws that govern the disintegration of naturally occuring radioactives elements.

While studying the transmutation reaction using cloud chamber Curie and Joliot observed boron to emit positrons while being bombarded by alpha-particles. Positron is a fundamental particle having a mass equal to the mass of electron and possessing positive charge. The existance of this particle was predicted by Dirac and proved experimentally by Carl Anderson in 1932 in the course of a study of photographs of cosmic ray tracks in a Wilson cloud chamber. The transmutation reaction proposed by Curie and Joliot leading to the emission of positron is given by

$$_5B^{10} +_2 He^4 \rightarrow_o n^1 +_1 e^+ +_6 C^{13}$$

Applying the principle of convervation of mass and energy to the above reaction. Curie and Joliot estimated the mass of neutron to be 1.012 a.m.u. Positron emission was also observed when aluminium and magenesium were bombared by α-particles from polonium. Curie and Joliot found that thin foils of B, Al and Mg, after a 10 minute exposure to α-rays from polonium, continued to emit positrons even after the α-ray source was removed. The rate of emission to positrons was found to decrease exponentially with time after the removal of the source of α-rays. The artificial radioactive nuclide contained in each foil exhibited a different half-period of decay. The energy of positrons emitted was different for the three cases. When Al was irradiated by α-rays of low energy, the yield of positrons was reduced, but their energy and half period remained unaltered. No electrons were found to accompany the positrons indicating that the source of positrons is not due to pair production. Curie and Joliot proposed the following reaction to take place, when B is irradicated by α-rays leading to the formation of N^{13} which is radioactive.

$$_5B^{10} +_2 He^4 \rightarrow_o n^1 +_7 N^{13}$$

$_7 N^{13}$ decays with a half-life of 10 min. leading to

$$_7 N^{13} \rightarrow \beta^+ +_6 C^{13}$$

The reactions when Al and Mg are bombarded by α rays are:

$$_{13} Al^{27} +_2 He^4 \rightarrow_o n^1 +_{15} P^{30}$$

$$_{15} P^{30} \rightarrow \beta +_{14} Si^{30}$$

$$T_{\frac{1}{2}} = 3.25 \text{min.}$$

$$_{12} Mg^{24} +_2 He^4 \rightarrow_o n^1 +_{14} Si^{27}$$

$$_{14} Si^{27} + \beta_+ +_{13} Al^{27}$$

$$T_{\frac{1}{2}} = 6.7 \text{min.}$$

The observations made by Curie and Joliot were established by radio chemical analysis. In the boron reaction, boron nitride was taken as the target. It is irradicated with α -rays for several minutes and then heated with caustic soda. All the nitrogen was librated as gaseous ammonia. The ammonia was found with other boron targets and was not found with isotope of nitrogen. Similarly, aluminium target after being irradiated with α -rays was dissolved in hydrochloric acid there- by liberating any phosphorus present as the gas phosphene (PH_3) while the aluminium remained behind in the solution. This solution was then evaporated to dryness and the residue tested for positron activity. No activity was found. The gas containing phosphorus showed the characterstic positron activity. This indicated that the activity was associated with an isotope of phosporus. The above tests provided the chemical evidence for the artificial transmutation of one element to another.

Nuclear reaction induced by protons, deuterons, neutrons and photons can also result in radioactive nuclides. For example, the reaction C^{12} (d, n) N^{13} proposed by Curie and Joliot was verified experimentally.

After the discovery of positron β activity, Fermi and his co-workers irradiated all available elements with neutrons and produced electron β activity in more than 40 elements. The reactions involved were (n, γ), (n,p) and (n, α) all of which increased the ratio of neutrons to protons.

The constitution of all known nuclides both stable and radioactive can be understood by plotting neutron number against proton number as shown in Fig.15.2. The stable nuclides are indicated by block squares and the radioactive nuclides are represented by open circles, open triangles and crosses as illustrated in the figure. The artificially radioactive nuclides lie above or below the stable nuclides. These nuclides have either too many neutrons or too few neutrons. A nuclide with too few protons tends to become stable by increasing the nuclear charge leading to electron emission (β -activity). A nucleus with too many protons, tends to become stable by decreasing the nuclear charge leading to the emission electron of positron (β + activity).Electron emission occurs in isotopes whose mass number is greater than those of stable isotopes.

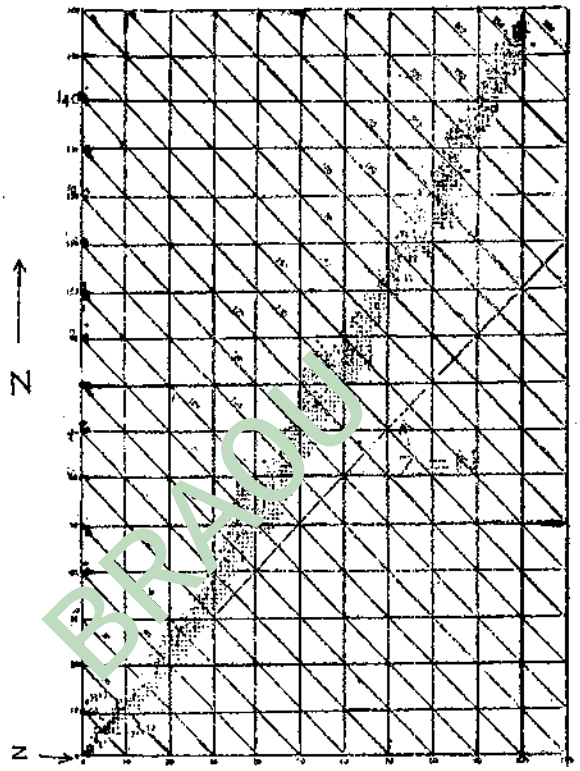


Fig.15.2 Neutron number (N) versus proton number (Z) for nuclides. stable nuclie; O-β emitter; +-beta$^+$ emitter; β^+, β 6 - emitters; Δ, $\rightarrow \alpha$ -emitters.

The charge of a nucleus can also be reduced if the nucleus captures an orbital electron. The extranuclear electrons in the course of their motion often come close to the nucleus and according to wave machanics, may even penetrate, the nucleus. The electrons most likely to do this are K-electrons in atoms with large value of Z. that is in proton-rich atoms. If an electron from the K-shell is captured the process is called K-capture, Less often an L-shell electron may be captured. It is called L-capture. The vacancy in the K or L shell, is usually filled by an electron from an outer shell with the emission of K or L, X-ray characteristic of the product nucleus. In this process no charged particle is emitted and the process can be observed only because of X-ray emission.

Orbital electron capture is sometimes accompained by the emission of electrons from the extra nuclear structure. This was discovered by Auger and these electrons are called Auger electrons. These Auger electrons are due to internal photoelectric effect. Here instead of the emission of K-X-ray an L-electron with kinetic energy equal to the difference between the K-X-ray energy and the binding energy of the L-electron may be emitted.

The processes of electron emission, positron emission and orbital electron capture, are called isobaric transformations because these involve only changes in charge but not in mass number. The question whether an artificial radioactive nuclide decays by electron emission, position emission or orbital electron capture can be understood in terms of energy available for the disintegration process.

Let us consider electron emission process. The general reaction is given by

$$_{Z}X^{A} \rightarrow Z+1^{Y^{A}} +_{-1} e^{o}$$

The Q-value of the reaction can be given by

$$Q = \left[M_n\left(_{Z}X^{A}\right) - M_n\left(_{Z+1}Y^{A}\right) - m\right]C^2$$

$M_n^1 s$ represent nuclear masses, m represents the mass of an electrons.

Interms of atomic masses

$$Q = \left[M_a\left(_{Z}X^{A}\right) - Zm - M_a\left(Z+_1 Y^{A}\right)(Z+1)m - m\right]C^2$$

$$Q = \left[M_a\left(_{Z}X^{A}\right) - M_a\left(Z+_1 Y^{A}\right)\right]C^2$$

The condition that electron emission is energetically possible is that the Q-value should be positive. Hence

$$\therefore \quad M_{aZ}X^{A} \triangleright M_a\left(_{Z+1}Y^{A}\right)$$

The atomic mass of the artificial radioactive nuclide must be greater than that of its isobar, with nuclear charge one unit greater.

For positron, the general reaction is given by

$$_{Z}X^{A} \rightarrow_{Z-1} Y^{A} +_{1} e^{o}$$

The Q-value of the above reaction is given by

$$Q = \left[M_n\left(_{Z}X^{A}\right) - M_n\left(_{Z-1}Y^{A}\right) - m\right]C^2$$

or $$Q = \left[M_a\left(_{Z}X^{A}\right) - Zm - M_a\left(_{Z-1}Y^{A}\right) + (Z-1)m - m\right]C^2$$

$$Q = \left[M_a\left({}_Z X^A\right) - M_a\left({}_{Z-1} Y^A\right) - 2m \right] C^2$$

If Q is positive, positron emission is more favourable.

Hence $M_a\left({}_Z X^A\right) \triangleright M_a\left({}_{Z-1} Y^A\right) + 2m$ 15.3

For positron emission to be energetically possible the atomic mass of the artificial radioactive nuclide must be greater than the atomic mass of its isobar with nuclear charge one unit smaller by atleast 2 electron masses.

For orbital electron capture, the general reaction is given by

$${}_Z X^A + {}_{-1} e^o \rightarrow {}_{Z-1} Y^A$$

The Q-value of the reaction is given by

$$Q = \left[M_n\left({}_Z X^A\right) + m - M_n\left({}_{Z-1} Y^A\right) \right] C^2$$

$$Q = \left[M_a\left({}_Z X^A\right) - Zm + m - M_a\left({}_{Z-1} Y^A\right) + (Z-1)m \right] C^2$$

$$Q = \left[M_a\left({}_Z X^A\right) - M_a\left({}_{Z-1} Y^A\right) \right] C^2$$

If Q is positive, orbital electron capture is possible and hence

$$M_a\left({}_Z X^A\right) > M_a\left({}_{Z-1} Y^A\right)$$

The mass of the artificial radioactive nuclide must be greater than that of its isobar with nuclear charge one unit smaller.

The two conditions represented by Eq.15.3 for positron emission and Eq. 15.4 for orbital electron capture indicate that orbital electron capture depends on the probability of penetration of the K-electron into the nucleus which is very small. Hence, if the energy available exceed $2mC^2$ positron emission takes place instead of orbital electron capture, K-capture takes place when the condition given by Eq.15.4 is satisfied. But, it is not observed always even when the condition is fulfilled.

The nuclides C^{11}, N^{13}, O^{15}, F^{17}, Ne^{19}, Na^{21}, Mg^{23} and Al^{25} decay by positron emission. The nuclides C^{14}, N^{15}, F^{20}, Ne^{23}, Na^{24}, Mg^{27} and Al^{28} decay by electron emission.

The isotopes of iodine, show all the three types of decay. I^{121} and I^{122} decay by positron emission. I^{123} captures an orbitral electron. I^{124} sometimes captures an orbital electron and sometimes decays by positron emission. I^{125} captures an orbital electron. I^{126}

sometimes captures an orbital electron and sometimes decays by electron emission. I^{127} is stable. I^{128} decays by emission of an electron 95% of the time and either by orbital electron capture or positron emission 5% of the time, Isotopes I^{129} to I^{130} decay by electron emission.

There exists some correaltion between the radioactivity of an artificial nuclide and the transformation by which it is produced. Electron emission is common for activities produced by (n, γ), (n,p), (n, α), and (d,p) reactions. In these reactions, the charge to mass ratio decreases. Positron emission is common for activities produced by (p, γ), (n,p), (n,p), (α, n), and (d,p) reactions. In these reactions, the charge to mass ratio decreases. Positron emission is common for activities produced by (p,γ), (p,n) (α,n), (d,n) and (γ,n) reactions. Here the charge to mass ratio increases, (α,p), (p, α,) reactions usually given out stable products.

15.5 NUCLEAR REACTIONS

In Unit 14, we have studied about the various types of nuclear reactions when a target nucleus is bombarded by nuclear particles taking into account the conservation of mass and energy, But, information about the relative probability of different reactions provided clues to understand the structure of the nucleus and the nature of nuclear forces.

In transmutation process, the target nucleus will be excited and the decay of the excited state then gives information about energy levels and decay schemes. The excitation of the nucleus can be caused by any kind of incident particle and may result in the emission of any kind of particle say proton, neutron, deuteron, alpha particle or gamma ray..

A nuclear reaction can be represented in most general way, as

$$X + X \rightarrow X + x \qquad 15.5$$

$$\rightarrow X^* + x \qquad 15.6$$

$$\rightarrow Y + y$$

$$\rightarrow Z + z \text{ etc.} \qquad 15.7$$

The reaction represented by Eq. 15.5 is called elastic scattering. The outgoing particle is the same as the incident particle, The total kinetic energy of the system after the collision is the same as that before the collision. Eq. 15.6 represents inelastic seattering process. The target nucleus is raised to an excited state. X*. The total kinetic energy of the system as a result of collision is decreased by an amount equal to the excitation energy of the target nucleus. Eq.15.7 represents different types of transmutation reactions. Here the product nucleus may be formed either in the ground state or more often in the excited state. The nucleus formed in the excited state decays very quickly to the ground state with the emission of gamma-rays.

The existence of excited states of the product nucleus was discovered by measuring the energies of the protons emitted in (α, p) reactions on light element. When a light element is bombarded by monoenergitic α-same energy could be observed as shown below.

$$\alpha + X \rightarrow Y + \left({}_1H^1\right)_A + \left({}_1H^1\right)_B + \left({}_1H^1\right)_C$$

Here the subscripts A,B,C represent groups of protons and each group contains protons of the same energy. Q-value can be a calculated for each energy group. The protons group with greater energy gives the greatest Q-value say Q_1 and this corresponds to the ground state of the product nucleus. A proton group of lowest energy gives a lower Q-value say Q_2. The difference between Q_1, and Q_2 gives the excitation energy of the product nucleus after the emission of lowest energy proton. Experimental results confirm the above ideas. For example consider the reaction.

$$ {}_2He^4 + {}_{13}Al^{27} \rightarrow \left[{}_{15}P^{31} \right] \rightarrow {}_{14}Si^{30} + {}_1H^1 + Q \qquad 15.8 $$

The above reaction, was carried out by bombaring an aluminium foil with α particles accelarated in a cyclotron to an energy of 7.3 MeV. The ranges of the protons emerging along the direction of the incident beam were measured by counting coincidences between protons and gamma-rays. Four groups of protons were observed. The range energy and the corresponding Q-values are given in Table 15.1

Table 15.1 Range, energy and Q-values of different protons groups observed as given by Eq. 15.8.

Range in air cm	*Energy Mev*	*Q-value Mev*
101.6	6.34	2.22
60.8	6.98	-0.06
40.8	5.55	-1.44
25-30	4.2-4.65	-2.44

The data presented in Table 15.1 indicates that there exist three excited state of Si^{30} with energies 2.28 MeV, 3.66 MeV and 4.6 MeV. It was also observed that no gamma-rays accompany the highest energy group of protons.

The above type of emission of different energy groups as particles was also found in other types of nuclear reactions.

During the investigation (α, p) reactions, it was found that when the energy of the incident α-particles was increased the yield of protons did not increase monotonically. Instead, the yield showed sharp maxima at certain discrete values of the energy.

In the transmutation reaction.

$$ {}_2He^4 + {}_{13}Al^{27} \rightarrow \left[{}_{15}P^{31} \right] \rightarrow {}_{14}Si^{30} + {}_1H^1 + Q $$

the yield of protons had peaks at γ-particle energies of 4.0, 4.49, 4.86, 5.25, 5.75 and 6.61 MeV, The occurence of maxima in the reaction rate at different energies is called resonance and the particular energies at which the maxima occur are called resonance energies. At each resonance energy of the incident γ-particles, different proton energy groups are observed. Resonance also takes place in all other nuclear reactions. The experimental results obtained at resonance energies are useful in understanding the excited levels of the compound nucleus.

15.6 TRANSURANIC ELEMENTS

Elements of atomic number greater than 92 are known as Transuranic elements. Such elements do not occur in nature but may be obtained by suitable nuclear reactions. They are all radioactive and members of the actimide group.

Teh capture of neutron by Uranium need not lead to fission as discussed earlier. In 1934. Fermi suggested that when low energy neutrons are bomarded at the Uranium nucleas, then elements of atomic number greater than 92 may be produced according to the following reaction.

$$_{92}U^{238} +_{o} n^{1} \rightarrow \left(_{92}U^{239}\right) \rightarrow_{92} U^{239} + \gamma$$

$$_{92}U^{239} \rightarrow_{93} U^{239} +_{-1} e^{o}$$

$_{92} U^{239}$ will decay into an element with atomic number 93 by the emission of an electron. In turn this new element may decay by the emission of another electron into another element with atomic number 94. Much study was made to identify such transuranic elements from 1934 to 1939.

1. The first element discovered was Neptunium with atomic number 93 and mass number 239 by bomarding $_{92}U^{238}$ with neutrons in 1940. The new element $_{92} U^{239}$ decays into $_{93}U^{239}$, known as Neptunium with a half life of 23 mins. according to the following reactions.

$$_{92}U^{238} +_{o} n^{1} \rightarrow_{92} U^{239}$$

$$_{92}U^{239} \rightarrow_{93} Np^{239} +_{-1} e^{o} \quad (T = 23 \text{ min.}).$$

This was established by chemical methods.

2. The second element discovered was plutonium with atomic number 94. When the element $_{93} Np^{239}$ decays by emitting a β particle a new element with atomic number 94 was found. The bomarded Uranium with deuteron 's also producer plutomin according to the following reactions.

$$_{92}U^{238} +_{1} H^{2} \rightarrow \left(_{93}Np^{240}\right) \rightarrow_{93} Np^{238} + 2_{o}n^{1}$$

$$_{93}Np^{238} \rightarrow_{94} Pu^{238} +_{2} He^{4}$$

Using accelerators many new isotopes of plutonium were descovered.

3. The Third element discovered was Americium with atomic number 95 by bombarding $_{92}U^{238}$ in the α particles from a 40 MeV cyclotron according to the following reaction.

$$_{92}U^{238} +_2 He^4 \rightarrow_{94} Pu^{241} +_o n^1$$

$$_{94}Pu^{241} \rightarrow_{95} Am^{241} +_{-1} e^o \qquad (T = 13 \text{ yrs.})$$

In this way curium with Z = 96, Berkelium with Z = 97, Californium with Z = 98, Einstenium with Z = 99 and Fermi with Z = 100, Mandelvinium with Z = 101, Nobeium with Z = 102, Lawrencium in the Z = 103, Kurchatonium with Z = 104 and Hahnium with Z = 105 were discoverd with established by Chemical studies.

The antificial transmutation fecilitated to discover the missing elements from the periodic table. They are Techinetium, Promethium, Francin and Astatime whose atomic number are 43, 61, 85 and 87 respectively.

15.7 SUMMARY

The mass of an atom is less than the sum of the masses of the particles, constituting the atom. This deficit mass is called mass defect. When an atom is formed this deficit mass gets converted into energy. This energy is responsible for binding the nucleons inside the nucleus. It is termed the binding energy of the nucleus.

The binding energy per nucleon increases rapidly for light nulei with notable peaks at A = 4n where n = 1,2,3,..... ($He^4, Be^8, C^{12}, O^{16}, Ne^{20}, Mg^{25}$) indicating the very high stability of these nuclides.

For A > 16, the binding energy per nucleon is the same around 7.3 to 8.7 MeV; indicating that the binding energy of a nucleus is proportional to A the mass number of the nuclide.

Artificial radioactivity was discovered by Curie and Joliot. In the transmutation reaction caused by the bombardment of boron target by α, particles, $_7N^{13}$ formed was found to decay by positron emission.

Nuclear reactions induceed by protons, deuterons, neutrons and photons may also result in radioactive nuclides.

Artificially produced radioactive nuclides decay by three processes, namely, positron emission, electron emission and orbital electron capture. These transformations are called isobaric transformations.

Electron emission is common for radioactive nuclides produced in (n, γ), (n,p), (n,d) and (d,p) reactions. Position emission is common for radioactive nuclide produced in (p,γ), (p,n), (α,n), (d,n) reactions.

In a nuclear reaction, the product nucles can form in different excited states. The product particles formed in groups have different energies. Within a group all the particles have the same energy.

There exist sharp maxima in yield of product particles versus the energy of incident particles. This phenomenon is called resonance, Resonance reactions are common to all

types of nuclear reactions.

15.8 MODEL EXAMINATION QUESTIONS

I. Answer the following question in detail.

1. What are isobaric transformations based on the Q-value of the reaction explain the condition under which each transformation takes palce.

II. Answer the following questions briefly.

1. Discuss the main features of the binding energy per nucleon versus mass number curve.

2. Explain the curie Joliot experiments which lead to the discovery of artificial radio activity.

III. Solve the following problems

1. Determine the binding energy per nucleon of ${}_{92}U^{235}$, given its atomic mass as 235.118 a.m.u masses of hydrogen atom and neutron respectively are 1.0078 a.m.u and 1.00867 a.m.u.

UNIT–16 : NUCLEAR MODELS

Contents

16.1 AIMS AND OBJECTIVES

In this unit various models for the nucleus are discussed After going through this unit.

- You will be able to describe the shell model and liquid drop model suggested for a nucleus.
- You will be able to explain the significances of magic numbers. You can predict the binding energy of unstable nucleus.
- You can evaluate the energy released in nuclear fission.

16.2 INTRODUCTION

The constitution of the nucleus has been well-established with the discovery of neutron by Chadwick in 1932. If A is the mass number and Z the atomic number of a nuclide there exists Z number of protons and (A-Z) number of neutrons inside the nucleus. The high stability of deuteron indicates that the force between proton and neutron is an attractive force. The exact nature of the nuclear forces is not yet fully understood. The force of attraction increases as the distance between the necleus decreases but when the distance of separation is equal to or more than the diameter of the nucleus the force becomes highly repulsive. The nuclear force is assumed to be spin-dependant. It is not completely central, but depends to some extent on the angle between the axis of the spins and the line joining the two nuclecs. The nuclear forces are assumed to be charge-independent. The approximate constancy of binding energy per nucleon indicates that the forces must have the property of saturation. But short-range nature of nuclear forces precludes the above feature. To account for proportionality of binding energy of the nucleus with the mass number, the nuclear force or part of the force, between two nucleons is assumed to be some-times, attractive and sometimes repulsive, depending on the state of the nucleons with respect to each other. That is, nuclear forces have been assumed to be exchange forces. Yukawa proposed meson theory of nuclear forces where the nucleons interact with the exchange, of mesons and a meson field exists between the nucleons. Different types of potentials like square well, exponential, Gaussian and Yukawa have been suggested as nuclear potentials to explain the short range nature of the nuclear forces. The assumption that the nuclear forces are charge independent is valid to explain the nucleon-scattering experiments at low energies. But on high-energy scattering the assumption fails to explain the experimental observations.

In the absense of a detailed theory of nuclear structure, different models have been proposed to explain the nuclear propertries. Barlet, Guggenheimer Elasser and others developed independent particle model involving closed shells of $2(2l+1)$ neutrons or protons

based on the analogy of extra nuclear electronic structure of the atoms. Here l represents the orbital angular momentum, quantum number. N. Bohr proposed the liquid drop model based on the analogy that exists between a nucleus and a liquid drop. This model is very much successful in explaining nuclear fission. Maria Mayer revived the shell model based on the experimental evidence for closed shells in nucleus at 50,82 and 126 indentical nucleons which cannot be explained by liquid drop model. Maria Mayer and independently Haxel, Jensen and Suess introduced j-j coupling by assuming strong spin-orbit forces for nucleons. The model was very much successful in explaining the experimental facts. A Bohr and Mottelson proposed collective model which is a generalization of the independent particle model. It also includes the aspects of liquid drop model. The collective model is successful in explaining the ground state characteristic of nuclei as well as making some-reliable predictions concerning some of the first few excited states at energies greater than 6 MeV. The nuclear reactions cannot be explained by independent particle model. The concept of compound nucleus formation and its decay can be explained by the statistical model. Optical model for the nucleus has been proposed to explain the results of high energy-scattering experiments. In this model, the nucleus is considered as uniform sphere having a certain refractive index for the incident particle. To expalin the reaction and absorption process it is assumed that the sphere is not perfectly transparent, but rather has an attenuation coefficient for particles passing through it.

No single model cited above can explain all the experimentally observed facts, But the shell model and liquid drop model are highly successful in explaining a mojority of experimental facts. In this lesson we shall study in detail the shell model and liquid drop model of the nucleus.

16.3 SHELL MODEL OF THE NUCLEUS AND MAGIC NUMBERS

The shell model of the nucleus has been conceived, based on certain experimental observations regarding the properties of the nucleus. The nuclear properties were found to vary periodically in a sense similar to that of the periodic system of elements. Most of the properties show marked discontinuities near certain even values of the proton or neutron number say 2, 8, 20, 50, 82, 126, which are referred to as magic numbers. The magic numbers of neutrons or protons have been interpreted as forming closed shells of neutrons and protons analogous to electron shells in atoms. The neutron and proton shells form independent of each other. Let us now study the experimental evidences supporting the pressence of magic numbers, before we discuss in detail the shell model of the nucleus.

1. Exceptional stability is associated with nuclei for which the proton and neutron numbers are 2 or 8 $_{2}He^{4}$ and $_{8}O^{16}$ are more stable than their neighbours.

2. A careful study of binding energy per nucleon versus mass number curve indicates the existence of several kinks which occur at $_{82}Pb^{208}$ (Z = 82, N = 126), $_{38}Ce^{140}$ (Z = 58, N = 82) $_{50}Sn^{120}$ (Z = 50, N = 70) and $_{38}Sr^{88}$ (Z = 38, N = 50). In these nuclides either Z or N corresponds to 50, 82 and 126.

3. A study of relative abundances of nuclei compiled from the data on the composition of the earth, sun, Stars and meterites by Brown indicates pronounced peaks at $_{8}O^{16}$ (N, Z = 8), $_{20}Ca^{40}$ (N,Z = 20) $_{50}Sn^{118}$ (Z = 50) : $_{39}Y^{89}$, $_{39}Zr^{89}$, $_{39}Zr^{89}$ (N = 50); $_{56}Ba^{138}$, $_{57}La^{139}$, $_{58}Ce^{140}$ (N = 82); and $_{82}Pb^{208}$ (Z = 82, N = 126).

4. The number of stable isotopes possed by any element varies considerably, being the greatest when Z = 20, 50, 82 and N=20, 28, 50 and 82.

5. The binding energy of the last nuetron added to a nucleus can be determined experimentally by measuring Q-values of (d,p), (n, γ), (γ ,n) reactions and can also be calculated using semiempirical binding energy formula. The difference between the observed and calculated values of binding energy ΔE of the last neutron versus (N+1) neutron number indicate sharp discontinuities at 50, 126 and 82 neutron numbers as can be seen from Fig. 16.1.

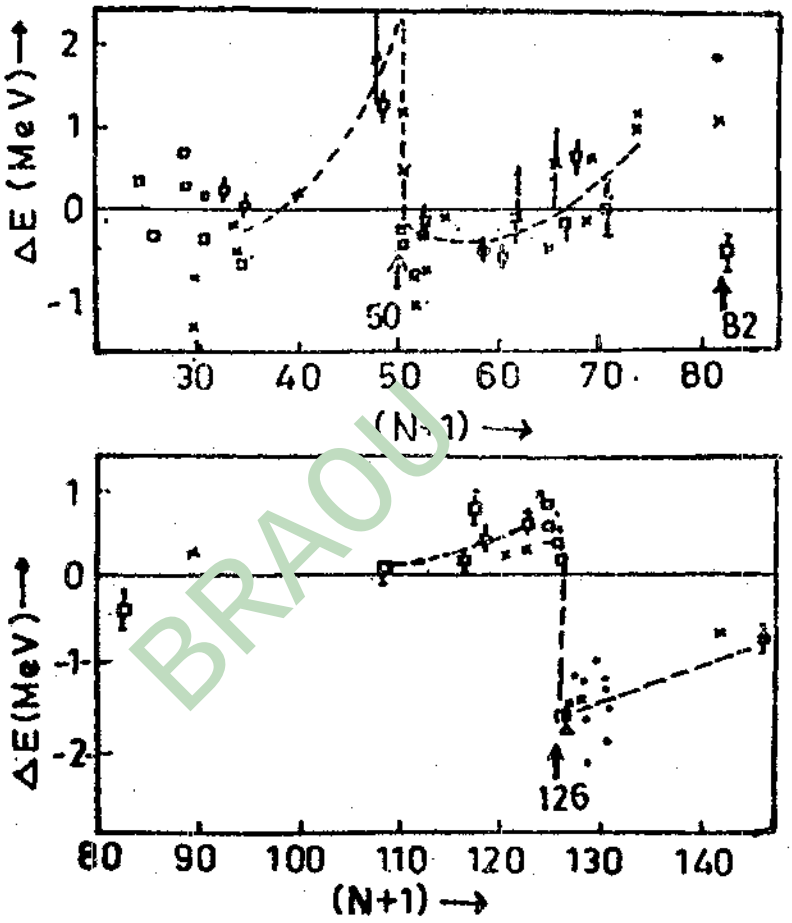


Fig.16.1 Binding Energy of the last neutron added to nucleus ΔE versus (N + 1)

6. The neutron capture cross section measured at 1MeV shows marked minima at neutron numbers 50, 82 and 126 as can be seen from Fig.16.2.

7. Natural radioactive series all end with a stable isotope of lead Z=82 out of the three isotopes $_{82}Pb^{208}$ with N=126 is the most abundant one.

8. The energy of α-particles emitted by radioactive nuclides is exceptionally large when the daughter nucleus has 126 neutrons. Similarly the energies of the emitted β particles are especially large when the number of neutrons or protons of the product nucleus corresponds to a magic number.

9. Nuclear isomers are particularly abundant when the odd nucleon number are close to 50, 82 and 126.

10. The electric quadrupole moments vary symmetrically changing from positive to negative when the odd nucleon number is close to 8, 15, 28 50, 82, 120 and form negative to positive, when the odd nuclear number is close to 10, 18, 30, 55, 90. That is the nucleus posses spherical symmetry only at certain nucleon numbers.

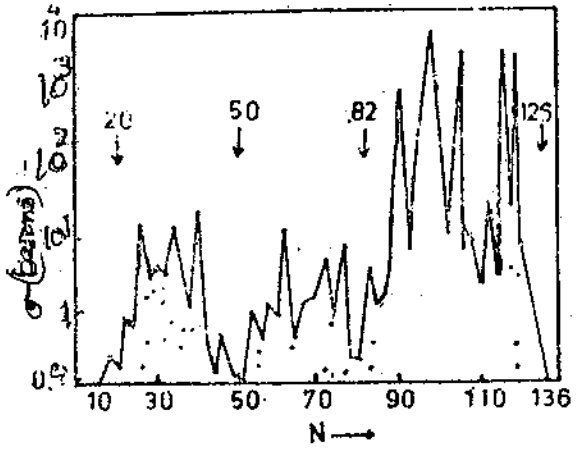


Fig.16.2 Thermal cross section σ Versus neutron number N for even A nuclei.

11. Experimental determination of masses of nuclieds in the neighbour-hood of Z=20 and 28 and N=20 and 28 indicate maxima for binding energy per nucleon for the nuclides with N or Z equal to 28 which is less marked than that for other magic numbers. The numbers 14, 28 and 40 are called semi-magic numbers.

The experimental evidneces, as detailed above, indicates the magic numbers 2,8,20,50,82 and 126. The nuclides having these nucleon numbers show extra stability and are spherically symmetrical. These doubly magic nuclides namely $_2He^4$, $_8O^{16}$, $_{20}Ca^{40}$, $_{82}Pb^{208}$ are particularly notable for their high stability. The independent particle model (also called a shell model) must explain the occurance of these magic numbers and account for the properties of the nucleus.

Assumption in the shell model

Inside the nucleus, each nucleon is supposed to move in the field produced by (A-1) nucleons. Hence each nucleon behaves as though it were moving independently in a central field which can be described by a short range potential well. This potential is also assumed to be the same for all values of e. In this central potential, each nucleon is imagined to be capable of describing an orbit of well-defined energy and angular momentum in a manner analogous to the behaviour of atomic electrons. In such a model, the inter-action between the nucleons is weak. But, this is contrary to the well demonstrated strong interaction between nucleons as seen from scattering experiments and in nuclear reactions. Weisskopf resolved this weak interaction pardox by making use of Pauli exclusion principle. Accordingly, amongest the individual nucleons within a nucleus in its ground level or a level having small excitation energy, the expected strong interaction may be present, but unable to manifest itself because all quantum states into which the nucleon might be scattered are occupied. Contrariwise, an incident nucleon can be scattered or captured into a previously unoccupied, and highly excited quantum state.

Prediction of closed shell configuration at the magic numbers

The value of the independent particle model lies in its ability to give a nearly correct energy sequence for nucleon states having different values of R. The order of the nucleon states is quite insensitive to the detailed-shaped of the potential so long as the potential decreases rapidly outside the nuclear radius. A simple rectangular well having a greater depth U and a width equal to the nuclear radius R as shown in Fig.16.3 and is a sufficiently good representation.

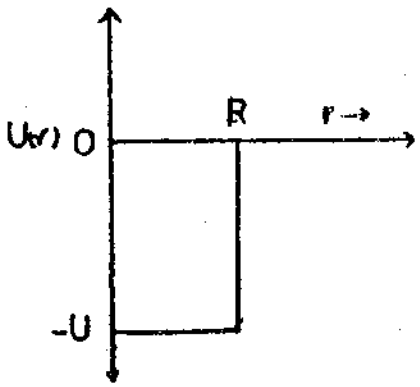


Fig.16.3 Nuclear potential U (r) versus r (Rectangular potential well)

of such a short-range force. The wave functions for independent particles with in such a potential well obey the radial wave equation.

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+K^2R-\frac{l(l+1)}{r^2}R-\frac{2m}{h^2}UR=0$$

where

$$K^2=\frac{2mE}{h^2}$$

In the above equation m represents the mass of the nucleon and E represents the energy eigen value. The wave functions are zero at r = R and r > R. The allowed energy states then correspond to the sequence of solutions of the radial which have zero values at r = R. The full sequence of energy levels in the infinitely deep potential well consists of the series.

1s,1p,1d,2s,1f,2p,1g,2d,3s,1h,2f,3p,1i,2g,3d,4s

with nearly constant spacing. The mean spacing between single particle levels is

$$\Delta E\cong\frac{h^2}{8ma^2}\cong 120A^{-2/3}MeV$$

The most important test of the individual particle model is its application to the problem of magic numbers. Let us suppose that each nucleon obeys Pauli's exclusion principle separately so that each level with an orbital number l can accommodate $2(2l+1)$ neutrons and the same number of protons. On this basis, the numbers at which the levels are filled up for each type of nucleon are as indicated below :

Order of the State	ls	lp	ld	2s	lf	2p	lg	2d	3s
Occupation number $2(2l+1)$	2	6	10	2	14	6	18	10	2
Aggregate numbers of nuclons $\sum 2(2l+1)$ Z or A-Z=N	2	8	18	20	34	40	58	68	70

The first two of the sequence of numbers are magic numbers. The

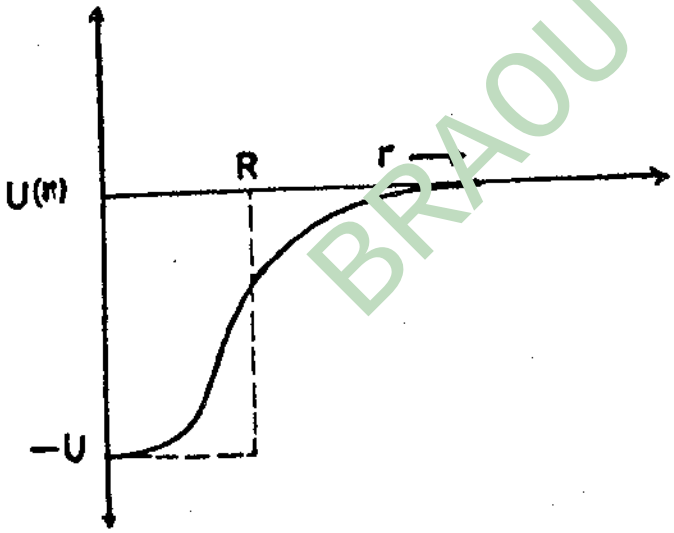


Fig.16.4 Rectangular potantial well rounded off at the edges.

sequences fail to give any indication of a closed shell at 50, or 82 or 126.

Some measure of improvement may be achieved by rounding off the edges of the rectangular potential well, as shown in Fig.16.4. Here the function U(r) rises less steeply than a step function to its value outside the well. This modification has the effect of raising the energy levels and the high l values are raised more than the low l values so that there occurs a grouping of the levels. The regrouping of levels then yields predictions about the magic numbers, as given below :

level	ls	lp	ld-2s	1f-2P	lg-2d-3s	1h-2f-3p
2(2l+1)	2	6	12	20	30	42
Total	2	8	20	40	70	112

This scheme gives the first three magic numbers correctly but the rest are wrong.

The modification necessary to make the theory fit the observed magic numbers was introduced by Mayor and by Haxel, Jenson and Suess, who pointed out that the single particle levels may be split considerly by spin-orbit interactions. If the interaction is of the type which couples the spin and orbital momenta parallel to each other, every level except 's' levels divides into two sub-states with total angular momentum.

$$i = l + \frac{1}{2} \text{ and } i = l - \frac{1}{2}$$

(1) The state with $i = l + \frac{1}{2}$ is more tightly bound than $i = l - \frac{1}{2}$ state.

(2) The magniude of the splitting increases with the value of l.

(3) Each sub-state contains (2i+1) nucleons of each type. The essembly of nucleons is as follows

Substate	$1s_{1/2}$	$1p_{3/2} - 1p_{1/2}$	$1d_{5/2} - 2s_{1/2} - 1d_{3/2}$	$1f_{7/2}$
(2i+1)	2	4+2	6+2+4	8
Total number	2	8	20	28

Substate

$$2p_{3/2} - 1f_{5/2} - 2p_{1/2} - 1g_{9/2}, 1g_{7/2} - 2d_{9/2} - 3s_{1/2} - 1h_{11/2}, 1h_{9/2} - 2f_{13/2} - 3p_{5/2} - li_{13/2}$$

(2i+1)	4+6+2+10	8+10+2+12	10+14+6+14
Total number	50	82	126

The above scheme is the basis of the present shell-model calculations and explain well the systematics of nuclear ground states. The agreement with the observed magic number shows that there is a define shell structure of nucleons within the nucleus. The sequence of energy levels for a nucleon under different potential wells and spin-orbit coupling is presented in Fig.16.5.

Nuclear shell model has been found to be successful in predicting the total angular momenta of nuclei, in understanding nuclear isomerism and in interpreting nuclear quadruple moment data.

16.4 LIQUID DROP MODEL OF THE NUCLEUS

The analogy that exists between a liquid drop and the nucleus led to the proposal of liquid drop model for the nucleus. For example, the saturation and short-range nature of nuclear forces are analogous to the properties of the forces which hold a liquid drop together. In a liquid drop the closely packed molecules interact in such a way that the liquid density is effectively constant. A nucleus may also be considered to be analogous to a drop of incompressible fluid of very high density of the order of 10^{17} kg/m^3.

In the liquid drop model, the interaction between the nucleons are asumed to be strong. Weizacker developed a semi-empirical mass formula also called as semi-empirical binding energy formula, based on the anlogue between the nucleus and liquid drop and also making use of other classical ideas such as electrostatic repulsion and surface tension. This formula was found to be extremely useful in predicting the binding energy of unstable nuclei and in evaluating the energy release in nuclear fission or α particle emission. The semi-empirical mass formula has been developed taking into account the various factors that affect the nuclear binding and weighing these factors with constants derived both from theoretical and experimental data. Let us now study the details in setting up the semi-empirical mass formula.

The mass M of a neutral atom containing Z protons and (A - Z) neutrons inside the nucleus, where A represents the mass number can be given by

$$M = ZM_H + (A - Z)M_n - B \qquad 16.1$$

In the above equation, B represents the binding energy. M_H and M_n represents the mass of hydrogen atom and mass of neutron respectively. B is made up of a number of terms each of which represents some general characteristics of the nuclei, and hence can be written as,

$$B = B_0 + B_1 + B_2 + \qquad 16.2$$

To arrive at a satisfactory description of the binding energy B, the following assumptions regarding the properties of the nucleus are made.

1. Teh nucleus is like a droplet made up of incompressible matter and all nuclei have the same density.

2. The distinction between the triplet (n-p) and singlet (n-p) force is ignored. The forces between nucleons are considered to be spin independent and charge independent. That is (n-p) = (n-n) = (p-p).

3. The nuclear forces are considered to be short range and are effective between nearest neighbours. Each nucleon is considered to interact with all its nearest neighbours.

The evaluation of various terms of the binding energy B are as follows :

(i) Volume energy. For all nuclei A > 16, the binding energy per nucleon is almost constant. This indicates that the main contribution to the binding energy of the nucleus comes from a term proportional to the mass number A; Since the volume of the nucleus is proportional to A, the energy term B_0 is called as volume energy and is given by

$$B_0 = a_v A \qquad 16.3$$

In the above equation, a_v is an arbitrary constant which is to be evaluated empirically. The volume energy is also called exchange energy in view of the exchange nature of nuclear forces.

(ii) Surface energy : The nucleons at the surface of the nucleus have fewer nearest neighbours than those nucleons existing deep inside the nuclear volume. Hence there will be a reduction in the binding energy. Since the volume energy has been calculated considering all the nucleons to be within the volume, we have to subtract from it a correction term for the nucleons which constitute the nuclear surface. This energy term is called surface energy analogous to surface tension of a liquid. The surface energy term B_1 is given by

$$B_1 = -a_s A^{\frac{2}{3}} \qquad 16.4$$

since the surface area of the nucleus is proportional to $A^{\frac{2}{3}}$. The arbitrary constant a_s, can be evaluated from empirical data.

(iii) Coulomb energy. There exists long-range coulomb repulsive force between protons. This coulomb energy tends to lower the binding energy. The coulomb energy term B_2 is given by

$$B_2 = -a_c \frac{Z^2}{A^{1/3}} \qquad 16.5$$

where

$$a_c = \frac{3e^2}{5R_o(411 \in_o)} \qquad 16.6$$

In this e, R_0 and $\in_o$ represent charge of the proton, radius of the nucleus containing single nucleon given by 1.5 x 10-15 m and $\in_o$ dielectric constant of the medium.

(iv) Asymmetry energy.

There is another contribution to the deficit in the binding energy of the nucleus which depends on the neutron excess (A - 2Z). This energy is called asymmetry energy B_3 and is proportional to $\frac{(A-2Z)^2}{A}$ It is given by

$$B_3 = \frac{-a_a(A-2Z)^2}{A} \qquad 16.7$$

where a_a is a constant which can be determined empirically.

(v) Pairing energy

It is well-known that nuclides containing even number of protons and neutrons are

more abundant and most stable where-as nuclides containing odd number of protons and neutrons are less stable. The nuclei for which either the proton or neutron number is even are intermediate in stability. This is waht is called the "odd-even" effect. These aspects fail to appear in the new terms discussed so far because we have considered the nuclear forces to be spin independent. The above odd-even effect can be represented by the pairing energy term B_4 given by

$$\begin{aligned} B_4 &= +\delta / 2A \text{ for even Z even N} \\ &\quad -\delta / 2A \text{ for odd Z odd N} \\ &= 0 \text{ for even, Z, odd N and odd Z, even N.} \end{aligned} \qquad 16.8$$

The binding energy formula is obtained by combining the terms represented by Eqns. 16.2 to 16.8 and is given by

$$B = B_0 + B_1 + B_2 + B_3 + B_4$$

$$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{\delta}{2A} \qquad 16.9$$

By substituting Eq. 16.9 in Eq. 16.1 we arrive at the following equation representing the atomic mass.

$$M(Z,A) = ZM_H + (A-Z)M_n - \frac{B}{C^2}$$

$$M(Z,A) = ZM_H + (A-Z)M_n - \frac{a_v A}{C^2} + \frac{a_s A^{\frac{2}{3}}}{C^2} + \frac{a_c Z^2}{C^2 A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \neq \frac{\delta}{2AC^2}$$

Where C represents the velocity of light. (given by $2.998 \times 10^8 ms^{-1}$).

The values of the constants can be determined by combination of theoretical calculations and adjustment to agree with experiemental values of the masses or binding energies. In principle the five constants a_v, a_s, a_c, a_a and δ can be determined from five masses or reactions and using these constants one can predict the masses of nuclides. The values of the constants, so evaluated, are

$$\begin{aligned} &a_v = 14.0 \text{ MeV}, \; a_c = 0.64 \text{ MeV} \\ &a_s = 13.1 \text{ MeV}, \; a_a = 19.4 \text{ MeV} \\ &\delta = 270 \text{ MeV forevenZ, evenN} \\ &\delta = 270 \text{ MeV foroddZ oddN} \end{aligned} \qquad 16.10$$

Using the masses of proton, neutron, hydrogen atom in terms of a.m.u say M_H = 1.008142 a.m.u, M_n = 1.008982 a.m.u and using the values of the constants given by Eq. 16.10 we can rewrite Eq. 16.9 as

$$M(Z,A) = 0.99395A - 0.00084Z + 0.0141A^{\frac{2}{3}}$$

$$+0.021\frac{(A-2Z)^2}{A} + \frac{0.00063Z^2}{A^{\frac{1}{3}}} - \frac{\delta}{A} \qquad 16.11$$

where

$\delta^{\frac{1}{2}}$ = 0.145 even Z, even N

0 for even Z, odd N, odd Z even N
-0.145 odd Z odd N

Several formulae similar to Eq. 16.11 have been developed with some what different values for the various constants depending on the amount of effort put into the determination of the constants. The semi-empirical mass or binding energy formula has been used to calculate the mass values corresponding to as many as 5000 pairs of Z, A values). The values are quite accurate, but the only draw back is that the formula does not account for all features of dependence of binding energy on mass number A and charge number Z.

Worked Example 1

Calculate from semi-empirical mass formula the energy available for alpha-decay of Po^{214}

The decay scheme of Po^{214} is

$$_{84}Po^{214} \rightarrow _{82}Pb^{210} + _2He^4 + Q$$

$$Q = \left[M\left(_{84}Po^{214}\right) - M\left(_{82}Pb^{210}\right) + \left(_2He^4\right)\right]931.48MeV$$

The atomic masses of $_{84}Po^{214}$, $_{82}Pb^{210}$ and $_2He^4$ can be calculated based on semi-empirical mass formula. Accordingly, since 1 a.m.u = 931.48 MeV we can express the mass of a nuclide in terms of a.m.u as

$$M\left(_ZX^A\right) = Z(1.0078252) + (A-Z)(1.0086654) - \frac{14.0A}{931.48}$$

$$+\frac{13.1(A)^{2/3}}{931.48} + \frac{0.64(Z)^2}{931.48A^{1/3}} + \frac{19.4(A-2Z)^2}{A(931.48)} - \frac{270}{2A(931.48)}$$

$$M\left({}_{84}Po^{214}\right) = 84(1.0078252) + (214-84)(1.0086654) - \frac{14.0\times 214}{931.48}$$

$$+\frac{13.1(214)^{2/3}}{931.48} + \frac{0.64(84)^2}{(214)^{1/3}(931.48)} + \frac{19.4(214-168)^2}{214(931.48)}$$

$$-\frac{270}{2(214)931.48} = 214.086 a.m.u$$

$$M\left({}_{82}Po^{210}\right) = 82(1.0078252) + (210-82)(1.0086654) - \frac{14.0\times 210}{931.48}$$

$$+\frac{13.1(210)^{2/3}}{931.48} + \frac{0.64(82)^2}{(210)^{1/3}(931.48)} + \frac{19.4(210-164)^2}{210(931.48)}$$

$$-\frac{270}{2(210)931.48} = 210.07791 a.m.u$$

$$M\left({}_{2}He^{4}\right) = 2(1.0078252) + 2(1.0086654) - \frac{14.4\times 4}{931.48} + \frac{13.1(4)^{2/3}}{931.48}$$

$$+\frac{0.64(2)^2}{4^{1/3}(931.48)} + \frac{1.24(1)}{4(931.48)} - \frac{270}{2(4)(931.48)} = 3.9738 a.m.u$$

$$\therefore \quad Q = \left[214.0864 - (210.07791 + 3.9738)\right]931.48$$

$$Q = \left[(0.03469)(931.48)\right] = 34.3\, MeV$$

As per semi-empirical mass formula the energy available for α decay of Po^{214} is 34.3 MeV. But, the experimental value is 7.83 MeV.

It is worthwhile to note here that the semi-empirical mass formula predicts the masses of nuclides fairly well only if A > 16. Hence the mass of $2He^4$ as predicted by semi-empirical mass formula is less than the experimental value i.e. 4.00387 a.m.u. Hence making use of this value we can write for

$$Q = \left[214.0864 - (210.07791) + 4.00387)\right]931.48\, MeV$$

$$Q = \left[214.0864 - 214.08178\right]931.48$$

$$Q = [0.00462]931.48 = 4.3 MeV$$

While semi-emipirical mass formula predicts a value of the order of 4.3 MeV for the energy available for α decay of Po^{214} the experimental value is 7.83 MeV

Worked Example 2

Calculate from semi-empirical binding energy formula, the binding energies of the last neutron in $_{82}Pb^{207}$

The binding energy of last neutron in Pb^{207} is given by binding energy of $_{82}Pb^{207}$ - binding energy of $_{82}Pb^{206}$

$\therefore$ Binding energy of $_{82}Pb^{206}$ as per semi-empirical binding energy formula is given by

$$B.E(_{82}Pb^{206} = 14(206) - 13.1(206)^{2/3} - \frac{0.64(82)^2}{(206)^{1/3}} - \frac{19.4(206-164)^2}{206} + \frac{270}{2(206)}$$

$$= 14(206) - 13.1(7.515) - 0.64(82)^2 - 19.4(42)^2 + \frac{270}{2(206)} = 1532.97 MeV$$

$$B.E(_{82}Pb^{207}) = 14(207) - 13.1(207)^{2/3} - \frac{0.64(82)^2}{(207)^{1/3}}$$

$$\frac{-19.4(207-164)^2}{207-0}$$

Since δ = 0 for odd A nuclides with even Z values

$$= 14(207) - 13.1(7.539) - \frac{0.64(82)^2}{2.746} - \frac{19.4(43)^2}{207} \Rightarrow 1538.93 MeV$$

$\therefore$ B.E. of the last neutron is Pb^{207} is (1538.93 - 1532.97) = 5.96 MeV

16.5 SUMMARY

Most of the properties of the nucleus exhibit marked discontinuties near certain even values of the proton or neutron number namely 2, 8, 20, 50, 82 and 126. These are called magic numbers.

The magic numbers of protons or neutrons have been interpreted as forming closed shells of protons and neutrons anlogous to electron shell in atoms..

The development of shell model is mainly due to the efforts of Maria Mayer and Hexel, Jensen and Suess by introducing strong spin orbit coupling, These people successfully predicted closed shell configuration corresponding to the magic numbers. Closed shells are assumed to be formed independently for portons and neutrons. There exists close analogy

between the liquid drop and the nucleus. The nature of nuclear forces binding the nucleons inside the nucleus is similar to the forces, binding the moleculues inside a liquid drop. The liquid drop model originates from this analogy. Based on the liquid drop model of the nucleus Weiszacker developed semiempirical mass or binding energy formula. The semi-empirical mass formula is highly successful in predicting accurately the masses of nuclides.

16.6 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Discuss how the shell model of the nucleus accounts for the shell structure at the magic number values for neutron and proton numbers.
2. Discuss the nuclear propertieis that support the importance of magic numbers for nucleous inside the nucleus.
3. Discuss how the shell model of the nucleus accounts for the shell structure at the magic numbers values for neutron and proton numbers.
4. Derive the semi-empirical mass formula based on the liquid drop model of the nucleus. Discuss the significance of this formula.

II. Answer the following questions briefly.

1. Mass defect and binding energy
2. Magic numbers

III. Solve the following problems

1. Estimate the mass of $_{87}Pb^{208}$ and $_{83}Bi^{209}$ based on semi-empirical mass formula.
2. Calculate from semi-empirical binding energy formula the binding energy of last neutron in Pb^{208}

(Ans: 7.03 MeV)

UNIT–17 : NUCLEAR FISSION AND FUSION

Contents

17.1 AIMS AND OBJECTIVES

This unit explains the phenomeno. of nuclear fission

After going through this unit

- You will be able to discuss the role of nuclear fission in the release of energy.
- You will be able to list out the conditions under which nuclear fission process can be controlled in a nuclear reactor.
- You will be able to explain the various fusion reactions taking place in the sun the stars.

17.2 INTRODUCTION

The application of Einstein's mass energy relation to nuclear reactions led to the conclusion that nucleus is a seat of power. The loss of mass when a nuclear reaction takes place gets converted into energy which is released in the form of heat, gamma radiation, kinetic energy of particles and excitation energy of product nucleus. The two reactions, nuclear fission and fusion are special types of nuclear reactions. When energy release is extremely high. Unless these reactions are controlled the (I) Processes proceed continuously leading to the release of tremendous amount of energy that would lead to the desbiluction of matter, material and culture. In this lesson we shall study how these nuclear processes have been discovered and how they can be controlled for useful proposes mentioned.

17.3 DISCOVERY OF NUCLEAR FISSION

Experimental investigations on the absorption of slow neutrons by uranium by Fermi and others indicated that transuraric elements could be produced, by beta decay of the nuclide U^{239} formed absorbing the neutron

$$_{92}U^{238} +_{o} n^{1} \rightarrow_{92} U^{239} + (\text{gamma rays}) \qquad 17.1$$

$$_{92}U^{239} \rightarrow {}_{-1}e^{o} + {}_{93}Np^{239} \qquad T = 23 \text{ minutes}$$

$$_{93}Np^{239} \rightarrow {}_{-1}e^{o} + {}_{94}Pu^{239} \qquad T = 2.3 \text{ days}$$

$_{94}Pu^{239}$ is an alpha-emitter with a half life of 25000 yrs. Starting from the reaction given by Eq.17.1 the β-decay processes leading to Np^{239} and Pu^{239} do take place but experimental data also indicated the occurrence of many other beta activities. By analysing the experimental data, in the year 1939, O. Hahn and F. Strassmann concluded that alkaline earth metals were produced in the irradiation of uranium by neutrons. These alkaline earth metals are beta active. Some of the material with intense beta activity was found to be associated with the element barium, as indicated below.

$$Ba \xrightarrow{\beta^-} La \xrightarrow{\beta^-} Ce \qquad 17.2$$

In the year 1939, to explain the observations made by O. Hahn and F. Strassmann, Frish and Meitner introduced the concept of fission as a process where-by the nuclei of heavy elements split into much smaller fragments. The credit for the discovery of nuclear fission goes to O.Hahn and F. Strassmann. It has been observed that the odd mass numbered nuclides U^{233}, U^{235}, Pu^{239} and so forth to undergo fission by slow neutrons where as the even mass numbered nuclides Th^{232}, U^{238} and so on to undergo fission by fast neutrons.

17.4 NUCLEAR FISSION REACTION

The fission reaction of U^{235} when it absorbs a slow neutron is given below.

$$_{o}n^{1} + {}_{92}U^{235} \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{o}n^{1} + Q$$

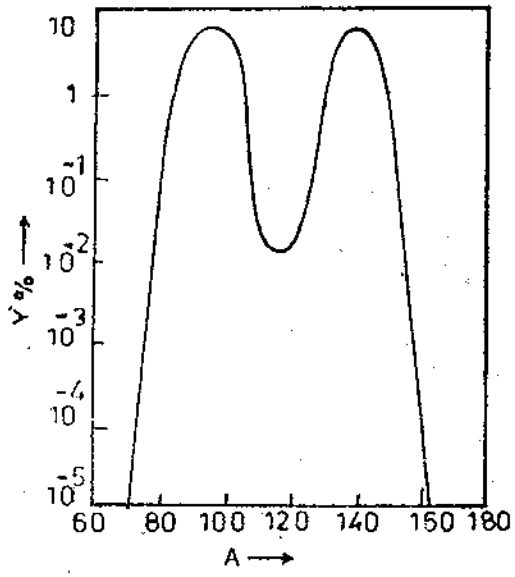


Fig.17.1 Fission mass yield in % (Y) versus-mass number A of fission products plotted on a semi-log scale for the fission of U^{235} bombarded by slow neutrons.

Q is the energy released in the fission process.

$_{56}Ba^{141}$ and $_{36}Kr^{92}$ are unstable and become stable either by beta emission or emission of one or more excess neutrons. The neutrons emitted after measurable time after the fission process are called delayed neutrons. These delayed neutrons play an important role in the control of nuclear reactors. The decay scheme of Ba and Kr is as follows :

$$_{56}Ba^{141} \xrightarrow{\beta^-} {}_{57}La^{141} \xrightarrow{\beta^-} {}_{58}Ce^{141} \xrightarrow{\beta^-} {}_{59}Pr^{141}$$

18 min. 3.7 hr. 28 days

$$_{36}Kr^{92} \xrightarrow{\beta^-} {}_{37}Rb^{92} \xrightarrow{\beta^-} {}_{38}Sr^{92} \xrightarrow{92} {}_{39}Y^{92} \xrightarrow{92} {}_{40}Zr^{92}$$

2.4s 2.7 hr. 3.5 hr.

When uranium nucleus undergoes fission it, splits up in many ways. The fission yield curve for U^{235} is shown in Fig.17.1. The fission products have mass number values, ranging from 72 to 160. About 97% of U^{235} nuclei undergoing fission yield products which fall into two groups, a light group with mass numbers from 85 to 104 and a heavy group with mass numbers from 130 to 149. The most probable type of fission, which occurs in about 7% of the total, gives products with mass numbers 95 and 139. The reactions corresponding to this type is given by

$$_{o}n^{1} + {}_{92}U^{235} \rightarrow {}_{92}U^{236} \rightarrow {}_{57}La^{139} + {}_{42}Mo^{95} + 2\,{}_{o}n^{1} + Q$$

The fission yield curve is symmetrical around the mass number A = 117. For U^{235} and Pu^{239} the general shape of the fission yield curve is similar to that of U^{235}. The fission yield curves will be a symmetrical for reactions initiated by fast neutrons and other particles like alpha particles and deutrons.

17.5 EXPLANATION OF THE NUCLEAR FISSION PROCESS, BASED ON LIQUID DROP MODEL OF THE NUCLEUS.

N. Bohr and J.A. Wheeler developed the theory of nuclear fission based on liquid drop model of the nucleus. In the case of a liquid drop the surface tension forces keep it in a stable from. The nuclear forces keep the nucleus in a stable state. For a liquid drop to be broken up into small drops or for a nuclues to undergo fission there must be considerable distortion which will be possible only if additional energy is available.

When a neutron is abosrbed by a heavy nucleus a compound nucleus is formed. The energy gained by the heavy nucleus is equal to the binding energy of the additional neutron plus the kinetic energy of the incident neutron. Because of this excess energy, there exists a distinct probability for the compound nucleus like liquid drop to undergo a varity of strong oscillations. These oscillations will distort the shape of the nucleus and may become ellipsoidal in nature (B) as shown Fig.17.2. If the gain in energy accpompanied by the absorption of the neutron is insufficient to cause further

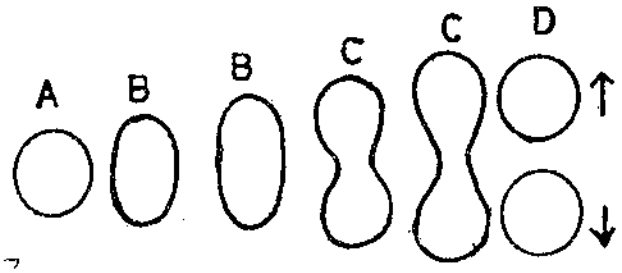


Fig. 17.2 Sequence of steps that occur in the process of nuclear fission based on liquid-drop model of the nucleus.

deformation beyond B, intranuclear forces will compel the nucleus to return to its orginal spherical form. The excess energy will be released in the form of gamma radiation or some particle. If the excitation energy is sufficiently large, the drop attains the shape of a dumbbell (C). Now the restoratation of the state A is very much improbable since the electrostatic repulsion between the positive charges on the two ends of C cannot be overcome by the relatively small nuclear binding force operating in the constricted region Consequently the system passes rapidly to D representing fission into two seperate nuclei which are propelled in opposite directions. The series of changes take place only if it is accompanied by a net decrease of mass.

According to liquid drop theory of nuclear fission, the most probable mode of division of the liquid drop nucleus is into two equal agments. But experimental observation is asymmetry associated with distribution of mass of the fragments. The observed assymetry can be accounted based on shell structure of the nucleus. The nucleues may be considered to consist of a number of layers of nucleons. On the a critical deformation of the nuclear surface is attained, the outer nuclear shells begin to break up in a symmetrical manner. The tightly bound inner core, however, does not break up and it ultimately moves off with the rest of the nuclues bound to the inner core. The net resul is, therefore, the production of two fission fragments with different masses.

17.6 DIFFERENT TYPES OF FISSION REACTIONS

Nuclear fission can take place not only with neutrons but also with other particles like high energetic photons, protons etc.

(a) Photo fission :- High energetic photons will induce fission reactions in ${}_{90}Th^{230}$, ${}_{92}U^{233}$, ${}_{92}U^{235}$, ${}_{92}U^{238}$, and ${}_{94}Pu^{239}$

(b) Fision of light nuclei :- Bombardment of Cu^{63} by protons will lead to fission reactions.

$$ {}_{29}Cu^{63} + {}_{1}H^{1} \rightarrow {}_{30}Zn^{64} \rightarrow {}_{17}Cl^{38} + {}_{13}Al^{25} + {}_{o}n^{1} + Q_1 \qquad 17.3 $$

$$ \rightarrow {}_{17}Cl^{38} + p + n + 6\,{}_{2}He^{4} + Q_2 \qquad 17.4 $$

$$ \rightarrow {}_{11}Na^{24} + {}_{19}K^{39} + {}_{o}n^{1} + Q_3 \qquad 17.5 $$

$$\rightarrow_{11} Na^{24} +_1 H^1 + 3_o n^1 + 9_2 He^4 + Q_4 \qquad 17.6$$

Reactions 17.4 and 17.6 are called spallation reactions in view of release of a large number of particles.

(c) Ternary fission

Some-times, three fragments of intermediate mass may be released in a nuclear fission. The type of fission is called ternary fission in a nuclear fission. $_{92}U^{235}$ bombarded by slow neutrons may give rise to 4.3 ternary fissions for every 10^6 binary fissions.

(d) Spontaneous fission.

Heavy nuclie with Z = 90, A = 230 to Z = 95, A = 241 undergo spontaneous fissions.

17.7 ENERGY RELEASE IN NUCLEAR FISSION REACTION

Consider the nuclear fission reaction.

$$_o n^1 +_{92} U^{235} \rightarrow_{92} U^{236} \rightarrow_{57} La^{139} +_{42} Mo^{95} + 2_o n^1 + Q$$

The atomic masses are

$$_o n^1 = 1.009 a.m.u$$

$$_{92} U^{235} = 235.118 a.m.u.$$

$$_{57} La^{139} = 138.950 a.m.u.$$

$$_{42} Mo^{95} = 94.936 a.m.u.$$

The energy release in the fission process Q can be evaluated by determining the mass defect ΔM.

(1) force between them

(2) at the cost of

$$\Delta M = \left[(1.009 + 235.118) - (138.950 + 94.936) + 2 \times 1.009\right] = 0.223 a.m.u.$$

since 1 a.m.u. = 931.48 MeV, the energy released due to the mass defect 0.233 a.m.u. associated with the fission reaction is given by

$$Q = \Delta M \times 931.48 = 208 MeV$$

There are about 30 different ways in which the nucleus can be devided and the mass defect associated with each process would be almost the same. Hence on an average, the energy release per single fission of U^{235} can be taken to be equal to 200 MeV. It is distributed among the products of the fission reaction as detailed below:

Kinetic energy of fission fragments	165 MeV
Kinetic energy of neutrons	5 MeV
Energy of instantaneous gamma-rays	8 MeV
Energy of β-decay in fission fragments	5 MeV
Energy of gamma decay in fission fragments	6 MeV
Energy of neutrons	11 MeV

17.8 NUCLEAR FUSION

A nuclear reaction in which a heavier element is built up from fusion of light elements is called nuclear fusion. The mass of the element formed in the fusion reaction is less than the sum of the mass of the light elements taking part in the fusion reaction. As a result, in fusion reactions energy is released. Fusion reactions take place at very high temperatures of the order of 10^8C. Hence fusion reactions are also called as thermonuclear reactions. At very high temperatures the atoms lose their electrons which continue to move freely among the naked nuclei. For fusion to occur, the nuclei have to be brought very close to each other. Since these nuclei are positively charged great speed is required to overcome the repulsive force between them. The kinetic energy required for nuclei of the isotopes of hydrogen to take part in fusion proces is of the order of 0.1 MeV. To attain this energy, the temperature required is 10^8 to 10^9C. This temperature is higher than that existing in the sun and the stars. However Maxwelian distribution of energies can account for a few nuclei to attain this energy even at temperatures of the order of 10^7 °c, the usual temperature of Sun and Stars. At 20 x 10^7 °c the average energy possessed by nuclei is 0.002 MeV but a few nuclei may have energy of the order of 0.1 MeV. It is these nuclie which take part in the fusion process.

Worked Example - 1

Calculate the energy released when four hydrogen atoms combine to form a Helium atom. The atomic mass by hydrogen atom is 1.00814 a.m.u. and Helium atom is 4.00387 a.m.u.

The reaction $_1H^1 +_1 H^1 +_1 H^1 +_1 H^1 \rightarrow_2 He^4 + Q$

Atomic mass of 4 hydrogen atoms = 1.00814 x 4 = 4.03256 a.m.u.

Atomic mass of Helium atom = 4.00387 a.m.u.

The mass defect = Δ M = 0.02869 a.m.u.

The energy release Q = ΔM x 931.4. MeV = 0.02869 x 931.48 MeV = 26.7 MeV

Among the various thermonuclear reactions possible, one of the best known is that of fusion of the nuclei of hydrogen, deutrium and tritium isotopes into the nucleus of helium., H.A Bethe suggested two sets of reactions to account for the source of energy of the suns and the stars.

These are

1. Carbon - Nitrogen cycle. Here Carbon acts as a catalyst in the fusion of 4 protons to form helium nucleus.

2. Proton-Proton chain. Here the first step is a combination of 2 protons into deuteron

The carbon - Nitrogen cycle take place as detailed below;

$$_6C^{12} +_1 H^1 \rightarrow_7 N^{13} + \gamma + Q_1$$

$_7N^{13}$ is radioactive with half life of 10.1 min. It decays as

$$_7N^{13} \rightarrow_6 C^{13} +_1 e^o + \nu$$

$+_1 e^o$ and ν are beta particle and neutrino respectively

$$_6C^{13} +_1 H^1 \rightarrow_7 N^{14} + \gamma + Q_2$$

$$_7N^{14} +_1 H^1 \rightarrow_8 O^{15} + \gamma + Q_3$$

$_8O^{15}$ is radioactive with half-life of 2.05 min. It decays as

$$_8O^{15} \rightarrow_7 N^{15} +_1 e^o + \nu$$

$$_7N^{15} +_1 H^1 \rightarrow_6 C^{12} +_2 He^4 + Q_4$$

Hence effective reaction is

$$4_1H^1 \rightarrow_2 He^4 + 2_+e^o + 2\nu + 2\gamma + Q$$

The proton - proton chain takes place as detailed below

$$_1H^1 +_1 H^1 \rightarrow_1 H^2 +_{+1} e^o + \nu + Q_1 \qquad 17.7$$

$$_1H^2 +_1 H^1 \rightarrow_2 He^3 + \gamma + Q_2 \qquad 17.8$$

$$_2He^3 +_2 He^3 \rightarrow_2 He^4 + 2_1H^1 + Q_3 \qquad 17.9$$

Effective reaction

$$4_1H^1 \rightarrow_2 He^4 + 2v + 2_{+1}e^o + 2\gamma + Q \qquad 17.10$$

Another proton proton cliam reaction takes place as detailed below

$$_1H^1 +_1 H^1 \rightarrow_1 H^2 +_1 e^o + v \qquad 17.11$$

$$_1H^1 +_1 H^2 \rightarrow_2 He^3 + \gamma \qquad 17.12$$

$$_2He^3 +_2 He^4 \rightarrow_4 Be^7 + \gamma \qquad 17.13$$

$$_4Be^7 +_{-1} e^o \rightarrow_3 Li^7 + v + \gamma \qquad 17.14$$

$$_3Li^7 +_1 H^1 \rightarrow_2 He^4 +_2 He^4 \qquad 17.15$$

Also

$$_4Be^7 +_1 H^1 \rightarrow_5 B^8 + \gamma \qquad 17.16$$

$$_5B^8 \rightarrow_4 Be^8 +_{+1} e^o + v \qquad 17.17$$

$$_4Be^8 \rightarrow_2 He^4 +_2 He^4 \qquad 17.18$$

It is thought that the proton-proton chain given by Eqs. 17.7 to 17.10 is important at lower temperatures i.e. when the sun was formed. The proton-proton chain given by Eqs. 17.11 to 17.18 is important at high temperatures, that is, in the present state of the sun.

The relative probabilities of the occurence of the carbon-nitrogen cycle and proton-proton chain depend on the temperature. At low temperatures, the proton - proton chain predominates. As the temperature is raised, the carbon-nitrogen cycle rapidly becomes of increasing significance. In the sun and the similar stars with interior temperatures of the order of 20 millian °C, the two processes take place at equal rates.

17.9 UTILIZATION OF ENERGY RELASED IN NUCLEAR FISSION AND FUSION

As per the discussion persented in section 17.6 when a Uranium nucleus undergoes fission, an amount of energy of the order of 200 MeV is released. In addition, 2 or 3 neutrons are also released. These neutrons take part in inducing further fission of uranium nuclei and this process builds up at a rapid rate. This type of growth of the fission process as illustrated in Fig.17.3 is called the chain reaction. Fig.17.4 shows the growth curve of fission in pure U^{235}. Tremandous amount of energy is released in this fission process. This released energy can be

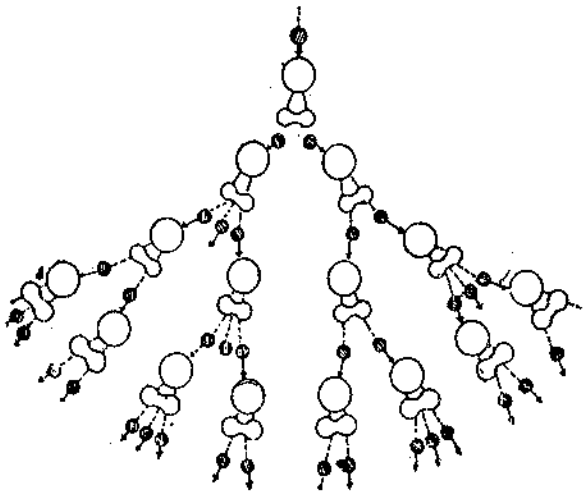


Fig. 17.3 Chain reactions in the fission atomic nuclei of heavy elements like U^{235}, Pu^{239} etc.
O - nucleus before fission ∫ shape of the nucleons of the instant fission.

used both for constructive and destructive purposes. The fission of one U^{235} liberates 200 MeV energy which is equal to 200 x 1.6 x 10^{-6} ergs. When one gram atom of P^{235} undergoes fission the energy released is given by 200 x 1.6 x 10^{-6} x 6.02 x 10^{23} = 1.93 x 10^{20} ergs. The energy released when 1 kg of U^{335} undergoes fission is equal to 821 x 10^{20} ergs or about 2 x 10^{10} Kcal. This energy is equal to the energy released in the explosion of 20,000 tons of TNT.

The energy released in the fission process can be expressed in terms of power units also. Since 1 MeV = 1.6 x 10^{-6} ergs 1.6 x 10^{-13} watt sec the fission of one U^{235} nucleus give out 200 x 1.6 x 10^{-13} watt-sec energy. Hence to obtain 1 watt of power, the number of fissions that should take place are 3.1 x 10^{10} per sec. The energy release in units of power when 1 gram of U^{235} undergoes fission is 8.2 x 10^{10} watt-sec or 2.3 x 10^{4} K watt-hour or nearly 1 M watt-day. If the energy release is spread over a day the complete fission of 1 kg of U^{235} will produce energy in the form of heat at the rate of 1000 M watts. If this heat is converted into electricity

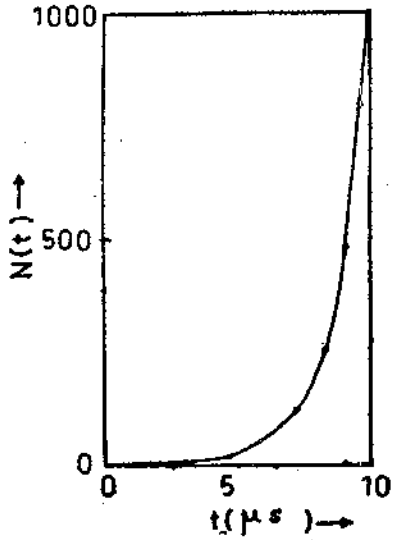


Fig. 17.4 Growth curve of fission U^{235}
N(t) - Number of atoms split at any instant of time.

at conversion efficiey of 30 percent then the electrical energy can be obtained at a rate of 300 M watts. This out-put is equivalent to the energy out-put from a plant which consumes 2500 tons of coal. It is this aspect that led to the development of nuclear power plants. In atomic bomb which contains U^{235} or Pu^{239} the chain reaction is an uncontrolled chain reaction and in a nuclear reactor, the chain reaction is controlled one. In section 17.10 we discuss in detail a nuclear reactor.

The Hydrogen bomb works on fusion reactions, which are uncontrollable ones.

The thermonuclear reactions can also be controlled and attempts all over the world are in progress to achieve controlled thermonuclear reactions at workable costs to use the fusion energy for constructive purposes.

17.10 NUCLEAR REACTORS

A system in which the fissionable and nonfissionable materials are so arranged that the chain reaction can proceed at a controllable rate is called nuclear reactor. The different functionaries of a nuclear reactor are discussed below.

(i) The criticality of the reactor determines whether a chain reaction will continue at a steady state, increase or decrease. The effective multiplication factor K_c is given by

$$K_c = \frac{P}{A+L}$$

Where P, A and L represent rate of production of neutrons, rate of absorption of neutrons and rate of leakage of neutrons respectively. If K_c=1 the reactor is said to be critical, if $K_c < 1$ the reactor is said to be sub critical and if $K_c > 1$ the reactor is said be supercritical. If F represents the rate of fission process and 'n' the average number of neutrons emitted per fission then P is given by nF Hence

$$K_c = \frac{nF}{A+L} = \frac{nF}{A}\left(\frac{1}{1+L/A}\right) \qquad 17.19$$

F/A depends upon the amount of fissionable and non-fissionable material and their cross-section for fission and neutron capture. L/A depends on the stability of the reactor to contain and absorb neutrons before they can escape through the surface.

(ii) **Size of the reactor** Case 1. If the size of the reactor is small then L increases and A decreases. This results in L/A increase. Hence as per Eq. 17.19 $K_c \to 0$. Case2:- If the size of reactor is increased, L decreases and A increases. This

results in $K_c \to \frac{F}{A}$

Case 3 : If the size is chosen such that, say $K_c = 1$ that is $\frac{F}{A} = 1$ then the size of the

reactor is called the critical size. The mass of the fuel used in the nuclear reactor to obtain critically using a critical size is known as the critical mass of the fuel. This fission processes for different values of K_c are illustrated it Fig.17.5.

(iii) **Types of nuclear reactors** : Depending upon the manner in which the fuel and moderator are mixed, reactors are classified. A homogeneous reactor is one in which the fuel and moderator form a mixture which has uniform composition. A common type of a homogeneous reactor consists of a solution of Uranyl nitrate in water with the enriched uarnium. The solution is put in a steel sphere and this is surround by a neutron reflector such as beryllium oxide and graphite. The entire reactor is shielded by lead, cadmium and conrete.

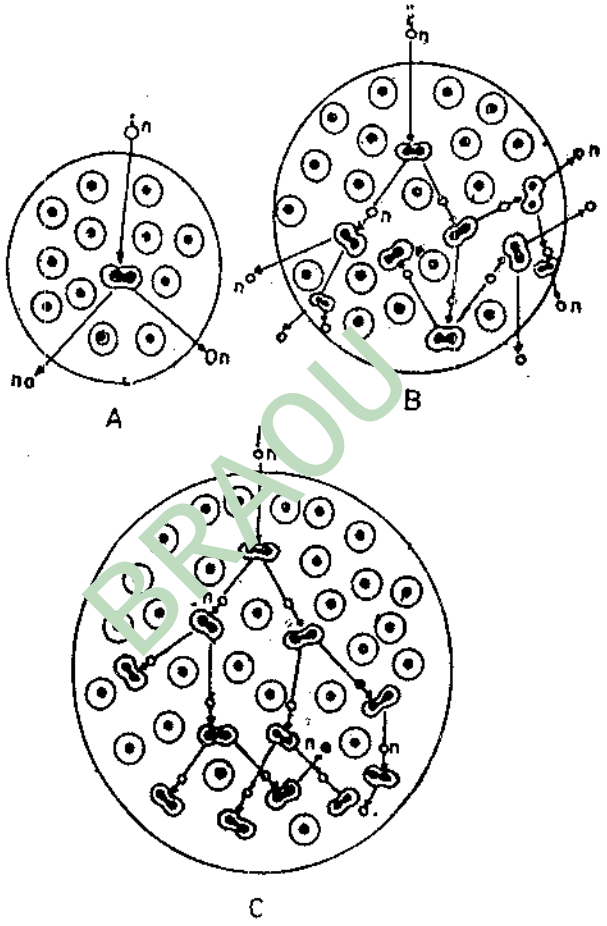


Fig.17.5 (A)-subcritical mass $K_c < 1$. Fission starts but no chain reaction develops, (B)-Critical mass $K_c = 1$ chain reaction develops steadily, (C)-critical mass $K_c > 1$, chain reaction grows in geometeriali progression.

The operation of the reactor can be controlled by means of cadmium rods which penetrate the beryllium oxide reflector.

Most of the nuclear reactors are of the heterogeneous type. Here the fissionable material is concentrated in containers suitably distributed throughout the moderator. In many cases, the fissionable materials in the form of cylinders of uranium or uranium oxide enclosed in aluminium tubes are spaced in a lattice net-work in the moderator.

The reactors are also classified based on the energy of neutrons used in the fission process. If fast neutrons are used for fission, the reactor is called is a fast reactor. If the

neutrons used are of intermediate energy the reactors are called moderate reactors. If slow neutrons are used in the fission process in a nuclear reactor, such reactors are called thermal reactors.

(iv) **Fuel employed in nuclear reactors**. 1) Natural uranium containing 0.72% of U^{235}
2) Enriched uranium containing more than 0.72% of U^{235}
3) Pu^{239}
4) U^{235}

(v) **Moderators employed in reactors** : A chain reaction in natural uranium can be kept up continuously in a Steady state if the fast neutrons ejected during the fission of U^{235} can be slowed down, which will cause further fission of U^{235} atoms, more quickly than they are absorbed by U^{235}. The material employed to slow down the fast neutrons rapidly, is called the moderator and the process is called thermalization. A moderator should not absorb the impinging large number of neutrons. It must slow down the fast neutrons quickly and scatter them to cause further fission. Thus, the absorption cross-section of the moderator for neutrons should be small. Heavy water is the best liquid moderator. It is also usable as a coolent and shield but is quite expensive. Ordinary water can be used as a moderator when enriched uranium is used as fuel. Beryllium, beryllium oxide and graphite are used when solid moderator is required.

(vi) **Reflectors** : Fission probability can be increased by scattering the neutrons back into the reactor prior to leakage. For this purpose, materials like graphite, nickel, tin and lead can be used as reflectors which will have large scattering cross sections.

(vii) **Reactor coolents** : Reactors operate at high power level and the heat developed in the reactor core should be removed by some form of continuous cooling. The reactor coolent should be a good conductor of heat, non corrosive to the materials in the reactor and stable to radiations to which it is exposed and have low capture cross-sections for thermal neutrons. Coolents with high coefficients of heat transfer are desired for power producing reactors, operating at high temperatures since the power output depends on the rate of removal of heat. The rate of heat extraction is much greater for liquid coolents and the thermal efficiency is higher for gas coolents. Helium is the best gas coolent, but is expensive. Carbondioxide can be employed satisfactorily. Liquid metals such as sodium, potassium mercury, lead and bismuth may be used. Heavy water and ordinary water may also be used. Alloy of sodium and potassium is the best liquid coolent.

(viii) **Structural materials** : The choice of the structural materials in a reactor is limited by their absorption of neutrons. They should not disintegrate under the heavy neutrons and gamma radiations present. The structural materials that can be used for thermal reactors are beryllium, graphite, aluminium and zirconium. For the fast reactors, chromium, nickel, and molybdenum may be used as structural materials.

(ix) **Reactor Shielding** : The main radiations from a nuclear reactor that must be shielded are prompt neutrons, delayed neutrons, alpha particles, gamma radiations etc. The material, used for shielding, depends on the type of application of the reactor. For a power plant, concrete is usually employed as, shielding material.

(x) **Control of the reactor** : There are two types of controlling methods, namely (1) Automatic control and (2) mechanical control. A homogenous reactor can be controlled by increasing the level of the moderator. A heterogeneous reactor can be controlled by inserting

materials such as boron or cadmium. There will be three types of control rods present in the reactor, namely, (1) Shim (2) fine control and (3) safety rods. As operation continues, small changes in temperature, hence small changes in reactivity is required to keep the reactor at steady state power. This can be accomplished by automatic operation of the control rods. During normal operation most of the excess reactivity is aborbed.

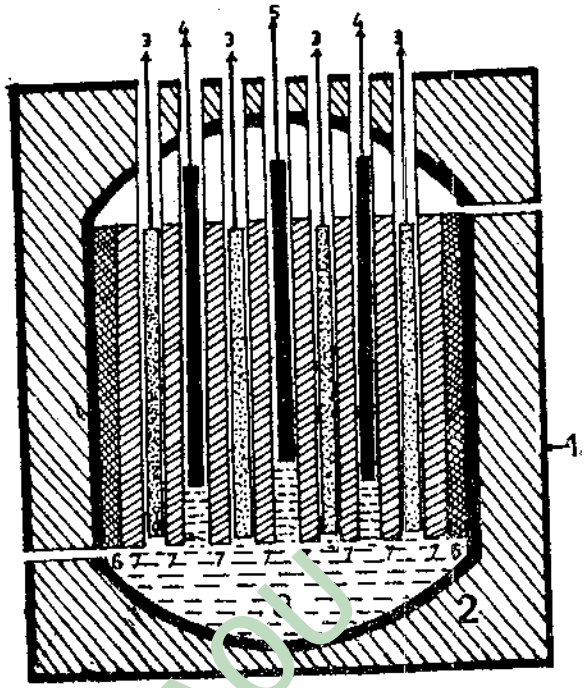


Fig. 17.6 Structure of nuclear reactor

1. Container of a reactor
2. Biological Protective Shield (Concrete)
3. Nuclear fuel (U^{235}, U^{238} Pu^{239})
4. Control rods (Cadmium, boron)
5. Safety rod (neutron absorber)
6. Neutron reflector (beryllium, graphite)
7. Moderator (heavy water, beryllium and graphite)
8. Cooling system (water).

in shim rods. The accumulated fission products will act as poison to the nuclear chain reaction if they are not removed at regular intervals and it is necessary to pull out the control rods gradually to keep constant reactor power. Under emergency cases the control rods are immersed to their maximum position, there-by reducing the multiplication factor well below unity and the chain reaction dies down.

The main parts of a typical nuclear reactor are schematically shown in Fig.17.6 and it is self explanatory.

(xi) **Applications of nuclear reactors** : The nuclear reactors are used for (a) research purpose (b) production of fissile material and (c) power production. Nuclear reactors are used in the production of radio isotopes. The chemical elements are inserted at varying depths in the special channels of the protective shield of the reactor. These are then subjected to bombardment of intense neutron beams and radioactive isotopes are produced. The radioactive isotopes find wide application in medicine, agriculture and industry. The radiations from the reactor are used to conduct research on the sturcture of matter

and to study the influence of radiations on physical, chemical and biological characteristics of materiais. Nuclear reactors can be employed to conduct reasearch on fuel elements for new reactors.

Worked Example - 2

The thermal nuclear reactor containing a mixture of U^{235} and U^{238}, operates at a power level of 1000 megawatts. Find the rate at which U^{235} is consumed by fission.

Energy released per fission of U^{235} = 200 MeV since 1 MeV = 1.6 x 10^{-6} erg. = 1.6 x 10^{-13} watt sec. Hence energy release, in units of power when 1 atom of U^{235} undergoes fission = 200 x 1.6 x 10^{-13} watt sec. The power released when 1 gram of uranium undergoes fission.

$$= \frac{200 \times 1.6 \times 10^{-13} \times 6.022 \times 10^{23} \times 1gm.}{235gm}$$

= 200 x 4.1 x 10^8 watt sec.

= 4.1 x 10^8 x 200 watt sec.

$$= \frac{8.2 \times 10^{10}}{60 \times 60 \times 24} = \frac{8.2 \times 10^8}{864}$$

= 0.949 x 10^6 watt-day.

To obtain a power of 1000 M watts, the amount of U^{235} which should undergo fission is given by

$$= \frac{1g}{0.949 \times 10^6} \times \frac{1000 \times 10^6 watt - day}{watt - day} = 1.054 \times 10^3 gm$$

$\therefore$ 1.054 Kg of U^{235} per day should undergo fission to obtain a power of 1000 Megawatts-day.

Worked Eample - 3

In a controlled thermonuclear reaction of the type

$$_1H^2 +_1 H^2 \rightarrow_1 H^3 +_1 H^1 + Q_1$$

$$_1H^2 +_1 H^3 \rightarrow_2 He^4 +_o n^1 + Q_2$$

1 gram of deuterium is spent. Evaluate the energy release given the atomic masses of $_1H^1$ = 1.00814 $_1H^2$ = 2.0147 a.m.u. $_0n^1$ = 1.0086654 a.m.u.

$_1H^3$ = 3.016997 a.m.u. and $_2He^4$ = 4.003873 a.m.u.

In the reactions

$$_1H^2 +_1 H^2 \rightarrow_1 H^3 +_1 H^1 + Q_1$$

Q_1 = [2(2.01474)-(3.016997 + 1.00814)] 931.48 MeV

Q_1 = (4.02948) - (4.025137) 931.48

Q_1 = (0.004343) 931.48 = 4.045 MeV.

In the reaction

$$_1H^2 +_1 H^3 \rightarrow_2 He^4 +_o n^1 + Q_2$$

Q_2 = [(2.01474+3.016997) - (4.003873 + 1.0086654)] 931.48 MeV.

= (5.031737 - 5.0125384) 931.48 MeV.

(0.0191986) 931.48 = 17.88 MeV.

Total energy release in the process = $Q_1 + Q_2$ = 21.93 MeV. No. of deuterium atoms undergoing fusion process = 3. If 1 gm deuterium is spent in the fusion process the energy release.

$$= \frac{21.93}{3} \times \frac{6.02 \times 10^{23}}{2} = 22.0031 \times 10^{23} MeV$$

In terms of power $= \frac{22.0031 \times 10^{23} \times 1.6 \times 10^{-13} wastt \sec.}{60 \times 60 \times 24}$

= 4.07 Mega watt-day.

Worked Example - 4

Assuming that a heavy nucleus like $_{92}U^{238}$ undergoes symmetric fission, evaluate the energy released in the fission process based on semi-empirical mass formula.

According to semiempirical mass formula, the energy Q released in the fission process is given by

Q = M (Z,A) - 2M(Z/2, A/2)

since symmetric fission indicates that the fragments are of equal change and equal mass.

We can write for Q as

$$Q = ZM_H + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}}$$

$$+\frac{a_a(A-2Z)^2}{A} - 2\left[\frac{ZM_H}{2} + \left(\frac{A}{2} - \frac{Z}{2}\right)M_n - a_v\frac{A}{2}\right.$$

$$\left. + a_s(A/2)^{2/3} + a_c\frac{(Z/2)^2}{(A/2)^{1/3}} + a_a\frac{A/2 - 2Z/2\ )^2}{A}\right]$$

neglecting the small pairing energy term for odd odd, even even nuclei involved.

$$Q = a_s A^{2/3} + a_c\frac{Z^2}{A^{1/3}} - 2a_s(A/2)^{2/3} - 2a_c\frac{(x/2)^2}{\left(\frac{4}{2}\right)^{1/3}}$$

$$Q = a_s A^{2/3}\left[1 - Z^{1/3}\right] + a_c\frac{Z^2}{A^{1/3}}\left[1 - \frac{1}{2^{2/3}}\right]$$

$$Q = a_s A^{2/3}(0.260) + 0.370 a_c\frac{Z^2}{A^{1/3}}$$

Using a_s = 13.1 MeV, and a_c = 0.64 MeV.

$$Q = -(13.1)0.260A^{2/3} + (0.64)(0.37)\frac{Z^2}{A^{2/3}}MeV$$

$$Q = -3.406A^{2/3} + 0.2368\frac{Z^2}{A^{1/3}}$$

$$Q = -3.406(238)^{2/3} + \frac{0.2368(92)^2}{(238)^{1/3}} = -130.8 + 323.4$$

= 192.6 MeV.

17.11 SUMMARY

In nuclear fission, a heavy nucleus like U^{235} undergoes disintegration due to absorption of a neutron into two fragments of nearly equal mass with release of 2 or 3 neutrons and energy of about 200 MeV. The phenomenon of nuclear fission was discovered by O.Hahn and F. Strassmann. The fission products have mass numbers ranging from 72 to 160. About 97% of U^{235} nuclei undergoing fission yeild products which fall into two groups namely a light group with mass numbers ranging from 85 to 104 and a heavy group with mass numbers from 130 to 149. The nuclear fission reaction is explained based on liquid drop model of the nucleus. In nuclear fission, light elements combine to form a heavy element with the release of energy. Fusion reactions take place at very high temperatures of the order of 10^{80}C. These reactions are called thermonuclear reactions.The energy release from the sun and stars is explained based on the fusion reactions specially called proton-proton chain reactions and carbon-nitrogen cycle reactions. The energy release in a nuclear fusion reaction or fission reaction can be used both for constructive and destructive purposes.

The neutrons produced in the fission process further initiate fission and the reaction builds up at a geometric rate. This is as termed chain reaction. In an atomic bomb explosion, the chain reaction is uncontrollable. In hydrogen bomb explosion, the fusion reactions are also uncontrolliable. In a nuclear reactor the chain reaction is controlled. The energy release can be used for production of power and for other useful purposes.

In a nuclear reactor if the multiplication factor K_c is 1, then chain reaction continues at a steady state. The reactor is said to be critical if $K_c = 1$, subcritical if $K_c < 1$ and super critical if $K_c > 1$. The fuel materials used in nuclear reactors are natural uranium containing 0.72% of U^{235}, enriched uranium containing more than 0.72% U^{235}, Pu^{239} and U^{233}. In a nuclear reactor moderator is used to slow down the neutrons, control rods are used to control the chain reaction. Nuclear reactors are used for research, for production of fissile material and for power production.

17.12 MODEL EXAMINATION QUESTIONS

I. Answer the following in briefly.

1. Describe the various functionaries of a nuclear reactor.

II. Answer the following in detail.

1. Discuss the nuclear fission reaction with details of decay process of fission fragments energy release and fission yield.
2. Explain why fusion reactions ae called thermonuclear reactions.
3. Give an account of the various fusion reactions responsible for the release of energy from the Sun and stars.
4. Uses of nuclear reactors.
5. On the basis of liquid drop model explain the nuclear fission process.

III. Solve the following problems

1. Determine the energy release in the thermonuclear reaction.

$$_3Li^6 + _o n^1 \rightarrow _1 H^3 + _2 He^4 + Q_1$$

Given the atomic masses $_3Li^6$ = 6.01703 a.m.u.

$_o n^1$ = 1.008665 a.m.u.

$_1 H^3$ = 3.0170 a.m.u. and $_2 He^4$ = 4.00487 a.m.u.

(Ans : Q_1 = 4.5 MeV.)

17.13 GLOSSARY

Luminiscene	:	Emission of light in the visible region by substances like Zns when irradiated by electrons, X-rays or ultraviolet radiatons.
Accelerator	:	Device used to increase the energy of nuclear particles.
Fluorescene	:	Emission of light by substances during irradiation by ultraviolet radiation or electrons or high energetic particles.
Phosphorscence	:	Emission of light by substance after the source of irradiation is removed.
Trapping centre	:	A localized energy level existing between valence and conduction bands in a semi-conductor crystal which can absorbed a hole or an electron.
Photomultiplier	:	A device used to detect feeble light signals. It works on the principle of photoelectric effect i.e., emission of electrons by material when irradiated by light.
Endoergic reaction	:	A nuclear reaction (chemical reaction) in which energy (heat) is absorbed.
Exeroergic reaction	:	A reaction in which energy is released.
Radiative capture	:	Absorption of neutron leading to emission of gamma radiation.
Mass Spectrograph	:	A device used to determine the atomic masses.
Meson field	:	The field existing between nucleons inside the nucleus under the influence of which π mesons are exchanged.
Excitation energy	:	Energy required to excite the nucleus from ground state to an excited state.
Quadrupole moment	:	This is a measure of the spherical symmetry of the nucleus. Quadrupole moment Q is zero for nuclie for which there exists closed shell configuration.
Pauli exclusion principle	:	This principal states that no two electrons in an atom can have identical quantum numbers. This principle also applies to nucleons inside the nucleus. This is the basic

		principle based on which the shell model of nucleus has been developed.
Triplet and singlet forces	:	The force between nucleons whose spin orientation is the same is called triplet force. The force between nucleons with opposite spin orientation is called snglet force.
Exchange force	:	The force acting between nucleons is exchange force. It has the property of being attractive or repulsive depending on the states of the particles.
Prompt neutron	:	Neutron emitted immediately after the fission.
Delayed neutron	:	Neutron emitted some time after the fission process occurs.
Maxwellian distribution of energies	:	The atoms or moleules of a gas may have any energy ranging from 0 to ∞ at any given temperature T. Such a distribution of energies among the particles is defined by Maxwell distribution.
Fissile material	:	Nuclear fuel that can undergo fission.

17.14 RECOMMENDED BOOKS

1.	Kaplan,	Nuclear physics	Addison-Wesley publishing Co., London
2.	Liversey, D.	Atomic and Nuclear Physics	Blaisdell Publishing Co., London.
3.	Burcham, W.E.	Nuclear Physics-An Introduction	Longman Group Ltd., London.
4.	Tiwari, P.N.	Fundamentals of Nuclear Science	Wiley Eastern Pvt. Ltd., New Delhi.
5.	Evans, R.D.	The Atomic Nucleus	McGraw-Hill Book Co., London.
6.	Eichholz, G.G. and Poston, J.W.	Principles of Nuclear Radiatiion Detection	Ann Arbor Science Publishers Inc. Michigan.
7.	Green, A.E.S.	Nuclear Physics	McGraw-Bill Book Co. Inc., New Yor.
8.	Russell Wehr, M., and Richards, J.R.	Physics of the Atom	Addison Wesley Publishing Co., London.
9.	White, H.E.	Modern College Physics	Affiliated East-West Press Pvt. Ltd., New Delhi.

BLOCK-4 : ELEMENTARY PARTICLES

UNIT–18 : ELEMENTARY PARTICLES (CLASSIFICATION)

Contents

18.1 AIMS AND OBJECTIVES

This unit explains the fundamental particles, properties and their classification.

After going through this unit you will be

- able to distinguish the properties of various fundamental particles. We use the physical qualities like Parameters mass, spin, ... achieving this object
- able to classify the fundamental particles by various methods.

18.2 INTRODUCTION

What are elementary particles?

All matter in the universe must have been built up with small particles. If we go on dividing and sub-dividing matter, we should arrive at a set of particles about which it can be said: "all nature is made from these". Such particles are called elementary or fundamental particles.

It was once though that the atoms were the ultimate, indivisible constitutents of matter. The word "atom" is derived from the Greek root "atoms", which means indivisible. It is an achievement of the 20th century Physics to reveal that the atom is divisible, that it is no longer elementary and that is has a complex structure. In 1911, Ernest Rutherford showed that the atom consists of a small dense nucleus surrounded by a cloud of electrons. It was subsequently revealed that the nucleus itself can be broken down into still smaller particles called Protons and Neutrons. Since then, many related particles have been indentified. During the last 25 years, these particles too were proved to be complex rather than elementary. These particles are now throught to be made up of the simpler things called Quarks. This hypothesis was first proposed in 1963. Though their independent existence is not experimentally proved as yet, it is belived to exist. It seems to be truly elementary in nature.

18.3 VARIOUS ELEMENTARY PARTICLES - FIRST AMONG THEM

In the early 1930's, the atomic drama had only four characters the electron, proton, neutron and photon. Protons and neutrons are the constituents of the nucleus and electrons in the space around it. These three are the building blocks of atoms. The electron is the lightest particle, having some rest-mass and a negative charge, which is the basic unit of electricity. Proton has a mass of $1836m_e$, where m_e is the mass of an electron. It has a unit positive charge. The neutron has a mass of about $1839m_e$ and no charge.

The photon is the quantum unit of radiation. It is the carrier of electromagnetic energy. It always travels with the velocity of light and as such posses energy. It therefore possesses mass also, according to the relation $E = mc^2$, respectively. It must be noted that the mass of the photon is only by virtue of its motion and that it has no rest-mass. It does not possess any electrical charge.

All these particles spin on their axes. The characteristic spin of the electron, proton and neutron is $\frac{1}{2}$. For a photon; the spin is 1. Another important property related to spin is their "Statistics". It is the behaviour of a population of identical particles. Electrons, Protons and Neutrons, which have a spin of $\frac{1}{2}$ obey Pauli's exclusion principle, i.e., there can be only one electron (for example) at a time spinning in a particular direction and revolving in a given orbit. They are said to obey Fermi-Dirac Statistics. They are hence called Fermions. Particles like photon, whose spins are whole numbers, do not obey Exclusion Principle. They have Bose-Einstein Statistics and are called bosons.

18.4 ANTI-PARTICLES POSITRON

The behaviour of electrons in electromagnetic fields was developed by many Physicists, notably P.A.M.Dirac of England. According to this theory, a fundamental particle has the properties of a wave. When Dirac's wave equation for the electron was solved, it yielded two frequencies negative and positive. He proved that it does have a physical significance and that it correspondends to an electron with positive charge. It was also predicted by theory that, if a positive electron collided with a negative electron, they would annihilate each other and their mass would be converted into photons with an equivalent amount of energy.

These remarkable predictions came true when Carl.D.Anderson of the California Institute of Technology discovered the positron. This has the mass of an electron and a unit of positive charge. This positron is called the anti-particle of the electron, because it cancels out an ordinary electron.

18.5 ANTI-PROTON

Dirac's general equation was slightly modified and applied to the proton also. It predicted the existence of an anti-particle an anti-proton identical to the proton but with a negative charge. Its properties also were predicted to be as follows :

1. It must have the same mass as a proton (1.6724×10^{27} Kgms).
2. It must have an equal charge of opposite sign.
3. It should be stable, lasting for ever in vaccum, without decaying spontaneously into a different particle.

4. When it meets a neutron or proton, it must disappear in mutual annihilation, liberating energy equivalent to the masses of the two particles.
5. It is never produced separately, but always in a pair with a proton or a neutron.
6. It must have an angular momentum (or spin) equal that of the proton.

Physicists began an intensive search for such a particle. It was estimated that an energy of about 6 BeV (billion electron volts) would be required for the production of a proton-antiproton pair. An instrument called Betatron was specially designed for this purpose. This instrument is capable of giving a kinetic energy of the order of BeV to protons. A target made of copper was bombared with 6 BeV protons. Many other particles also, having the same momentum as anti-protons, were generated in this process. With great difficulty, they could detect about 4 anti-protons per each hour of operation of the Betatron. This happened in October, 1955.

18.6 THE NEUTRINO

The next addition to the list of particles was possible a study of the behaviour of the neutron. Inside the nucleus, a neutron can live indefinitely. But when the particle is observed outside, it was found to be very unstable. In an average time of about 18 minutes, it spontaneously emits beta particle (i.e., an electron) and turns into a proton. Let us represent this in the form of a reaction as follows.

$$Neutron \rightarrow Proton + Electron$$

The Proton and electron together are about 1.5 electron masses (m_e) lighter than the neutreron. This difference in mass is equialent to about 7.8×10^5 ev. It means that the kinetic energy of proton and electron should be of this order. But, in fact, they did not have so much of energy. To explain this discrepancy, Pauli suggested that another particle with zero rest-mass is formed in the decay and that this carries off the missing energy. Fermi pursued this idea further and named the invisible particle the "neutrino". He worked out in detail the properties of the presumed particle as follows :

1. The particle must have no charge.
2. The particle must be practically weightless. For simplicity it is considered to have zero mass.
3. The particle must have a spin of one-half unit.
4. It must have its birth in beta-decay i.e. a reaction in which a beta particle is one of the decay products.

Formi constructed a complete theory of beta-decay. According to this, a neutron continuously loses and regains an electron and a neutrino by emission and absorption. Such a particle was actually detected later.

18.7 THE PION

The next particle to be added to the list was predicted by another analogy with the Dirac process. The proton and the neutrons which constituted the nucleus are called nucleons. When so many protons are crowded in a nucleus, there must have been large repulsive forces. Then, how does the nucleus remain stable? What holds protons and neutrons together in the nucleus? The answer to these questions was provided by the Japanese Physicist, Yukawa. He proposed that nucleons emit and absorb a quantum called a meson. Since the nuclear forces extend over a very short range only, he predicted that

the meson would have a finite rest mass. He calculated this to be between 200 and 300 times the mass of an electron. Further, there were various reasons to suppose that there would be both charged and neutral mesons.

Yukawa's brilliant prediction came ture 12 years latter. In 1947, a particle weighting about 270 electron masses was discovered and this was found in three forms positive negative and neutral. These particles were called the π-messons, or the pions. The neutral pion has 264 times the mass of hte electron, whereas the charged pion has a mass of 273. Pions are emitted by the nucleons. They are readily absorbed by nuclei. They are unstable. For example, the charged pion decays into a lighter particle (called the μ meson) and a neutrino with a half-life of a new 10^{-4} seconds. The neutral pion decays much faster, with a half-life of about 10^{-15} seconds, into two gamma rays. These reactions can be represented as follows :

$$\pi^{+} \rightarrow \mu^{+} + \nu^{+}$$

$$\pi^{-} \rightarrow \mu^{-} + \nu^{-}$$

$$\pi^{o} \rightarrow \gamma + \gamma$$

18.8 THE MUON

This is perhaps the first unstable particle to be discovered (with the exception of the free neutron). The muon was discovered in 1936 in the cloud-chamber photographs of cosmic radiation, by Anderson and Neddermeyer of the California Institute of Technology and independently by J.C. Street of Harvard University. The μ messons (or the muons) in cosmic radiation travelled easily through the atmosphere, penetrated lead plates and cloud even be detected in deep mines. They have a relatively long life, about 2 micro-seconds. Each muon has a mass 207 times that of the electron. There are both positive and negative muons, but there are no neutral muons. The muon's angular momentum (i.e.spin) appears to be $\frac{1}{2}$. The unstable muon if it is negative decays into an electron, a neutrino and an antineutrino. Similarly, the positive muon decays into a positron, a neutrino and an antineutrino. Here, it must be noted that the antineutrino is the anti-particle of a neutrino. Both are not electrically charged. They are distinguished by their spin only. Their spins would be oppositely directed.

Thus, it can be stated that the muon is an oversized electron. It is identical with the electron in all its properties except mass.

18.9 STRANGE PARTICLES

Around 1950, a large of new particles were added to the list. Many of them were discovered in the cloud-chamber photographs, when high-energy cosmic rays strike a lead plate. Among the tracks of showers, two-pronged or V-shaped patterns were observed. Physicists concluded that some unknown neutral particle must have decayed into two charged particles, which appear in the form of V in the photographs. The original neutral particle does not leave any track in the cloud-chamber.

A large number of V-photographs were collected and studied. It become clear that there should be two new neutral particles. One of them, called the lambda particle (represented by $\wedge$), decays into a proton and a negative pion. The other one, called the K

particle of Kaon, decays into a positive and a negative pion. The lambda, particle has a mass of 2181m_e. It has a spin of $\frac{1}{2}$ and hecne, it can be shown to be a fermion. It decays into a proton and a neutral pion. Sometimes, it may also decay into a neutron and a neutral pion. Its mean life is 2.7×10^{-10} second.

The Kaon decays into two pions I_{es} spin is zero. It has a mean life of 1.2×10^{-8} second. It has a mass of $965m_e$. It was first discovered in high-altitude cosmic-ray experiments with emulsions.

In the same category with the lambda are Sigma particles ($\sum$) charged and neutral, and the negative (χ) particle. All these are called "Strange Particle", because whenever they decay, there was a large discrepancy between the mass of a particle and the total mass of its decay products.

Among the other unstable particles are the anti-neutron ($\bar{n}$), which is a particle having the same properties as those of a neutron, except that the spin is oppositely directed.

18.10 CLASSIFICATION OF ELEMENTARY PARTICLES

There are now more than 100 elementary particles. They are classified, based on different criteria.

I. Classification on the basis of Slability

One such basis is stability against decay into other particles. The stable particles are the electron, proton, photon and the neutrino and their anti-particles, namely, positron, anti-proton and anti-neutrino. All these particles are stable and they do not decay.

Among the unstable particle are the neutron n, the negative muon μ^-, the three pions π^+, π^-, π^o and their anti-particles, namely, the anti-neutron $\bar{n}$ and positive muon μ^+.

Symbol	Particle	Charge in terms of basic units e	Mass in terms of electron-mass m_e	Spin quantum number	Life time in second	Anti particle
			Stable Particle			
e^-	Electron	-1	1	$\frac{1}{2}$		e^+
v	Neutron	0	$\simeq 0$	$\frac{1}{2}$		$\bar{v}$
p	Proton	+1	1836.12	$\frac{1}{2}$		$\bar{p}$
γ	Photon	0	0	1		γ
			Unstable Particle			
n	Neutron	0	1836.65	$\frac{1}{2}$	10^3	$\bar{n}$
μ^-	Muon	-1	206.8	$\frac{1}{2}$	2×10^{-6}	μ^+
π^+	Pions	+1	273.2	0	3×10^{-8}	π^-
π^o	Pions	0	264.2	0	2×10^{-6}	π^o

II. Classification according to the nature of particle inter-action:

Another way of classifying the elementary particles is in terms of the main types of interaction between them.

1. The gravitational force, which is, for example responsible for planetary systems. Such a force exists between fundamental particles also. This varies inversely as the square of the distance between the interacting objects. But this is considered to be very feeble, when compared to the forces binding the nucleons inside the nucleus.

2. There is the electromagnetic interaction. An electron kept in an electromagnetic field continuously emits and absorbs photons. This is the means by which the field and electron exert a force on each other.

3. Some particles decay, giving rise to lighter particles, such as beta particles or electrons. The forces responsible for these decays are very weak. This is called the weak interaction.

4. The nucleus of an atom contains protons within very small distances of the order of 10^{-13} cm from each other. There must be a force of attraction, which is greater than the electro static repulsion between the protons. This interaction is called the nuclear or strong interaction.

Each of the particles shown in the table above, interacts with other particles listed there by means of one or more of the four basic interactions. All the particles are subjected to gravitational interactions. All the charged particles interact electromagnetically, by virtue of their charge. The interaction between a proton and a neutron is a strong interaction. Wherever a beta particle or a neutrino is given out in a decay process, there is weak interaction.

III. Classification by Spin

Some elementary particles have a spin of zero or one and some other have a spin of $\frac{1}{2}$. Hence they can be classified on the basis of their spin. particles such as electrons, protons and neutrons have a spin of $\frac{1}{2}$. They obey Pauli's exclusion principle. This says that only one particle can occupy a given quantum state. For example, there can be only one electron at a time spinning in a particular direction and revolving in a given orbit around a nuclear. Such particles are governed by Fermi-Dirac statistics. Hence they are called Fermions.

Particles like the photons, whose spins are whole numbes, do not obey the exclusion principle. They are governed by Bose- Einstein statistics and are called Bosons.

IV. Classification on the basis of mass

Another method of classification is to distinguish between the heavier particles which are involved in the strong nuclear interaction and the other lighter particles. The heavy particles are called baryons and the lighter particles are called leptons. Neutron and proton belong to the baryon category and the electron, muon, neutrono belong to the lepton category. Other unstable particles such as lamda, sigma and xi particles, having masses greater than that of the nucleon, are called hyperons.

Mesons and baryons are strongly reacting particles and collectively they are called hadrons.

It can be said that the elemetnary particles are classified into groups according to their mass, as noted below :

1. The photon with zero rest mass.
2. The leptons or light particles. These are the electrons, muons and neutrinos and their anti particles. They are all fermions and interact weekly with other particles.
3. The mesons or intermediate particles, so called because their masses are between those of the muons and the nucleons.
4. The baryons. These are the heavy particles of nucleon mass and above. Nucleons and the hyperons belong to this category. All these particles are shown in the following Table :

Table : Elementary Particles

	Group	Particles	Symbols	mass int erms of m_e
I	Fermion of spin $\frac{1}{2}$	Baryous a) Proton	p	1836.12
		i) Nucleous b) Neutron	n	1838.6
		ii) a) Xi	x	2572.4
		b) Sigma	Σ	2333.6
		c) Lambda	λ	2182.4
		d) Omega	Ω	3272.0
		iii) Leptons		
		a) electrons	e^-	1
		b) Neutrino	ν	0
		c) Muon	μ	206.8
II.	Bosons of spin 0,1	a) Kaons	K	974.2
		b) pions	π	264.2
		c) photons	γ	0

18.11 SUMMARY

There are nearly more than 100 elementary particles. They can be classified on the basis of 4 different classes.

18.12 MODEL EXAMINATION QUESTIONS

I. Answer the following in detail.

1. Discuss the different methods in the classifications of fundamental particles.

II. Answer the following questions briefly.

1. What is meant by fundamental particle? Give examples.
2. "Fundamental particles are not truly funamental, "Discuss.
3. How the fundamental particles are classified on the basis of their masses.

UNIT–19 : CONSERVATIONS LAWS

Contents

19.1 AIMS AND OBJECTIVES

In this unit you study the main conservation laws to be followed in the production of elementary particles. After going through this unit you will be able to understand different conservations laws to be followed in elementary particle reactions.

19.2 INTRODUCTION

For any reaction to happen some basic conservation laws are to be followed. They are to be followed, mainly in different types interaction like strong interactions, Electromatic interaction weak interaction. A few interactions will follow certain other conservation laws which are not discussed here.

19.3 CONSERVATION LAWS

The behaviour of elementary particle depend upon number of conservation laws like any other physical quantity. Now the different conservation laws followed by elementary particles in all types of interactions namely, strong, electromagnetic, weak will be discuss.

a) Conservation of Linear momentum

The invariance of linear momentum of a physical system under translation in any direction or during a reactions is known as conservations of linear momentum in that direction. This we come across in many physical interactions, a familiar example is compton effect.

b) Conservation of angular momentum

This involves both types of (orbital and spin) angular momenta together. The orbital angular momentum is the motion of the particle about all axis of rotation. The spin angular momentum is the intinsic angular momentum of each particle about an axis through its own centre of mass. During the process of interaction, both angular momenta individually and also as a whole should be conserved. Strongly interacting fermions have a half integer spins. $s = \frac{1}{2}$ for $\sum$, λ, n and p; $s = \frac{3}{2}$ for Ω. Strongly interacting bosons have '0' spins s = o for η, K and π; weakly interacting fermions (leptons) have a spin equal to $\frac{1}{2}$. i.e. $s = \frac{1}{2}$ for μ, e, ν. Massless bosons, electromagnetic interacting γ rays have s = 1 and gravitons have spin of $\frac{1}{2}$.

c) Conservation of energy

This is more complicated in elementary particles as the large fraction energy is

interchanged between rest mass energy and kinetic or potential energies. The sum of these three is the total energy which is to be conserved in any reaction. For example the decay reaction K = $\pi^+ + \pi^- + \pi^6 + \pi^o$ is forbidden since the rest mass energy of k is not equal to the rest mass energies of pions. But K = $\pi^+ + \pi^- + \pi^o$ is allowed since the rest mass energies on both sides are equal. Another example is

$\pi^- + p^+ \rightarrow 2\pi^o + n$, is forbidden.

and $\pi^- + p^+ \rightarrow \pi^o + n$ is allowed.

d) Conservation of charge

This is the most familiar of the conservation laws. The charge is conserved in all processes and no exception is known. All elementary particles have a charge of +1, 0 or -1. In the examples $p^+ + p^+ \rightarrow p^+ + p^+ + \pi^o$ and $p^+ + p^+ \rightarrow p^+ + p^+ + \pi^+ + \pi^-$ the charge is conserved. But in $p^+ + p^+ \rightarrow p^+ + p^+ + \pi^+$ the charge is not conserved.

e) Conservation of Lepton number

The Lepton number in an elementary particle reaction is also to be conserved. Electrons, negative muons and neutrons have a lepton number of +1. Their antiparticle or antileptons have a lepton number of Y. In the reaction $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$, the lepton number is conserved and in the reaction $\mu^+ \rightarrow e^+ + \nu + \nu$ the lepton number is not conserved.

f) Conservation of Baryon number

The Baryon number in any process in elementary particle reactions is to be conserved. All normal baryons such as $p^+, n^o, \lambda^o, \Sigma^+, \Sigma^- \Sigma^o$ have a baryon number of +1 and their antiparticler, known as antibaryons have a baryon number -1. All the nervous have a baryon number as 'o'. In the example $\lambda^o \rightarrow p^+ + \pi^-$ the baryon number is conserved. But in teh example. $\lambda^o \rightarrow p^- + \pi^+$ the baryon number is not conserved.

The reaction $n^+ \rightarrow p^+ + e^- + \bar{\nu}$ is an allowed reaction as the both the laws of baryon's and leptons are conserved. No exception is found for the conservation of leptons and baryons in many elefuentary particle interactions. There are other conservation laws which are not applicable to weak interactions. The isospin property in conserved in only strong interactions. Other properties which are important and also not applicable to all there types of interactions are hyper charge and strangeres. Let us know about these three namely conservation of isospin, conservations of hyper charge and conservation of strangeness.

g) Conservation of isospin

Each nuclear particle possesses a certain spin T, know as isotopic spin or isospin. Each possible projection of this along a certain axis appears as a separate charge state of the corresponding particle. In the case of proton it is $+\frac{1}{2}$, in the case of neutron it is $-\frac{1}{2}$ and in the case of pion it 1. Number of states are (2T+1). In the case nucleons number of states are 2 that is protons, anti-protons and neutrons, antineutrons. But in the case of

pions, it is 3. That is +1, 0 and -1.

These isospin numbers are also asociated in the hadrons.

h) Conservations of hyper charge

This is twice average charge of the particle group. This is conserved in strong and electromagnetic interactions. Considering $\pi^+\pi^o$ and π^-, its average charge is zero and so its hyper charge is also zero. But the hyper charge for the pair K^+K^o is +1 and their antiparticles it is -1.

i) Conservation of strangeners

Strangeners is a quantum number which is so chosen that for all know non strange particle particles it is zero. Each strange elementary particle is assigned a "Strangeners number" such that, elementary particles are produced following the principle of conservation of strangeners number. The strangeners number assigned to $\sum^+$ is -1, to K^+ is +1, for π mesons and protons it is '0' The reaction $\pi^+ + p \rightarrow \sum^+ + K^+$ is allowed as the strangeners number is conserved. But $\pi^+ + p \rightarrow \sum^+ + \pi^+$ is forbidden as the strangeners number is not conserved.

19.4 SUMMARY

For the three types of elementary particle interaction different types of conservations laws are discussed. That is for commonly obsend particle interactions.

19.5 MODEL EXAMINATION QUESTIONS

I. Answer the following question in about 30 lines.

1. Explain the conservation laws which are observed in all three types of interaction.

II. Answer the following questions 10 lines.

1. Explain the conservation of isospin.

2. Explain the conservation of strangeners.

UNIT–20 : QUARK MODEL

Contents

20.1 AIMS AND OBJECTIVES

This unit explain Quark Model for the explanations of the structure of hadrons.

After going through this unit you will be able to explain the quark Model and different types of quarks.

20.2 INTRODUCTION

The Kaons, pions together with Baryons are grouped into a strongly interacting particles called hadrons. The structure of hadrons was explained with the help of quark's Model.

20.3 QUARKS MODEL

In 1963, an explanation was proposed by Gell-Mann and by George Zweig for the study of elementary particles. They assumed that all hadrons were constructed from more fundamental units called quarks. The quarks are supposed to have peculiar properties. All observed particles, without exception, have integral multiples of the electron's charge. Gell-Mann named the three quarks u,d and s, denoting up, down and sideways. According to this model, all the fundamental particles can be assumed to be made up of the three quarks and their anti-particles called the anti quarks. The quarks u, d and s have electric charges of +2e/3, -e/3 and -e/3 respectively, where e is the charge of an electron. A meson is made of one quark and one anti-quarks. Baryons are made of three quarks and anti-baryons are made of three anti-quarks. Similarly, every combination of quarks permitted by the rules gives rise to a known particle. Just two of the quarks, the u and s are enough to explain the sturcture of all the hardrons found in ordinary matter. Teh proton, for example, can be described by assembling two u quarks and a d quark. Its compsition is written as uud. Its charge is equal to $\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$. Similarly, a positive pion is composed of a u quark and $\bar{d}$ anti-quark. It is represented by u$\bar{d}$. Its charge is $\frac{2}{3}+\frac{1}{3}=+1$.

The third quark s is needed only to construst strange particles. This psovides an explicit definition of strangeness. A strange particles can now be difined as a particle that constains at least one s quark or s anti-quark. For example, the lambda particle represented as uds has a strangeness number of -1. Here, it must be understood that strangeness is a property like matter and energy, which is conserved when strange particles enter into strong inter-actions. When strange particles enter into weak interactions, strangeness is not conserved.

Another strange particle, neutral k Meson represented by ds has a strangeness of

+1. In both these cases, the presence of s quark or $\bar{s}$ antiquark is to be specially noted.

Until quite recently, these three kinds of quarks were sufficient to describe all the known hardons. But experiments conducted during the year 1966, created hadions, whose properties cannot be explained in terms of the original three quarks. These experiments could be interpreted, only by assuming the existence of a fourth kind of quark, called the charm, represented by c.

It is to be noted that quarks have not been detected either in cosmic radiation or in nuclear colliosions in high energy laboratories. The reason may be that quarks have no independent existence outside the hadrons. Intensive experimental search was conducted to find the independent existence of a quark outside a hadron. But the evidence suggests that quarks do not exist independently.

In spite of the unsuccessful searches the quarks may still exist because of the following reasons.

1. The quarks may be very heavy and the present accelerators may not have sufficient energy to produce them.
2. The production of quarks might be prevented by some unknown factors.
3. Quarks may exists within integrally charged particles, but individual quarks are not observed because of charge quantization.
4. Even though quarks exist within observed particles, perhaps they can never be isolated.

Man's knowledge of the universe is still very small. The more extensive the researches, more questions seem to be springing up. Quark theory has given answers to some problems, but provoked many fresh questions. Do quarks exist? Are they truly fundamental? Can we find one in collision experiments at very high energies? Is Physics becoming a less and less exact Science? Future alone can provide the answers.

20.4 SUMMARY

All matter is built up of small particles called quarks.

20.5 MODEL EXAMINATION QUESTIONS

I. Answer the following question in detail.

1. Explain quark Model in detail to describe the structure of hadrons.

20.6 RECOMMENDED BOOKS

1.	Atomic and nuclear physics	T.A. Littlefield & Thorley
2.	A contemporary view of Elementary physics	Borowitz & Bornstein
3.	Elementary particles	Yuan
4.	Fundamental particles	Bransden, Evans & Major
5.	Atomic Physics	J.B. Rajam
6.	Physics of the Atom	Wehr and Richards
7.	Modern University Physics	Richards Sears wehr Zemansky
8.	Where do Cosmic rays come from?	by Bruno Rossi (Scientific American-September 1953).

Scientific American" Articles:

Elementary particles	Gell-Mann Rosenbaum-July 1957
Pions	Marshark-January 1957
Muon	Sheldon Penman-July 1961
Neutrino	Phillip and Morrison January 1956
Anti proton	Serge Wiegand-June 1956
Neutron	Philip & Morrison-October 1951
Quaskswith Color and Flavor	Sheldon Lee Glashow 1967

BLOCK-5 : MAGNETIC PROPERTIES OF MATTER

UNIT–21 : DIA, PARA AND FERROMAGNETISM

Contents

21.1 AIMS AND OBJECTIVES

This unit describes the three types of magnetism namely Para, Dia and Ferro magnetism.

After going through this unit you will be

1) able to distinguish the three types of magnetic substances
2) able to evaluate the value of mass susceptibility

21.2 INTRODUCTION

From our experience we know that only a substance can be converted into magnets. But experiments showed that more or less all the substances behave as magnets, because all the substances possess electrons. They move in orbits, electrons in motion produce magnetic fields. In this unit we are going to study the types of magnetism. In this unit it is explained that, nucleus also possess magnetism, and this lead to the new technique namely Nuclear Magnetic Resonance (NMR) and also molecular structure of compounds can be determined using NMR Technique.

21.3 THE THREE TYPES OF MAGNETISM

Till the advent of nineteenth century, it was supposed that magnetic properties were confined to few elements like iron. But, in 1845, faraday discovered that all substances were magnetic to greater or lesses extent and that could be divided into two main classes, para-magnetic and dia-magnetic. When substances are freely suspended between the poles of a magnet, some of them tend to set themselves along the magnetic filed. They are also attracted by a magnet. They are called paramagnetic. Elements such as Fe, Ni, Co, Mn, Al etc. belong to this category.

The other class of substance is diamagnetic. They are repelled by the magnet. When suspended in a field, they set themselves at right angles to it. Bi, Sb, Hg, Zn, Cd etc.,

belong to this category.

Among the paramagnetic substances, some elements like iron are strongly attracted by magnetic fields. They acquire a relatively high magnetization in weak fields. This is called ferro-magnetism. It is to be noted that the ferro-magnetics loose their characteristics above a certain temperature and then behave as paramagnetics. This critical temperature is called the "Curie Point". Hence, ferro-magnetism is probably regarded as special manifestationof paramagnetism.

In order to explain the Para and Dia-magnetism, weber proposed the "molecular current" theory. Molecules of all substances are built with atoms, which contain electrons in complicated orbits. A revolving electron is equivalent to an electric current and hence Weber postulated the existence of molecular currents. In paramagnetic substances, the intrinsic molecular currents give the molecules a permanent magnetic moment. The diamagnetism was due to molecular currents induced by an external field. A magnetic moment is given to the molecules in a direction opposite to the that of the field. This was called a negative magnetic moment. Nearly 50 years later Langevin applied the electron theory to explain para and diamagnetism, about which we shall study in the later articles of this lession.

21.4 PROPERTIES OF PARA, DIA AND FERROMAGNETIC MATERIALS

21.4.1 Paramagnetic Substances

Paramagnetic substances are those which tend to move from weaker to stronger parts of the magnetizing field. A non-uniform magnetic field is represented in Fig. 21.1

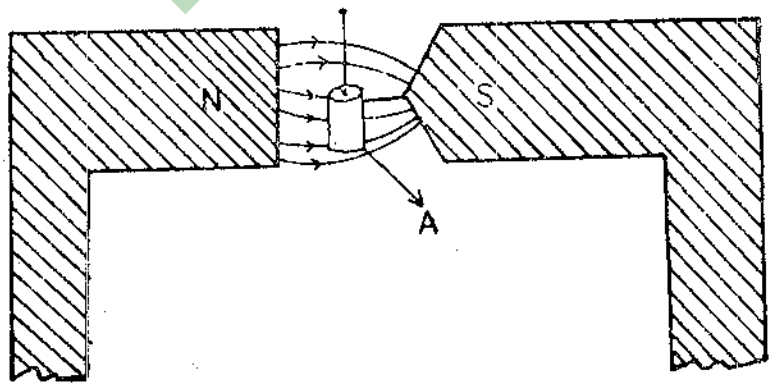


Fig 21.1 Non-uniform magnetic field, where the north pole has a flat face and south pole has a sharply pointed face, used for studying the types of magnetic substances. A-pice of material

The magnet has one sharply pointed pole piece and one flat piece. The magnetic field is much stronger near the pointed pole than near the flat pole. NS is an electro magnet. If a small piece of paramagnetic substance like Aluminum is suspended between the poles, it will move towards the pointed pole. If the substance suspended is in the form of a thin bar it sets itself with its longer axis parallel to the direction of magnetic field. Magnetic lines of force tends to pass through the material of the specimen. The permeability of paramagnetic material hence greater than that of free space. Permeability is the property of the medium and is a masure of its tendency to allow lines of force to pass through it. This is represented by μ.

There is another costant called susceptibility of a medium. This is represented by K and is defined as the ratio of the intensity of magnetism I in a body to the strength H of the magnetizing field. K is thus a measure of the magnetic quality of a substance. it is also called volume susceptibility. Mass susceptibility is equal to $\frac{K}{\rho}$ Where ρ is the density of the body. This is represented by χ

Susceptibility of paramagnetic substance is a small positive value. Its value is found to be inversely propotional to absolute temperature.

Al, Pt, Cr, O_2, Mn, solution of salts of Ni, Fe, and O_2 are some of the paramagnetic substances.

21.4.2 Dia-magnetic Substances

Diamagnetic substances such as Bi, Cu, Hg, Sb, Au, H_2 water, alcohol and air move from stronger to weaker parts of magnetic field. When suspended, in a non uniform filed shown in Fig. 21.1, A thin bar, when suspended sets itself at right angle to the direction of the field. The magnetic line of force tend to travel outside the specimen, hence the permeability of a diamagnetic material will be less than that of the free space. The susceptibility will be a low negative value.

21.4.3 Ferromagnetic Substances

Ferromagnetic substances are those which are attracted by magnets and can also be magnetized, to a large degree. At a temperature beyond the Curie point, the substance behaves as a paramagnetic specimen. Their susceptibility is a large positive value and permeability is also very large.

21.5 LANGEVIN'S THEORY OF DIAMAGNETISM

On the basis electron theory Langevin was able to explain both dia and paramagnetism. We shall first deal with diamagnetism.

An Electron revolving in an orbit of an atom constitutes an electron current A current flowing in a closed circuit is equivalent to a magnetic shell. The orbits of the various electrons may be so directed that the atom as whole may posses a resultant magnetic moment, on account of lack symmetry. Hence the atoms possess a permanent magnetic moment. When a magnetic field is applied to such substances , the atoms get rotated in direction of the field and as a result magnetic induction increases. This is the position in paramagnetic substances.

In diamagnetic substances, the electronic orbits are so oriented that the atom as whole has no resultant magnetic moment. If an external magnetic field id applied, it will not have any rotating effect upon the atom. But it will have some effect on the electron orbits and a magnetic moment is induced in a direction opposite to the field. Hence the magnetic induction tends to decrease.

Consider a single electron moving in circular orbit and the nucleus in an atom as represented in Fig 21.2 suppose m and e are mass and charge of the electron v and ω its linear and angular velocities respectively, let R be the radius of the orbit. The forces of attraction F counterbalances the centrifugal force acting on the revolving electron.

Therefore $F = \frac{mv^2}{r} mr\omega^2$

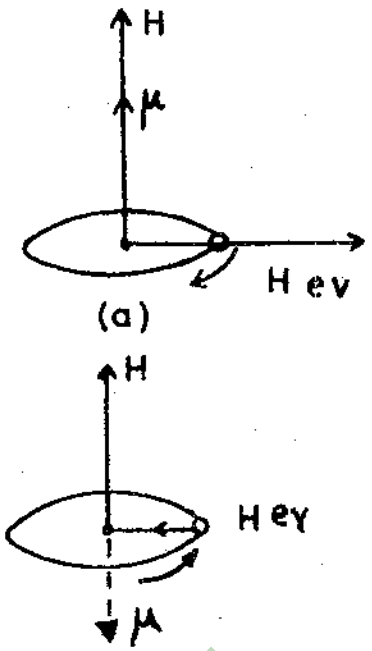


Fig 21.2 Electron revolving in circular orbit is subjected to magenetic field H (a) When electron revolves in a clock wise direction, force acts on it away from the centre (b) when electron revolves in an anti-clockwise direction, force acts on it towards the centre.

Suppose H is the intensity of the magnetic field in a direction perpendicular to the plane of the orbit. Under its influence, a force begins to act on the electron, whose magnitude is here and whose direction is given by Fleming 's left hand rule. According to this, if the electron is moving in the clockwise direction as in Fig. 21.2(a) the force here will be directed outwards. If the electron moves in the anticlockwise direction as in the Fig. 21.2(b) the force here will act inwards.

Larmor proved that the effect of applied field is precession of the orbit without any change in its form. This is called the Larmor Precession. Because of this, the angular velocity of the electron slightly changes. If $\delta\omega$ is the change in angular velocity, for an electron with a clockwise motion.

$$F - HeV = mr(\omega + \delta\omega)^2$$

Since $F = mr\omega^2$ and $v = r\omega$, we get

$mr\omega^2 - Her\omega = mr(\omega^2 + 2mr\delta\omega)$ neglecting the term involving $(\delta\omega)^2$, since $\delta\omega$ is small compared to ω.

Therefore $\delta\omega = \frac{-He}{2m}$

The negative sign here indicates that the change in angular velocity acts in sense opposite to that of the electron.

This change in angular velocity produces a change in the magnetic moment, represented by $\delta\mu$.

If an electron of charge e revolves in a circle at the rate of n revolutions per second, it is equivalent to current *ne*. If A is the area of the circle, the magenetic moment of electron orbit is equal to *ne*. A change in the induced magnetic moment.

$$\delta\mu = \in A\delta n = eA\left(\frac{\delta\omega}{2\pi}\right), \text{ Since } n = \frac{\omega}{2\pi}$$

In this expression $A = \pi r^2$ and $\delta\omega = \frac{-He}{2m}$

Therefore $\delta\mu = e(\pi r^2)\left(\frac{-He}{4Hm}\right) = -\left(\frac{He^2r^3}{4m}\right)$

From this equation it can be seen that the change of magnetic moment is always opposite to that of field.

In an atomic number Z there are Z electrons. Hence the change of magnetic moment for the entire atom will be equal to $Z\delta\mu$.

Since the electrons revolve in different orbits, the term r^2 should be replaced by a suitable mean value R^2

The induced magnetic moment of atom is given by

$$\delta\mu = -\frac{ZHe^2}{4m} \times \frac{2R^3}{3} = -\frac{ZHe^2R^2}{6m}$$

If *N* is the Avegardo number change in the magnetic moment of gram molecule is

$$\frac{\delta\mu}{A} = -\frac{NZHe^2R^2}{6m}$$

If A is the molecular mass and ρ is the density the volume will be $\frac{A}{\rho}$

$$\therefore \text{ Intensity of Magnetisation I} = \frac{\text{Change in the mag. Moment of a mole}}{\text{Volume}}$$

$$= -\frac{NZe^2R^2}{6m} \cdot \frac{\rho}{A} \qquad 21.1$$

$$\text{Susceptibility } K = \frac{T}{H} = -\frac{NZe^2R^2\rho}{6mA} \qquad 21.2$$

This is also known as volume susceptibiltiy

$$\text{Mass Susceplibility } \chi = \frac{K}{\rho} = -\frac{NZe^2R^2}{6mA} \qquad 21.3$$

The susceptibility per unit mass is also known as mass susceptibility which is $\frac{K}{\rho}$.

Significant Features of the Theory

1. From the expressions derived for susceptibilities, (equations 21.2 and 21.3), it can be seen that they are independent of temperature and field strength. This is in agreement with experimental results obtained with substances in ionic state for slat solution and in the atomic state for the inert gases.
2. The negative sign in all the equations shows that diamagnetism is an induced opposite effect, which disappears as soon as the field is removed. In the equations for $\delta\mu$ if H = 0; then $\delta\mu = 0$
3. Every electron aquired an induced moment in a direction opposite to that of the applied field, so the moment of the whole atom gets modified in the same sense, whether the initial atomic moment is zero or not. It means that all atoms should possess diamagnetic properties. It must be noted that this diamagnetic effect is very small, even in very strong field. This is something temporary due to an external agent.
4. In the case of paramagnetic substances also, this diamagnetic induction affect is present but paramagnetic orientation process is dominant and hence they behave differently.
5. using the equation 21.3 $R^2$the mean square of the radius of the atom of any element can be calculated, if susceptibilities are measured. These values are of R^2 agreed well with those given by kinetic theory.

21.6 LANGEVIN'S THEORY OF PARAMAGNETISM

Langevin explained Para magnetism on the basis of Kinetic Theory. The molecules of a paramagnetic gas, for example, are small permanent magnets because of electrons revolving in the orbits and their spin. Where there is no external magnetic field, the moments of the various molecules will be oriented in all random directions and resultant moment along any line of reference is zero.

If an external magnetic field is now applied, each molecule will tend to orient itself with its magnetic axis along the direction of the field. But because of perpetual motion of the molecules and consequent collisions between them, the process of alignment along the direction of the field gets disturbed. If applied field increases, more and more atoms will be

opposed. There will be thus an equilibrium distribution of the axes with reference to the direction of the field.

Suppose μ is the magnetic moment of each molecule and H is the intensity of the applied field. If θ is the angle between the axis of the molecular magnet and the direction of the field, the potential energy of each molecular magnet $= \mu HCos\theta$.

According to the kinetic theory, the number of molecules d N with their axis at an angler θ with a reference line is proportional to $Sin\theta.d\theta$. The number of molecules whose potential energy is W is proportional to $e^{-w/Kt}$ where K is the molecular gas constant and T is the absolute temperature.

i.e., $dn\alpha Sin\theta d\theta$ and also

$$\therefore \quad dn\alpha e^{\mu HCos\theta/KT}.Sin\theta d\theta \qquad 21.4$$

If $\frac{\mu H}{KT} = a$, eqn. 21.4 becomes

$$dN = c.e^{aCos\theta} Sin\theta d\theta$$

Where c is the proportionality constant

Since all orientations lie., between 0 and π, the total no. of molecules is given by

$$N = \int_o^{\pi} dN = \int_o^{\pi} c \times e^{aCos\theta}.Sin\theta,d\theta \qquad 21.5$$

If $Cos\theta = x, Sin\theta d\theta = -dx$. Substituting these values in eqn 21.5 we get

$$N = \int_{+1}^{-1} c.e^{ax}(-dx)$$

$$= \int_{-1}^{+1} c.e^{ax}.dx$$

$$= C\left[\frac{e^{ax}}{a}\right]_{-1}^{+1}$$

$$= \frac{C}{a}\left(e^{a} - e^{-a}\right)$$

$$\therefore \quad C = \frac{Na}{\left(e^{a} - e^{-a}\right)} \qquad 21.6$$

The intensity of magnetization I is called as follows. The component of the moment of each molecular magnet along the direction if field is $\mu\cos\theta$. The resultant magnetic moment due to all the dN molecules is equal to $\mu Cos\theta dN$. Since there are N molecules in unit volume, intensity of magnetization is given by

$$I = \int_{o}^{\pi} \mu\cos\theta.dN$$

$$= \int_{o}^{\pi} \mu\cos\theta.ce^{a\cos\theta}Sin\theta d\theta$$

As before, assuming $Cos\theta = x$ and $Sin\theta d\theta = -dx$, the above equation

$$I = c\mu.\int_{-1}^{+1} x.e^{ax}\,dx$$

Intergrating by parts, we get $I = c.\mu\left[\left(\frac{e^{a} + e^{-a}}{a}\right) - \frac{1}{a^{2}}\left(e^{a} - e^{-a}\right)\right]$

Substituting the value of c from 21.6

$$I = \frac{Na}{\left(e^{a} - e^{-a}\right)}\mu\left[\frac{1}{a}\left(e^{a} + e^{-a}\right) - \frac{1}{a^{2}}\left(e^{a} - e^{-a}\right)\right]$$

$$= \mu N\left[\frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} - \frac{1}{a}\right]$$

$$= \mu N\left[\coth a - \frac{1}{a}\right]$$ where cotha is a hyperbolic Function.

μN represents the magnetic moment per unit volume, when all the molecules are oriented in the direction of the field. therefore it represents the saturation value of I, which is represented by I_s.

$$I = I_s\left[\coth a - \frac{1}{a}\right]$$

$$\frac{I}{I_s} = \left[\coth a - \frac{1}{a}\right] = L(a)$$

Hence L(a) is known as Langevin function.

If a is small, the value of Langevin function be proved to be equal to $\frac{a}{3}$.

Therefor $\frac{I}{I_s} = L(a) = \frac{a}{3} = \frac{\mu H}{3kT}$ 21.6a

The volume Susceptibility $K = \frac{I}{H} = I_s\left(\frac{\mu}{3kT}\right) = N\left(\frac{\mu}{3kT}\right) = \left(\frac{\mu^2 N}{3kT}\right)$

The mass susceptibility χ_M is equal to $\frac{K}{\rho}$.

If N_m be the number of molecules in unit mass, the mass of one molecule is $\frac{1}{N_m}$

Since there are N molecules in unit volume, the volume of one molecule is $\frac{1}{N}$

Therefore $\rho = \frac{mass\ of\ a\ molecule}{volume\ of\ a molecule} = \frac{\frac{1}{N_m}}{\frac{1}{N}} = \frac{N}{N_m}$

Substituting this value of ρ, we get

$$\chi_M = \frac{K}{\rho} = K\frac{Nm}{N} = \frac{\mu^2 NNm}{N3kT} = \frac{\mu^2 .Nm}{3kT}$$ 21.7

The susceptibility of a gram molecule of the substance is called molar susceptibility and is represented by χ_M.

If N_0 represents the Avogadro number, the molar susceptibility is given by

$$\chi_M = \frac{\mu^2 N_0}{3kT}$$

Significant Features of Langevin's Theory

(1) The expressions for the susceptibilities derived above show that they are inversely proportional to the absolute temperature. This agrees with the experimental observations. The only condition to be satisfied is $a = \frac{\mu H}{kT}$ should be small. This is possible whenever low temperatures and very high magnetic fields are avoided.

(2) Langevin, while giving his theory assumed that all possible orientations of a molecular magnet and applied field are permissible. Thus his treatment of the problem was classical . however, quantum theory stipulates that only certain discrete orientations are permitted.

(3) Although Langevin's theory applies only to gases, it can be extended to dilute solutions of paramagnetic salts in which the ions move about independently of each other. this is verified experimentally also.

(4) All paramagnetic substances become solids at low temperature at which the magnetic molecules will not be free to move in spite of this Langevin's theory was tested and confirmed by Kamerlingh Onnes for solid substance, Gadolinium Sulphate.

21.7 DEFECTS OF LANGEVIN'S THEORY AND WEISS MODIFICATION

(1) Langevin's theory could not explain the dependence upon temperature shown by several paramagnetics such as highly compressed and cooled gasses, concentrated slat solutions, solid slats and crystals.

(2) All experimental observations lead to relation $\chi = \frac{c}{T-\theta}$ where C is a constant. This is known the Curie Weiss law. But Langevin's theory shows only that $\chi = \frac{c}{T}$.

(3) The Theory did not explain the intimate relation observed between para and ferro-magnetism.

In 1907, weiss modified Langevin's treatment by introducing a new concept of internal molecular field. when Langevin considered an ideal gas, he assumed that the mutual effect of the molecular magnets is negligible. When the external field tends to turn the molecular magnets, he had assumed that this will be opposed by thermal agitation.

But Weiss assumed that in a real gas, the molecules are mutually influenced by their magnetic moments. So at any point within the gas, a molecular field exists, produced by all the molecules in the neighborhood. This internal molecular field was assumed to be proportional the intensity of magnetization and act in the same direction. If the internal molecular field is H_m, I is the intensity of magnetization and β is a constant, $H_m = \beta I$

Then the effective filed H_e will be the vector sum of the applied filed H and the

internal molecular filed H_m.

Therefor $H_e = (H + H_m) = H + \beta I$

Substituting this value in equation 21.7, it can be proved

that $\chi_M = \frac{CM}{T-\theta}$

This is the Curie-Weiss law and θ is the Curie point which is equal to $\frac{\sigma^2 \beta \rho}{3RM}$ at this temperature ferr0-magnetics become paramagnetics. According to this law, susceptibility varies inversely as the excess of temperature above curie point, but not above absolute zero.

Many paramagnetic substances were found to obey Curie-Weiss law, as prredicted by theory. But this law could not explain why paramagnetic susceptibility of many alkali metals is partically independent of temperature. Pauli could give a satisfactory explanation for this, by applying quantum statistics to the free electrons in a metal.

21.8 WEISS THEORY OF FERRO-MAGNETISM

The Magnetism is observed in ferromagnetic materials is much larger than in paramagnetic substances. Elements like Fe, Co, Ni, Gd and a number of alloys are ferromagnetic. In order to explain the relation that exists between Para and Ferro-magnetism, Weiss introduced the idea of "domains". A region in which all the molecular magnets have only one direction of magnetization is called a domain. For these domains also weiss assumed thet eqn 21.8 could be applied.

i.e., $He = H + \beta I$

When β is positive, these domains get spontaneously magnetized, even in the absence of an external magnetic field. Particularly below a certain temperature, a ferromagnetic material should become spontaneously magnetized and all magnetism would be in the same direction. The magnetic moments of the individual atoms are coupled to neighboring moments every strongly through an interaction called the exchange interaction. Because of this interaction, all the moments are aligned in the same direction. If this is the case, why is it that every piece of iron or nickel is not a magnet?

The reason is that an ordinary piece of unmagnetized ferromagnet consists of a large number of domains. In any one domain, all the atoms are aligned with their axes in one direction. But the direction changes form one domain to the other in such a way that the domain axes from a series of closed loops, as shown in Fig.21.3 (a). In a Crystal, these magnetic fields of the domains cancel out and virtually no external or resultant field can be detected. In this condition, we say that the specimen is unmagnetised.

If now it is placed in a magnetic field, couples act on the atoms tending to rotate them along the direction of the field. For small fields, the domain boundaries get shifted such that proportion of the aligned atoms increases. (See Fig. 21.3 (b)). this shifting of boundaries is reversible, because when the magnetizing field is removed, boundaries return to their original positions.

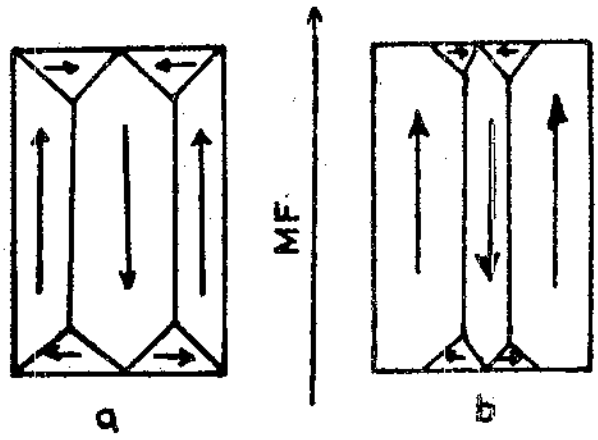


Fig.21.3 (a). Each arrow mark in a domain indicates the direction along which atoms are aligned, In the unmagnitised specimen, the domain form loops. (b). Where the Specimen is placed in a magnetic field, domain boundaries get shifted. If more atoms are aligned in the direction of the field, the area of that domain.

When the applied field is increased the axes of domain rotate abruptly one after another and magnetization increases rapidly. The changes are mostly irreversible and hence the material retains much of its magnetization, even when the field is removed.

In single large crystals of iron and other ferro-magnetic materials, the domains are as shown in Fig 21.3. But generally, these substances solidify in a polycrystalline from ie., infinite number of microscopic crystals, oriented at random throughout the specimen. Each crystal will then be separately divided into domains. They respond independently to the applied field. The domains in this case are represented in Fig.21.4

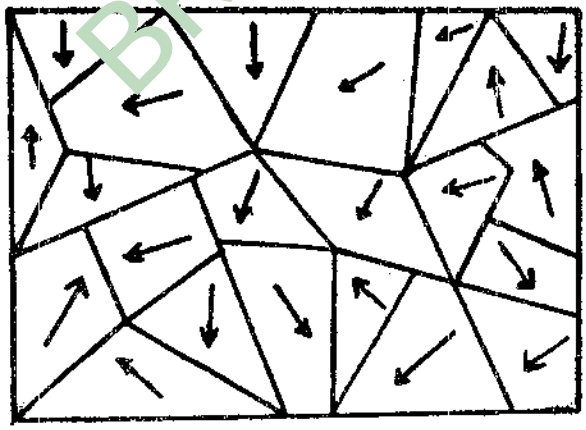


Fig 21.4 Random arramgement of domains in the unmagetised crystalline substances.

The observed properties of the whole specimen are thus the resultant effect of the random arrangement of the crystals.

Let us consider a gram molecule of the substance of molecular weight M and density ρ. Suppose σ is the gram molecular magnetic moment and σ_o its saturation value.

Since the domains are assumed to obey the general theory of paramagnetism.

Where $a = \frac{\mu He}{kT}$ 21.9

when the external field is zero, $H_e = 0 + \beta I = \beta I$

$$a = \frac{\mu He}{kT} = \frac{\mu \beta I}{kT} = \frac{\mu \beta \sigma \rho}{kTM} \qquad \text{Since } I = \frac{\sigma \rho}{M}$$

$$= \frac{\sigma_o \beta \sigma \rho}{RTM}$$

$$\sigma = \frac{RTM}{\sigma_o \beta \rho} \times a$$

$$\frac{\sigma}{\sigma_o} = \frac{RTM}{\sigma_o^2 \beta \rho} . a \qquad 21.10$$

The curves corresponding to the two equation (21.9) and (21.10) are shown in Fig. (21.5) Graph (1) represents equation (21.9) and graph (2) represents equation (21.10). The two curves intercept each other at the origin 'o' and at another point A.

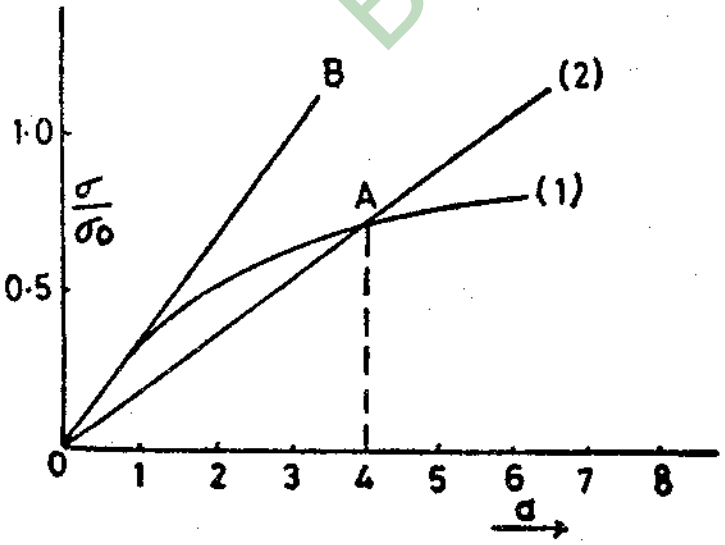


Fig. 21.5 Graph drawn between a and $\frac{\sigma}{\sigma_o}$

Graph (1) represents equation 21.9 and Graph (2) represents equation 21.10

Since we have assumed that the external field is zero, the molecules in the domains get spontaneously magnetised. The point A, which is the point of intersection of both the curves represents a stable of spontaneous magnetisation. OB in the Figure represents the tangent at the origin to the curve (1) also called the Langevin curve. If slope of graph (2)

coincide or becomes greater than that of the tangent OB, the two graphs will not intercept and hence spontaneous magnetisation will not occur.

According to equation 21.6(a), L(a) = $\frac{a}{3}$. Therefore $\frac{\sigma}{\sigma_o} = \frac{a}{3}$

For a stable spontaneous magnetisation

$$\frac{RTM}{\sigma_o^2 \beta\rho}.a < \frac{1}{3}.a$$

$$\frac{RTM}{\sigma_o^2 \beta\rho} < \frac{1}{3} \quad \text{or} \quad T < \frac{\sigma_o^2 \beta\rho}{3RM}$$

Since $\frac{\sigma_o^2 \beta\rho}{3RM} = \theta$ the curie point, T < θ.

So below the Curie point θ, when there is no external field, the domains are spontaneously magnetised, depending on the temperature.

Above the Curie point θ spontaneous magnetisation no longer occurs. The ferro-magnetic properties disappear and the substance becomes para-magnetic. At high temperatures, as a paramagnetic, it obeys Curie-Weiss law.

21.9 EXPERIMENTAL CONFIRMATION OF WEISS THEORY

As already explained in the last article, an ordinary piece of iron consists of many little crystals, with their axes pointing in different directions. Within each of these crystals, there are domains. When a small magnetic field is applied to the iron piece, the domain walls begin to move and some domains grow larger.

In every small crystal of the material, there will be impurities, dirt and imperfections. When a certain magnetic field is applied, the domain wall begins to move and in this process get stuck to the impurities, dirt or the imperfections. If the magnetic field is further increased, the domain wall suddenly moves in jerk. This process jerky motion of the domain walls continues, until the applied field is very high. This phenomenon by which the domain walls begin to jump, snap and jiggle as the field in increased, is called the Barkhausen effect. This can be phsically demonstrated by using the arrangement shown in Fig.(21.6). A coil of a few thousand turns is connected to a amplifier and a loud-speaker. A few silicon steel sheets are placed at the centre of the coil. If a bar magnet is brought slowly towards the steel sheets, sudden changes in magnetisation occur in jerks as explained above. As a result, sudden impulses of e.m.f. are induced in the coil. These can be heard as distinct clicks in the loud-speaker. As the magnet is brought nearer, one can hear a whole rush of clicks. The sound resembles the noise of sand grains falling down, when a can of sand is titled.

When the magnent is withdrawn, a similar sound is heard denoting that the domains are trying to get back to the original positions.

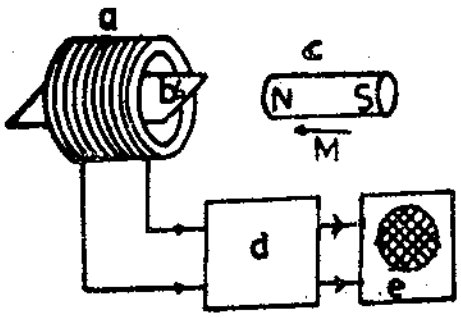


Fig.21.6 Experimental arrangement to demonstrate Barkhausen effect. When the bar magnent NS moved either towards or away from the steel strips, distinct clicks are heard, signifying the movement. a - Coil, b - Silicon steel strip, c - Bar magnet, d - amplifier, e - Speaker, M - motion.

This experiment lends support to the idea of spontaneously magnetised domains.

The susceptibilities of ferro-magnetic substances were determined at various temperatures above the Curie point. The experimental results were in confirmity with the theory, by obeying Curie Weiss law for paramagnetics.

Criticism of the Weiss Theory

(1) It was found that the Curie Weiss law breaks down at temperatures very near the Curie point. This could be explained only by assuming that the molecular magnetic moment is variable. But Weiss theory assumed it to be constant and independent of temperature.

These discrepancies were explained later by stipulating quantum conditions, according to which variation of magnetic moment takes place discontinuously.

(2) It is not possible to identity the domains, which are part of the individual crystals. On the basis of quantum theory, it was explained that the magnetic properties of a substance are determined by group of atoms which do not necessarily determine the crystal structure.

(3) The nature of the molecular field remians unknown, Here again, Heisenberg used a new quantum mechanics and showed that the molecular field is of purely electrical origin. Ferro-magnetism was attributed to the spin moments of electrons.

21.10 SUMMARY

There are three types of magnetic substances, they are, para, dia and ferro-magnetic substances.

Langevin explained para and dia magnetism on the basis of electron theory. This was further modified by Weiss in respect of paramagnetism as well as ferro-magnetism. Weiss theory of ferromagnetism found experimental supports is what is known as Barkhausen effect.

21.11 MODEL EXAMINATION QUESTIONS

I. Answer the following in detail.

1. Derive an expression for the mass susceptibility of a diamagnetic substance on the basis of Langevin's Theory.
2. Derive an expression for the subsceptibility of a paramagnetic substance.
3. What are the salient features of Langevin's Theory of paramagnetism? How was it modified by Weiss?
4. Describe Weiss Theory of Ferromagnetism. What was the experimental evidence in its favour? On what counts did it fail?

II. Answer the following in briefly.

1. What are the three types of magnetism? How do you distinguish between them?
2. Explain the Weiss concept of Internal molecular field.
3. Describe the domain theory with reference to a large crystal and a polycrystalline substance.
4. Describe the experimental set-up to demonstrate Barkhausn effect.
5. What are Curie law and Curie-Weiss law? Under what conditions are they applicable?

UNIT–22 : NUCLEAR MAGNETISM

Contents

22.1 AIMS AND OBJECTIVES

This Unit explains the technique of Nuclear Magnetic Resonance. In order to make you understand the technique the unit explains the magnetic nature of the nucleus.

After going through this unit you will be

- able to determine the molecular structures of complex compounds both organic and inorganic substances using NMR technique
- able to describe the magnetism possessed by nucleus is due to the orbital and spin motions of the charged particles.

22.2 INTRODUCTION

In the Bohr atom model, it was assumed that the electrons move in selected orbits, obeying the laws of classical mechanics. In 1925, two Dutch Physicists, Uhlenbeck and Goudsmit introduced the concept of spinning electron, according to which the electron revolves not only in the orbit, but also about its own axis, like a planet in the Solar System. So, the electron has a mechanical angular momemtum. Since a revolving charge is equivalent to a current, this spin gives rise to a magnetic moment. The total angular momentum as well as the total magnetic moment of the atom are due to both the orbital and spin motions of the electron. Every atom is, thus the resultant of two magnets, one arising from the orbital motion and the other from the spin motion of electron.

The magnetic moment due to the orbital motion of the electron is given by

$$\vec{\mu}_l = \frac{eh}{4\pi m}\vec{l}$$

where e = charge of the electron
h = planck's constant

m = mass of the electron
ℓ = orbit angular momentum

$\frac{eh}{4\pi mc} = \mu_B . \mu_B$ is the unit in terms of which atomic magnetic moments are measured. This unit is called the Bohr magneton.

Similarly, the magnetic moment due to the electron spin is given by

$$\mu_s = \frac{eh}{4\pi m} = \mu_B$$

22.3 NUCLEAR SPIN AND NUCLEAR MAGNETISM

A number of experimentally observed facts such as α-rays and γ-ray spectra etc., show that particles such as protons and neutrons inside the nucleus are also in continuous motion in discrete quantised orbits. So, these particles possess mechanical angular momentum. The electrically charged particles among them will have magnetic moment also. In addition to these orbital motions, nuclear particles of finite dimensions will spin around their own axes like the spinning electrons. This gives rise to a spin angular momentum and a consequent magnetic moment.

Thus, the nucleus possesses an intrinsic angular momentum, which is the resultant of the orbital and spin momenta of the various constituent particles. The total resultant angular momentum is called the nuclear spin.

Similarly, a nucleus possesses a magnetic moment, which is the resultant of the moments due to the orbital and spin motions of the constituent charged particles. This total resultnat angular momentum is called the nuclear magnetic moment.

The nuclear spin is represented by the quantity I. In other words, I is the angular momentum vector of the nucleus. That is, the resultnat angular momentum of the orbital and spin angular momenta of different particles is $\frac{Ih}{2\pi}$.

The atomic electrons will have the angular momentum vector represented by J. If I and J are combined vectorially we get total angular momentum F=I+J of the atom. That is the total angular momentum of the atom will be $Fh/2\pi$.

The nucleus possess a magnetic moment associated with the nuclear spin. The nuclear magnetic moment is given by

$$\mu_N = \frac{eh}{4\pi m} F$$

Where e = Charge of the nucleus
h = Planck's constant
M = Mass of the nucleus
F = Hyperfine quantum number

Since M is always greater than the mass of the electron, it will be very small compared to the Bohr magneton.

22.4 STERN GERLACH EXPERIMENT

For every atomic system, the angular momentum along any particular axis is always an integer or half-integer times $\frac{h}{2\pi}$ (also represented as $\hbar$). It means that the angular momemtum is quantised. This fact is surprising, because according to classical theory, angular momentum can take all possible values. The experiment of stern and Gerlach can be considered as a direct justification for the quantisation of the angular momentum. They devised an expriment for measuring the magnetic moment of individual Silver atoms.

Principle of the experiment

The magnetic moment of an atom arises from the arbital and spin motions of the electrons in it. Hence, the atom can be considered as an elementary magnet. If this is placed in a magnetic field the field acts on the atomic magnet.

If the field is homogeneous, the atomic magnet will experience a couple which will rotate its axis into the direction of the field. This is so because of the forces on the poles of the atomic magnet due to the external field are equal and opposite. When the atomic magnets move in such field along a direction perpendicular to the field, they just travel along a liner path without any translational displacement.

If the field is non-homogeneous, the forces on the poles will be unequal. As a result, the axis of the atomic magnet is not only rotated into the direction of the field, but there is a translatory displacement of the atom as a whole.

Apparatus : The experimental set-up used by Stern and Gerlach is shown in Fig.22.1.

The substance whose magnetic moment is to be determined is heated in an electric oven. Atomic rays of the substance are sent out in all directions with a velocity corresponding the temperature of vapourisation. The hole acts as a slit and a sharp linear beam of atoms is obtained by using a series of holes. This passes through the pole-pieces of an electro magnet. By using specially shaped pole-pieces, a non-homogeneous magnetic field is produced.

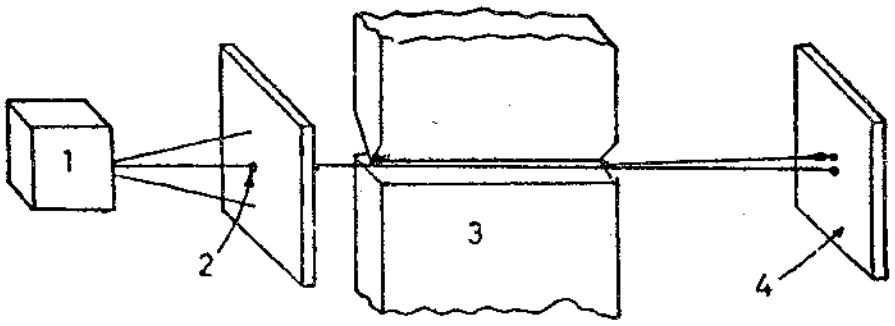


Fig 22.1(a) Stern-Gerlach experiment. Silver produced in the oven, through a hole and get divided into two separate beams represented by two dots on the glass plate.
1 - Oven, 2 - Hole, 3 - Magnet, 4 - Glass plate.

One pole is shaped as a knife-edge and the other has a flat face, provided with a groove. The magnetic lines of force crowd together at the knife-edge where the field strength is greater.

The beam is made to strike a glass plate P. The entire apparatus is in an evacuated chamber.

Experimental procedure

Stern and Gerlach produced a beam of silver atoms by evaporating

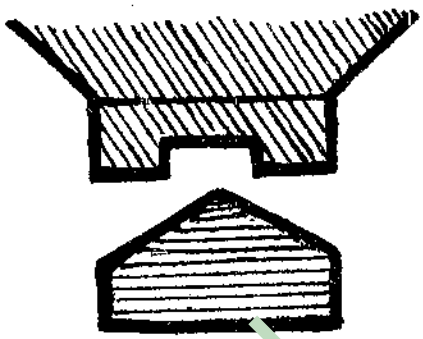


Fig.22.1(b) Pole pieces of a non homogeneous magnetic field employed in Stern Gerlach. One pole has flat face with a groove and the other has the shape of a knife edge.

silver in a hot oven. The beam was directed along the sharp edge of the electromagnet, so that the atoms are subjected to a vertical force in non-homogeneous field. A silver atom with its magnetic moment along the horizontal direction will not experience any force and hence would go straight. An atom with its magnetic moment, directed vertically up, would experience a force pulling it upwards. An atom with its magnetic moment pointing downwards would experience a downward push.

According to classical theory, all directions are possible for the magnetic moments. Hence, the atoms would be spread out according to their vrtical components of magnetic moment. So one should expect a deposition of silver atoms on a glassplate along a vertical line. But, Stern and Gerlach found on the glass plate two distinct spots only, thus revealing the complete failure of the classical theory.

Precautions to be followed while doing the experiments

1. As the deflections obtained are small, various parts of the apparatus must be carefully aligned.
2. The oven must be capable of withstanding a high temperature.
3. The chamber must be exhausted completely, so as to avoid collisions of the silver atoms with the atoms and molecules of any residual gas.

Fig.22.2 Traces obtained with Silver atoms, (a) Without field, (b) With field

Results

When the field is off, a sharp line was traced by the atomic beam on the plate. On establishing the non-homogeneous field, a double trace was obtained (as shown Fig.22.2b), and on either side. The traces are a little diffused because of the Maxwellian velocity distribution of the atoms.

The displacement D along the field direction is given by

$$D = \frac{1}{2}\left(\frac{\mu}{m}\right)\frac{dH}{dx}\left(\frac{L}{v}\right)^2 \qquad 22.1$$

Where μ = magnetic moment

m = mass of the atom

$\frac{dH}{dx}$ = rate of change of field with distance along the x-direction;

L = length of the field

v = velocity of the atom

It was calculated by using equation 22.1 and in the case of Silver, it was found to be equal to one Bohr magneton. Same was the case with H, Li, Na, K, Cu and Au, all of which gave 2 traces and μ was calculated as $\pm$ 1 Bohr magneton.

In the case of Zn,Cd,Hg, Sn, Pb = no effect was produced by the field. Hence μ=0.

In the case of Iron and Cobalt, μ was found to be greater than or equal to 6 respectively.

Significance of the results

These results verify the conculsions of the Spatial quantisation theory, namely (1) that only a certain discrete number of settings are possible for the atomic magnets with

respect to the field direction, i.e. (2J+I) orientations alone are permitted where J is the total angular momentum quantum number.

(2) that the electron has a spin of $\frac{1}{2}$

Thus, the essential characeristics of the vector atom model are confirmed by the experiments of Stern and Gerlach.

22.5 NUCLEAR MAGNETIC RESONANCE

In the Stern-Gerlach experiment, the deflection of atoms is very small and hence the measurement of the magnetic moment is not very precise. Rabi and his collaborators improved the appratus which resulted in a fantastic precision. Atoms, in general, have only certain discrete energy levels. Similarly, when atoms, are kept in a magnetic field, their orignal energy is split up into a finite number of energy levels.

When an atom has two levels which differ in energy by the amount ΔE, it can make a transition from the upper level to a lower level by emitting light of frequency ν where

$$\Delta E = h\nu \qquad 19.5$$

The samething can happen with atoms in a magnetic field. But the energy differences are so small that the frequency does not correspond to light, but to micro-wave or radio-frequencies. These are the lowest-energy form of electromagnetic radiation lying between 10^7 and 10^8 Hz. The transitions from the lower energy level to an upper energy level of an atom can also take place by the absorption of energy. Thus, if we have an atom in a magnetic field, we can cause transitions from one state to another by applying an additional electromagnetic field of the proper frequency. Such transitions take place only for a frequency called the resonance frequency. For any other frequency of the electromagnetic field, the chance of causing a transition is very small. This phenomenon is called nuclear magnetic resonance (NMR). Here, the radio-frequency (RF) waves induce transitions between the energy levels of nuclei of molecule, when kept in a magnetic field.

The quantity of energy involved in Rf radiation is very small and it is not sufficient either to vibrate, rotate or excite an atom or molecule. But this energy can affect the nuclear spin of the atoms of a molecule.

If a nucleus having a magnetic moment is introduced into a magnetic field H_o the two energy levels become separate corresponding to $m_I = +\frac{1}{2}$ and $m_I = -\frac{1}{2}$, parallel and antiparalle to the direction of the field. I represents the spin quantum number of the nucleus. These two energy levels E_1 E_2 are represented in Fig 22.3.

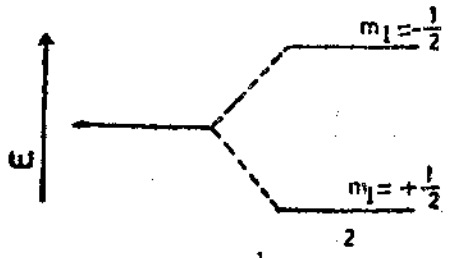


Fig 22.3 Energy-levels of a nucleus with $I = \frac{1}{2}$ in the absence and presence of a magnetic field.

When the nucleus absorbs energy, it will be promoted from the lower energy state E_1 to the higher energy state E_2 ie. the magnetic moment changes from the parallel state $m_I = \frac{1}{2}$ to the anti-parallel state $m_I = -\frac{1}{2}$

If the nucleus is in the higher energy state E_2 and radiation of energy ΔE is incident on the system, the nucleus will come down to the lower energy state E_1 and in this process, it emits energy corresponding to ΔE. The frequency at which energy is absorbed or emitted is given by

$$\nu = \frac{E_2 - E_1}{h} = \frac{\Delta E}{h} \qquad 19.6$$

This frequency is directly proportional to the applied magnetic field.

Thus, when a nucleus absorbs energy, it gets excited. It then emits energy while returning to the unexcited state. It absorbs energy again from the system in which it is placed and gets excited. The nucleus which thus oscillates between the excited and the unexcited states is said to be in a state of resonance.

This technique of NMR is very useful for investigating nuclear structure. The first observations of NMR signals were observed independently by Purcell and Bloch in 1945. A study of the structure of ethyl alcohol was undertaken in 1951, using the NMR technique. In 1952, both Purcell and Bloch were awarded the Nobel Prize physics.

22.6 PRINCIPLE OF THE MAGNETIC RESONANCE METHOD

If a particle having an angular momentum J and a magnetic moment μ is placed in a magnetic field H_o it will experience a torque equal to μ Ho, which makes it precess about an axis parallel to the magnetic field.

The angular velocity of precession is given by $\omega_p = \frac{\mu Ho}{J}$ 19.7

The resonance method consists in adjusting the strength of the magnetic field until the precession is in resonance with the super-imposed oscillating magnetic field H_1 whose frequency is in the RF range. suppose the angular velocity of the oscillating field is ω if $\omega = \omega_p$, the particle will suffer re-orientations. The spins of the atoms will change and, as such, they will be deviated from their original paths. They will not reach the detector (See the dotted lines a and b in Fig.22.5)

If ω is appreciably different from ω_p it will not cause any spin changes. So, the atoms will follow their original paths to the detector. The value of the current registered by the detector will not change. When ω is in resonance with ω_p a decrease in the detector current will be observed, as sown in Fig.22.4 knowing ω_p from the graph, nuclear magnetic moment μ can be calculated.

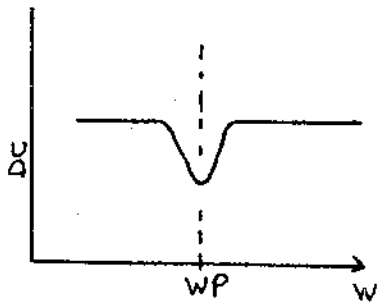


Fig.22.4 Where $\omega_p = \omega_s$ resonance conditions occur and a decrease in the detector current is observed. d. Dc - Detector current.

The resonance frequency can be determined by another method. As the magnetic field H_o is varied, for a certain value when $\omega = \omega_p$, transition occurs between nuclear energy states. The energy absorbed in this process produces a signal at the detector. This signal is amplified and recorded as a band in the spectum. This is called an NMR spectrum. A graph is

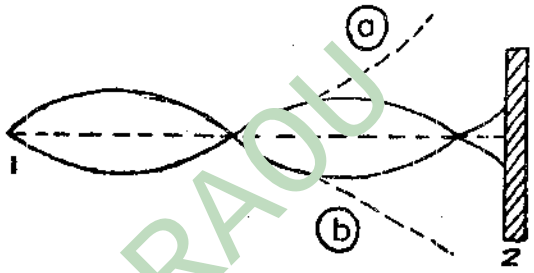


Fig.22.5 Atoms starting from the oven pass through a magnetic field over which another oscillating field is superimposed its the precession of the atoms is in resonance with the oscillating magnetic field, the atoms will be deviated along the dotted lines (a) and (b) will end reach the detecter (1) Electric oven (2) Deflector

drawn between absorption signal at the dectector and frequency of RF radiation is shown in Fig.22.6 from which the resonance frequency can be noted.

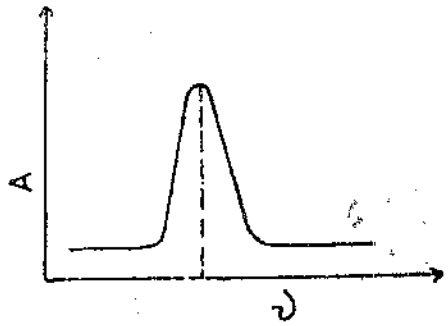


Fig.22.6 Graph between absorption signal at the detector and the frequency of R.F. radiation the peak in the figure indicates the A - Absorbance; ν - Frequency.

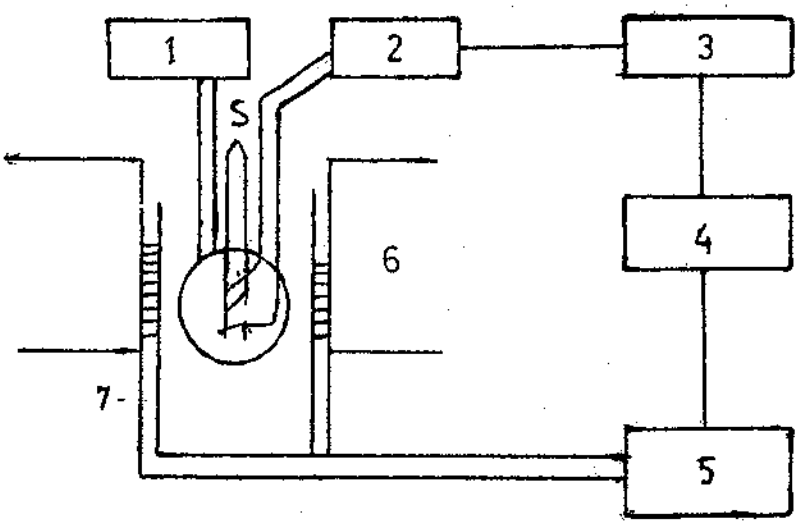


Fig.22.7 Schematic diagram of a N.M.R. spectometer. 1 - Transmitter, 2 - RF receiver, 3 - Deflector and Ampliper, 4 - Recorder, 5 - Sweep generator, 6 - Magnet, 7 - Sweepcoils

22.7 NMR SPECTROMETER

The first commercial NMR Spectrometer was sold by the varian associates in 1953. Second-generation instruments were transistorized. The third generation instrument with integrated circuits and mini-computers was introduced in the early 1970's.

A schematic diagram of NMR spectrometer is shown in Fig.22.7. A typical spectrometer consists of the following components.

1. Sample Holder

The folder should be chemically inert and durable. Cheap and sturdy glass tubes about 7.5 cm. long and 0.3 cm in diameter are used as sample holders (S in the Figure)

2. Magnet

This may be permanent or an electromagnet. It is improtant that it should give a homogeneous magnetic field. i.e., the magnitude and direction of the magnetic field should not change from point to point. The strength of the magnetic field should be of the order of 20,000 gauss and it should be constant. It is very important that the magnets are so designed as to have a uniform and constant field.

Conventional electromagnets are found to be less stable. They can be made stable by using stabilising devices. Permanent as well as electromagnets are used upto 100 Mhz. But electromagnets consisting of super conducting solenoid operating in liquid Helium Crystals are highly stable. These are used upto 230 M Hz.

3. Sweep generator

To create the resonance conditions, the precession frequency of the nucleus should be equal to the frequency of the applied rf radiation. Usually, the rf radiation is kept constant and the applied magnetic field H_o is varied so as to bring about resonance in nuclei. But, changing the magnetic field of a large stable magnet was found to be technical problem.

so, a pair of Helmholtz Coils are fixed to the pole faces of the main magnet. The current flowing through them is changed and as a result the magnetic field induced by these coils also changes. Since this induced field is in the same direction as the main field it gets added to it.

4. Transmitter or rf generator

To generate rf radiation, rf oscillator is used. The coil of oscillator is wound round the sample container, so that the sample is subjected to rf radiation. The oscillator coil T would be perpendicular to the applied mgnetic field H_o so that it does not change the effective magnetic field.

5. rf receiver

When the rf radiation is passed through the magnetised sample, two types of signals called absorption and dispersion signal arise, by observing which the resonance frequency can be determined, A detector is used to separate the two signals.

6. Ampliffer and Recorder

The absorption signal received from rf receiver is extremely weak. So, it is considerably amplified and then fed to a Recorder.

22.8 APPLICATIONS OF NMR SPECTROSCOPY

22.8.1 Chemical Shift

The meagnetic resonance phenomena have been used in many ways as tools for finding out new things about matter-especially in chemistry and nuclear physics. The numerical valves of the magnetic moments of nuclei tell us something about their structure. In chemistry, much has been learned from the shape of the resonances. Because of magnetic fields produced by neaby nuclei, the exact position of a nuclear resonance is shifted somewhat, depending on the environment in which any particular nucleus finds itself. Measuring these shifts helps us to know which atoms are near which other ones and enables us to understand the structure of molecules. These shifts are called chemical shifts.

The chemical shift is directly proportional to the total applied field and is measured in relative units of parts per million (PPM). The resonance frequencies of nuclei in a sample are measured relative to the resonance frequency of a nucleus in a reference compound and the frequencies are quoted relative to the reference frequency. The chemical shift, is defined as

$$\frac{H_o(reference) - H_o(Sample)}{H_o(reference)} \times 10^6$$

The chemical shift indicates what type of hydrogen atoms are present, e.g. methylene, methyl groups, olefins, others etc.

22.8.2 Spin-spin Coupling

The inter-action between the spins of the neighbouring nuclei in a molecule may cause the splitting of lines in the NMR spectrum. This is known as spin-spin coupling. This contributes to a better understanding of the electronic structure of molecules

3. NMR spectroscopy has been used to determine the molar ratio of the components in a mixture: (a) to study the hydrogen bonding in organic compounds.

(b) to study polyethylene, which has wide applications in our daily life as well as in industry.

(c) to determine the total concentration of a given kind of magnetic nucleus in a sample. For example, the total hydrogen in an organic mixture can be determined.

22.9 LIMITATIONS OF NMR SPECTROSCOPY

The various limitation of NMR spectroscopy are as follows:

1. One of the problems is its lack of sensitivity. The minimum size of the sample is about 0.1 ml with a miniumum concentration of about 1 percent.
2. In some compounds, two different types of hydrogen atoms resonate at similar resonance frequencies. This results in an overlapping of spectra, whose interpretation becomes difficult.
3. While determining the structure of organic compounds, no information about molecular weight is given. The relative number of different protons present are only known.
4. In most of the cases, only liquids can be studied by NMR spectroscopy.

Inspite of these limitations. NMR is one of the most useful analytical tools forstudying the structure of many inorganic and organic compounds.

Worked example

1. Caluclate the value of the Bohr magneton:

Bohr Magneton $M_B = \frac{eh}{4\pi m_e}$

Planck constant h = 6.63 x 10^{-34} J.S.

Specific charge $\frac{e}{m_e}$ = 1.760 x 10^{11} c. kg-

$$M_B = \frac{1.76 \times 10^{11} \times 6.63 \times 10^{-34}}{4\pi}$$

= 0.9281 x 10^{-23}

= 9.281 x 10^{-24} A-m^2

22.10 SUMMARY

It is established that:

A nucleus possesses magnetism because of the orbital and spin motions of the charged particles in the nucleus.

The atomic magnet possesses only a certain number of discrete orientations with respect to a reference directions.

The electrons have a spin of $\frac{1}{2}$

The technique of nuclear magnetic resonance is a very useful tool to investigate molecular structure of complex compounds, both organic and inorganic.

22.11 MODEL EXAMINATION QUESTIONS

I. Answer the following in detail.

1. Explain Stern-Gerlach experiment. State the principle of the method and its signifance.
2. Describe NMR Spectrometer. What are its applications.

II. Answer the following briefly.

1. What is meant by nuclear magnetic moment and nuclear spin?
2. Explain nuclear magnetic resonance.

22.12 RECOMMENDED BOOKS

1. Spectroscopy - By Gurdeep Chatwal and Sham Anand
2. Feynman lectures on Physics Vol.II.
3. Atomic Physics - J.B. Rajam
4. Introduction of Modern Physics - by Richtmyer - Kennard - Cooper
5. Mcgraw-Hill Encyclopedia of Science and Technology - Vol. 9.

BLOCK-6 : SEMICONDUTOR PHYSICS

UNIT–23 : SEMI CONDUCTORS

Contents

23.1 AIMS AND OBJECTIVES

This unit explains the classification of materials based on the band theory of solids.

After going through this unit you will be

- able to distinguish between the conductors, insulators and semi conductors
- familiarised with the simple semi conductor devices and
- able to explain the carrier density and determinations of energy gap.

23.2 INTRODUCTION

Considerable attempts were made by Sommerfeld to develop the free electron model in order to explain certain physical properties of solids like electrical conduction, specific heat, thermal conduction etc. The theory assumes that the valance electrons pertaining to the atoms are relatively free and the conglomaration of these electrons is known as Fermi gas. Many of the physical properties of metals can be easily explained by applying the Fermi-Dirac statistics to this type of system of particles and it can be extended to obtain the electrical conduction in metals. But later discoveries showed that the free electron theory fails to explain certain physical properties of solids like, increased conductivity in impure semiconductors and decreased resistivity of intrinsic semiconductors with rise in temperature, and existance of hole in semiconductors etc. This discrepancy was mainly due to the fact that the valance electrons in solids are not completely free as proposed by the free electron model. The behaviour of the electrons completely change when the interaction of positive ion cores and the neighbouring electrons are taken into consideration. The electron-lattice interaction revealed certain new facts. This new theory tried to explain the differences in properties of conductors, semiconductors and insulators. The theory developed based on the wider understanding of the materials in known as band theory of solids. Many factors inherent in this helped to pave the way for satisfactory explanation of the solid state behaviour.

23.3 BAND THEORY OF SOLIDS

According to band theory of solids one has to examine the motion of an electron under the influence of the surrounding electrons and the positive ion cores embeded in the crystal lattice. The fundamental aspects of this problem is the periodic nature of the potential energy function of the moving electron in the crystal. The electron energy level diagrmas that have been adopted for single, free atoms will no longer apply to the same atoms in a crystal. The energy levels of the inner electrons will be changed by a small amount since the inner electrons (electrons that are close to the nucleus) are not affected very much by the presence of neighbouring atoms. However, the levels of outer (valence) electrons are greatly influenced since these are the electrons which participate in chemical bonding. As the atoms are densly packed interaction between the outer electrons of the atom increase. Instead of obtaining a single descreate energy line two atoms will produce two lines (Fig.23.1) and N atoms will produce N lines.

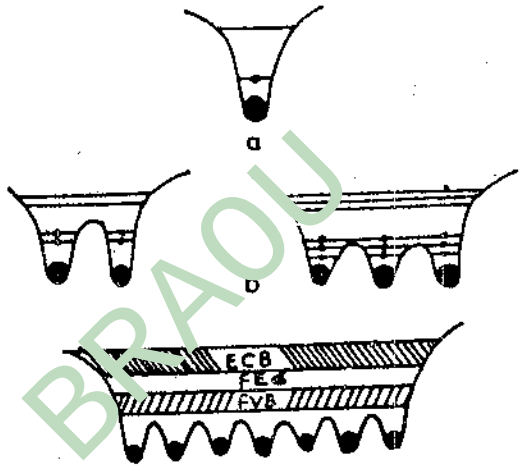


Fig.23.1 Energy level diagram
a) Isolated atom, b) two and three atoms close together, c) Solid crystal.
ECB - Empty Conduction band, FEG - Forbidden energy gap and FVB - Filled valence band

It is found that there is now a band of energies instead of descrete energy level as in the single atom. The energy of the outer electrons of any particular atom in the crystal must lie within this band, and it is not possible to specify an individual level any more. The width of the energy band depends upon the coupling between the outer electrons of the atom that is, upon the interatomatic distance of the crystal. If the interatomic distance is decreased the coupling increased and the band width increases. Fig.23.2 shows a theoritical curve, as it is not possible to vary continuously the interatomic distance in the crystal. But it is introduced here for a better understanding of the band theory of electron energies in crystals.

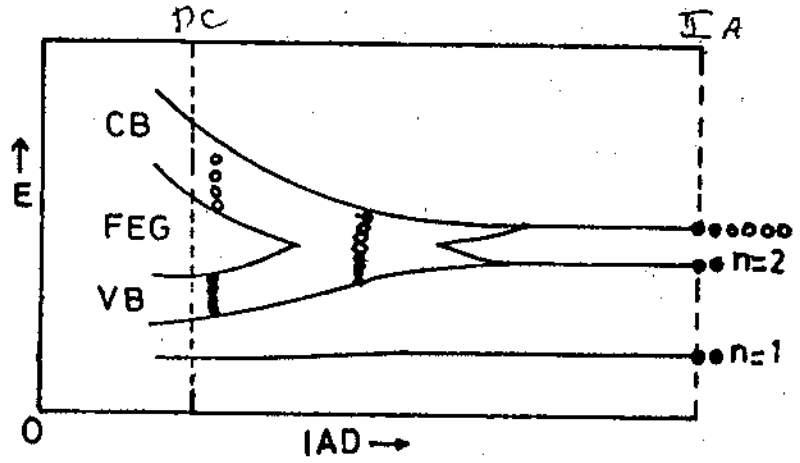


Fig.23.2 theoretical curve of the energy bands in carbon as a function of interatomic distance DC - diamond crystal, CB - Conduction band, IAD - Interatmatic distance, VB - Valence band, IA - Isolated atom, FEG - Forbidden energy gap, O - Occupaied state, o - state per atom

In carbon, there are two electrons in the n = 1 shell and four in n = 2 shell. However, there are eight energy states in the complete n = 2 shell. In the isolated atom shown on the right hand side of Fig. 23.2 six electron states have been shown filled and the remaining four states in the n = 2 shell are completely empty. As the interatomatic distance is decreased, the descrete levels broaden out into bands, and eventually the bands corresponding to the two sub-shells of n = 2 overlap as shown in the centre of the diagram. As the interatomic distance is further increased, a split occurs between the four upper states and the lower four states. At the point marked with dotted line the two bands are widely separated. The dotted line indicated refers to that of diamond crystal. The electron level corresponding to n = 1 hardly changes as the interatomic distance is decreased. The upper part of the energy band for n = 2 is known as the Conduction band, and the lower part is the valance band. In between these two, there is a region where no electron energy levels are permitted, and this region is known as the forbidden gap. Fig.23.3 shows an energy band diagram for a diamond crystal plotted in the same way as the energy level diagrams that have been discussed previously. At the bottom of this diagram is shown a narrow band corresponding to n = 1.

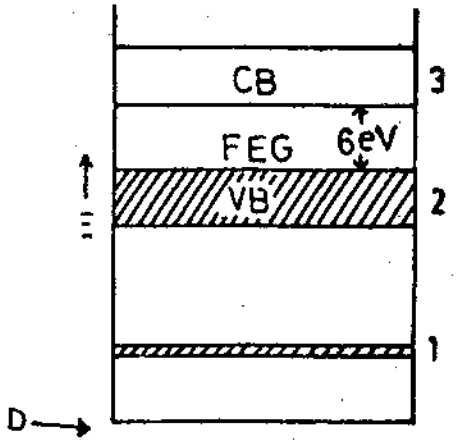


Fig.23.3 Band energy diagram for a diamond crystal.
E - Energy; D - distance, 1) Two levels per atom (filled); 2) Four levels per atom (filled); 3) Four levels per atom (unfilled)

A carbon atom has six electrons and so the lowest levels per atom will be filled. Thus in Fig.23.3, the two levels corresponding to n = 1 and the four levels per atom in the valance band are filled leaving the conduction band completely empty. But in practice, this is only approximately true since we have neglected the effect of any heat energy which will be imparted to the electrons in the crystal at any given temperature. If this energy is so small that it has no effect on the distribution of electrons in the levels, conditions are exactly as shown in Fig.23.3 The reason for the names valance and conduction bands now becomes very clear. The four outer electrons per atom, which are in the valance band, are the four electrons that determine the valance of the carbon and that are participating in covalent bonds. These electrons, which are shared with the neighbouring atoms, are closely found to the nuclei and so their energy is lower than the energy of the corresponding state of free atoms. If one electron is removed from a covalent bond in a diamond crystal, it will have acquired sufficient energy to move into an energy state above that of the bound electron. This may be represented as the passage of an electron from valance band and the bottom of the conduction band. This is equivalent to the valance band to the conduction band. The miniumum is the energy difference between the top of the valance band and the bottom of the conduction band. This is equivalent to the width of the forbidden energy gap (4 eV is a diamond crystal). When one of the electrons in the conduction band is removed from a covalent bonds it can move freely from atom to atom since the conduction band is empty. An electron once removed from the valance band to the conduction band, is available to move in any electric field that may be applied across the material.

23.3.1 Insulators

The requirements for a crystalline substance to be insulater at room temperature are that the valance band must be full, the conduction band must be normally empty, and the forbidden gap must be greater than about one electron volt. Under these conditions, only a minute fraction of total number of electrons in the material can gain enough energy to cross the forbidden energy gap and reach the conduction band. The average energy possessed by an electron at room temperature (300°K) is only 26 x 10^{-3} eV whereas it requires 6 eV of energy to move an electron from the valance band to conduction band in diamond. When an electric field is applied across a crystal, there is a flow of electrons in the conduction band. Diamond with very few electrons in the conduction band, is therefore an extremely good insulator. If the temperature of the material is raised, the average energy of the electrons raises, and the number of electrons with enough energy to cross the gap also increases.

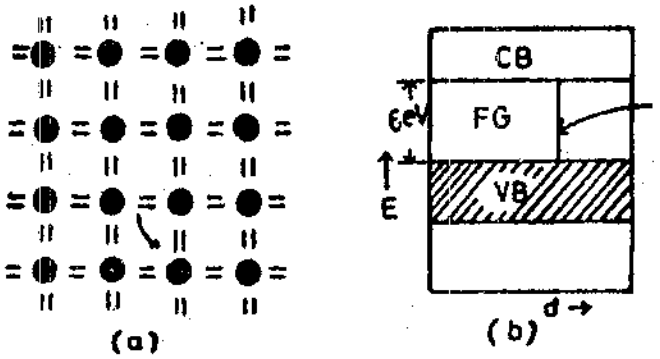


Fig.23.4 Simplified representation of an electron breaking from a covalent bond in diamond crystal.
CB - Conduction band; E - Energy; FG - Forbidden gap; d - distance;
1 - Electron moving from VB to CB.

Fig.23.4(a) shows a two-dimensional, simplified picture of a diamond crystal where the electrons that are held in covalent bonds are indicated by short lines. The arrow in this diagram indicates that one electron is about to break away from its bond, and it will then be free to move about in the crystal in the energy band diagram of Fig.23.4(b) this process is indicated by the arrow from the valance band to the conduction band In Fig.23.4(a) the two inner electrons are not shown since they play an insignificant part in the process of conduction; the energy band corresponding to these two electrons also has been omitted in Fig.23.4(b) Typical insulators include Mica; Quartz, Marble etc.

23.3.2 Conductors

The outer electrons in a conductor are loosely bound to the parent nucleus and the permissible electron levels are indicated in Fig.23.5. In this diagram, only the highest energy band is shown. This may be a single bond or it may be the result of the overlapping of two energy bands. However it is formed, it has more permitted electron levels than there are available electrons and so there are empty levels immediately above the

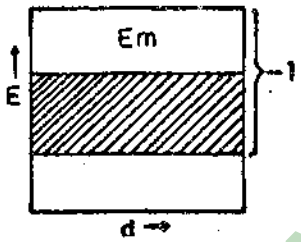


Fig.23.5 Band structure of a metal
E - Energy, D - distance; Em-Empty (1) Total band width; Band only partially fill.

highest filled levels. Consider an electron which has an energy corresponding to the top of the filled portion of the band. If this electron gains a small amount of energy from any source it can move up into the empty part of the band. It is then free to move about in the crystal because the energy states at this level are not occupied. Since both conduction band and valance band overlap on each other there is no forbidden energy gap in the energy band just above the filled levels, electrons are always free to move through the crystal under the influence of an electric field. A material of this type is known as conductor. Copper, silver, gold etc. belong to this category.

23.3.3 Semiconductors

A semiconductor has the energy band structure similar to that of an insulator. However an isulator has a forbidden gap which is so wide that very few electrons can cross it at room temperature; while a semiconductos possesses a narrow forbidden gap which allows a considerable amount of conduction at room temperature. Germinium and silicon are the two most commonly used semi-conducting elements and the widths of the forbidden energy gap are 0.72 and 1.12 eV respectively. These two materials have a resistivity intermediate between a conductor (such as copper) and an insulator (such as diamond). When the conductivity in a crystalline semiconductor is due soley to the breaking of covalent bonds the substance is said to be an intrinsic semiconductor.

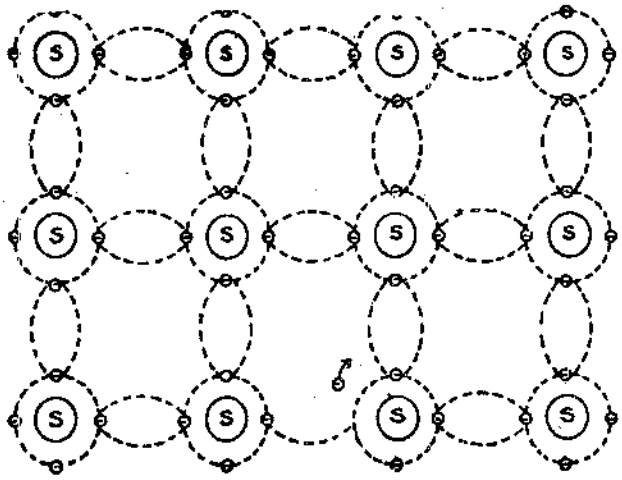


Fig.23.6 Diagramatic representation of Instrinsic semiconductor showing bonding of valuence electrons and the electron hole pair formation.
S - Germanium or Silicon

The atoms in a single crystal of either pure silicon or pure germanium are joined together by covalent bonds, and teh valance electrons are shared between the atoms as shown in Fig.23.6. At absolute zero temperature (i.e., at0°K), all the electrons are at their lowest energy levels, and the valance electrons are taken up in the covalent bonds of the structure. As a result there are no free electrons at this temperature and both silicon and germainium act as perfect insulators. However, this does not occur in practice, since it is not possible to manufacture a perfect crystal and impurities exist within the crystal (a crystal is said to be pure if the impurity content is less than 1 part in 10^{10})

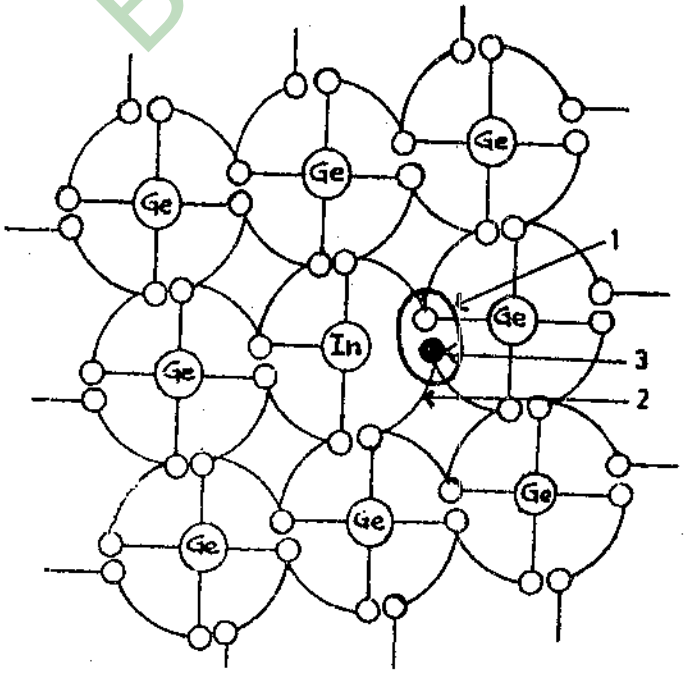


Fig.23.7 The structrure of a P-type semiconductor
1. Incomplete bond, 2.Acceptor atom, 3. hole (Electron deficiency)

At room temperature some valance electrons aquire sufficient energy to break away from the parent atom to become free (Fig.23.7) simultaneously, a hole is said to be generated, in the crystal structure (hole is equivalent to a positive charge carrier). This process is known as Thermal generation of electron-hole pair. Electron-hole pairs are generated at many points within the crystal with the result that an electron from one part of the crystal lattice recombines with a hole from another part of the strucrture. The average time that either charge carrier exists is known as the life time of the carrier, which is typically 10^{-2} to 10^{-3} second. When a hole and a electron recombine, they cease to exist as charge carriers. Thermally generated electronhole paris give rise to intrinsic conduction. When a potential difference is applied between the ends of the sample both electrons and holes contribute to the processes, of conduction. An increase in temperature (or incident radiation) causes the number of electron-hole pairs generated to increase. In turn this results in an increase in conductivity (i.e., reduction of resistance) with increase in temperature. For this reason most semiconductor materials have a negative temperature coefficient of resistance.

Check Your Progress - I

What is an intrinsic semiconductor?

23.4 EXTRINSIC CONDUCTION IN SEMICONDUCTORS

The deliberate introduction of a small quantity of impurity of the order of 1 part in 10^8 modifies the electrical properties of the semiconductor. It results in a new type of conduction known as extrinsic conduction. The process is called doping. If the impurity atom has a valancy of three, i.e., trivalent, a hole is generated in the host (silicon or germanium) lattice due to incomplete bonding as shown in Fig.23.7. This hole can be filled by any free electron in the crystal lattice causing the hole to appear at some other point. This type of impurity is known as p-type impurity (p for positive charge carrier) and the impurity atom is known as acceptor atom, since it results in the aceptance of an electron into the lattice. Typical acceptor atoms are boron (B), aluminium (Al), gallium (Ga) and indium (In). The mobile hole has all the charracteristics of positive charge carrier and holes are mojority charge carriers in P-type semiconductors. Electrons generated by thermal and other sources of energy are minority charge carriers.

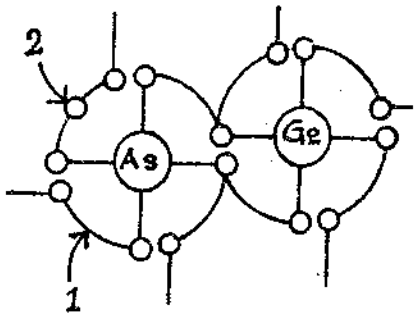


Fig.23.8 The structure of an n-type semichonductor
1. Donar atom, 2. Electron in Excess of Covalent bonding requirements.

If the imputity atom has a valency of five (i.e., penta valent) it has one more electron in its outer shell then have the host atoms (assuming them to be either silicon or germanium) as shown in Fig.23.8. Since there are only four electrons in the valance shell of the parent tetravalent atoms, the net charge on these atoms if the valence electrons were stripped off would be +4 electronic units. The additional electron orbiting the pentavalent atom is surplus

lattice than the valence electrons of the host atoms. This type of impurity is known as n-type impurity (n for negative charge carrier) and the impurity atom is a donar atom since it donates an electron for the process of conduction.

In n-type semiconductors, free electrons are the majority charge carriers. Holes generated by thermal and other means also act as charge carriers but since they are relatively few in number when compared with majority charge carrers they are called minority charge carriers. Typical donor elements include antimony (Sb) arsenic (As) and phosphorous (P).

There are compound semiconductors (intermetalic compounds) like gallium arsenide (GaAs which are used in many electronic devices, Gallium has a valance three and arsenic has a valancy of five. The same general rules doping apply to gallium arsenside as to silicon and germanium. Typical compound semiconductors include indium antimonide (InSb), indium arsenide (InAs), gallium phosphide (Gap), gallium arsenide (GaAs) etc.

Check Your Progress - II

What is doping

Table 23.1 Forbidden energy gap of typical semiconductors

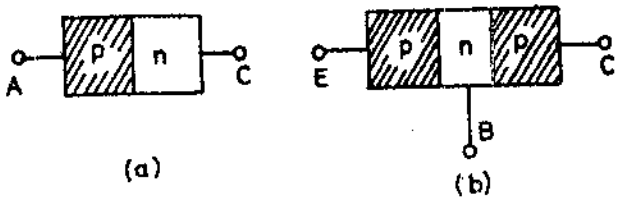
Name	Chemical symbol	Forbidden energy gap ev
Germanium	Ge	0.72
Silicon	Si	1.12
Indium antimonide	InSb	0.18
Gallium arsenide	GaAs	1.34
Cadmium sulphide	CdS	2.45
Zinc oxide	Zno	3.30

23.5 SEMICONDUCTOR DEVICES

By properly combining p and n materials in different ways, we get the semiconductor devices used in modern electrons. The remaining units discuss these devices and their application in detail. For now, here is a breif account of the basic semiconductor devices.

23.5.1 The Diode

Fig.23.9 shows the diode, it combines p and n materials. The diode is the most basic solid state (semiconductor) device. It lets majority charge carriers flow easily only in one direction. In other words, it is just like a one way traffic road. Because of this one way action, the diode is used in power supplies circuits that convert ac line to dc voltage suitable for driving electronic equipment.



a) Diode b) Transistor

Fig.23.9

23.5.2 The Transistor

A transistor, is a device with three doped regions Shockley and his coworkers worked out the theory of the transistor. The basic device has the ability to amplify weak signals. Besides starting the semiconductor industry, the transistor has spawned all kinds of related inventions like integrated circuits (ICs), optoelctronic devices, and microprocessors.

23.6 EXPRESSION FOR CARRIER DENSITY

Already we have studied about the motions of electrons and holes in a semiconductors. Now let us try to know about the concentration of electrons and holes which are the charge carriers in intrinsic semiconductors at thermal equilibrium. The total current in a semi-conductor is due to the motion of both electrons and holes.

Consider an intrinsic semiconductor in thermal equilibium at temperature T°K. Let all the conduction electrons have an energy E_c and all the valence electrons have an energy E_v. Let n be the density of the electron hole pairs in the semi conductor.

At thermal equilibrium the rate of creation of electron hole pairs on absorption of energy must be equal to the rate of distruction of the same by the process of recombination. The probability of occurrence of such recombination is propoitional to the number of collisions occuring between the free electrons and holes. Hence in an intrinsic semiconductor the number of recombinations per unit volume per unit time is directly proportional to the density of free electrons n_i and also to that of free holes Pi.

The rate of distruction of electron hole pairs recombination is

$$-\frac{dn}{dt} \alpha n_i P_i \qquad 23.1$$

The negative sign indicates that the number of electron hole pairs is decreasing with time. Equation 23.1 can be written as

$$-\frac{dn}{dt} = Rn_i P_i = Rni^2 \qquad 23.2$$

Since $n_i = P_i$ at thermal equilibrim.

Here R is a proportionality constant which is also known as recombination coefficient. If N is the number of covalent electrons the rate of creation of electron hole pairs per unit volume is directly proportional to

(i) $(N-n_i)$, the density of available electrons for thermal excitations and

(ii) $e^{-(E_c-E_v)/KT}$ known as Boltzman factor.

But $(E_c - E_v) = E_g$, the forbidden energy gap width.

$$\therefore \quad e^{-(E_c-E_v)/KT} = e^{-Eg/KT}$$

The rate of recombination of electron hole paris

$$-\frac{dn}{dt}\alpha\left(N-ni\right)e^{-Eg/KT}$$

or $$-\frac{dn}{dt}=G\left(N-n_i\right)e^{-Eg/KT} \qquad 23.3$$

where G is the constant of proportionality

From equations 25.2 and 25.3

$$Rn_i^2=G\left(N-n_i\right)e^{-Eg/KT}$$

Since $n_i >> N$

$$ni^2=\frac{G}{R}Ne^{-Eg/KT}$$

$$n_i=Ce^{-Eg/2KT}$$

where C is another constant

We can say that the density of the electron hole pairs in an intinsic semi conductor is determined by the forbidden energy gap and the temperature T°K.

23.7 DETERMINATION ENERGY GAP

Already we have studied that the band of energies occupied by the valency electrons in known as valence band and the free electrons which contribute to the flow of current are known as conduction electrons and occupy a band of energies known is conduction band. The energy difference between these bands is known as "Forbidden energy gap" or simply "Energy gap". For an insulator the energy gap will be around 6eV and in the case of intrinsic semi conductors will be around 1eV.

Let us derive an expression for the energy gap in a semi conductor speciman.

The resistivity of a semiconductor varies with the temperature according to the relation

$$-\rho_T=\rho_o e^{B/T}$$

where ρ_T and ρ_o are the resistivities at temperature T°K and O°K.

and B is a constnat

Similarly at another temperature. T_o the above expression can be written as

$$\rho_{T_o} = \rho_o e^{B/To}$$

Derviding one with the other

$$\frac{\rho_T}{\rho_{To}} = e^{B\left(\frac{1}{T}-\frac{1}{To}\right)}$$

In terms of resistances, this expression can be, written as

$$\frac{R_T}{R_{To}} = e^{B\left(\frac{1}{T}-\frac{1}{To}\right)}$$

or $$R_T = R_{To} e^{B\left(\frac{1}{T}-\frac{1}{To}\right)}$$

Taking logarithms on both sides to the base 'e', it becomes

$$\log_e R_T = \log_e R_{To} + B\left(\frac{1}{T} - \frac{1}{To}\right)$$

Converting these logarithms to the base 10, we get

$$2.303\log_{10} R_T = 2.303\log_{10} R_{To} + B\left(\frac{1}{T} - \frac{1}{To}\right)$$

$$\log_{10} R_T = \frac{B}{2.303}\cdot\frac{1}{T} + \left(\log_{10} R_{To} - \frac{B}{2.303To}\right)$$

This equation represents a straight line, equations of the form f = mx + C whose slope is $\frac{B}{2.303}$.

In the case of intrinsic semiconductor $B = \frac{E_g}{2K}$

using the value of B; the energy gap Eg can be calculated.

23.8 SUMMARY

The materials can be classified based on the band theory as conductors, semiconductor and insulators.

The materials whose conductivity lies between that of insulators and conductors are called semiconductors. Silicon and germanium are good semiconductors.

Addition of chose impurities in controlled amounts to pure semiconductor materials is called doping.

Covalent bond breaking in semiconductors liberate electrons from the bonding there by leaving behind a hole (positively charged) in the crystal. The absence of an electron, in semiconductor terminology, in a covalent bond in known as hole.

Doping of pure semiconductor with a trivalent impurity like boron, aluminium, gallium or indium leads to the formation of p - type semiconductor. These dopants are called acceptor impermities.

n-type semiconductor is formed by doping the instrinsic semiconductor with donor impurities (pentavalent impurity like arsenic, antimony, phosphorus).

23.9 MODEL ANSWERS

Check Your Progress - I

If the conductivity in a crystalline semiconductor is soley due to the break down of covalent bonds the substance is said to be an intrinsic semiconductor.

Check Your Progress - II

Addition of chosen impurities in controlled amounts to pure semiconductor material is called doping.

23.10 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detial.

1. Discuss the classification of materials based on the band theory of solids.
2. Characterize conductors, insulators and semiconductors on the basis of conductivity scale.
3. Discuss the characteristic features of intrinsic (pure) and extrinsic semiconductors. Discuss the structure of p and n type semi conductors.

I. Answer the following question briefly.

1. Donar and acceptor impurities Give some examples of semiconducting devices and explain their action.

UNIT–24 : SEMICONDUCTOR DEVICES

Contents

24.1 AIMS AND OBJECTIVES

This unit explains the behaviour of a pn junction diode and junction transistor.

After going through this unit

- you will be able to discuss the functioning of a junction transistor
- you will be familiarised with the transistor parameters & Characteristics

24.2 INTRODUCTION

In this unit we shall study the behaviour of a Pn juction diode and junction transister.

24.3 PN JUNCTION DIODE

A diode is formed when a p-type semiconductor is brought into physical and electrical contact with an n-type semiconductor. The juction that forms between p-and n-type semiconductor is the essential building block for a variety of semiconductor devices.

Let us now consider the behaviour of these devices especially pn diodes. During the manufacturing process if a germanium or a silicon crystal is doped so as to make it up p-type at one end and n-type at the other end, a very importnat semiconductor device (called pn juction diode) can be obtained. At the junction that is formed, one side (the p-side) it has a large number of electrons. If the conditions are quite favourable the diffusion takes place. Holes from the p-side diffuse across the junction into the n-side and electrons from the n-side diffuse across the junction into p-side as shown in the Fig.24.1a. As a result of this diffusion, the p-side develops a net negative charge and the n-side develops a net positive charge. This diffence is due to the fact that the n-side lost electrons and gained holes at the same instant the p-side lost holes and gained electrons. But still the diode is as whole electrically neutral. So the net positive charge in the n-side must be exactly equal to the net negative charge in the p-side. Diffusion of charge carriers continues till an equilibrium state is reached

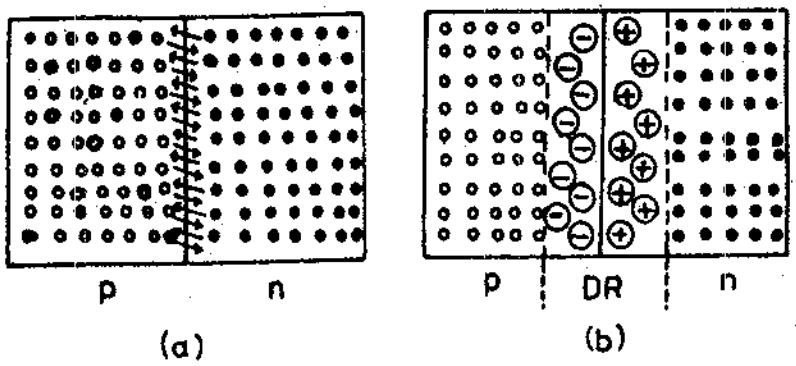


Fig.24.1 Depletion region formation. a) Diffusion of carriers across the juction, b) Diffusion continues till equilibrium is reached. DR Depletion region

when no additional carriers have enough energy to over come the electric field which has built up at the junction as a result of the redistribution of charge on both the sides.

This situation is shown in Fig.24.1b The nagatively charged acceptor ions in the p-side repel electrons while positively charged donor ions in the n-side repel holes.

The region on either side of the junction where the stationary charges appear is called the depletion region. This name has originated because of the fact that this region has been depleted of mobile charge carriers (free electrons and holes). This region is some times called as transition region or space charge region. The difference in potential at the junction formed due to the diffusion process is usually on the type of the semiconductor material (germanium or silicon) used and as well as on the doping concentrations of the p- and n-regions.

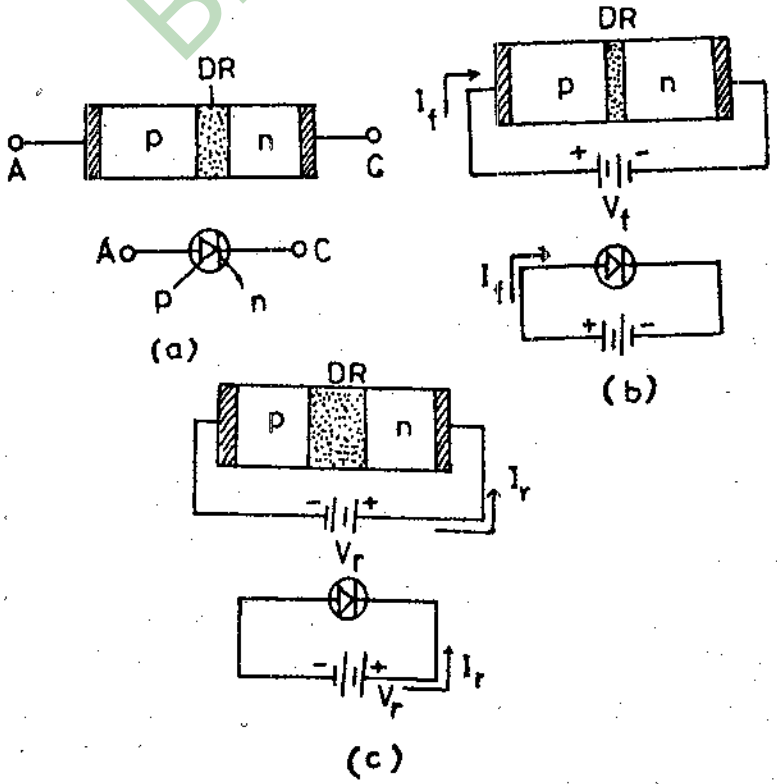


Fig.24.2 pn junction diode
a) no bias, b) Forward bias, c) Reversebias. A - Anode, C - Cathode; DR - Depletion region

Fig.24.2 a shows the diode under equilibrium conditions with no externally applied voltage. The circuit sysmbol is also shown in the same figure. Now let us examine the action of the diode under the influence of externally applied field, or when the diode is biased. Then the negative end of a battery is connected to the n-side (called the cathode) and the positive end of the battery to the p-side (called the anode,) a fairly large current is observed. This current is called the forward current, and the diode is said to be forward biased as dipicted in Fig.24.2 b. If the polarity of the battery is reversed and the negative end is connected to the anode and the positive end to the cathode, a minute current is noticed. This current is called the reverse current and the diode is said to be reverse baised as shown in Fig.24.2 c.

Check Your Progress - 1

Depletion region is the _____ appear.

Under forward bias condition, the externally applied voltage in opposition to the contact potential, there by lowering the effective or net potential at the junction. So the thickness of the depletion region reduces considerably, causing a high current flow. When the diode is reverse biased the externally applied voltage acts in the same direction as the contact potential thus increasing the effective potential barrier across the junction, while at the same time widening the depletion region itself. This result in a low current in the reverse bias condition.

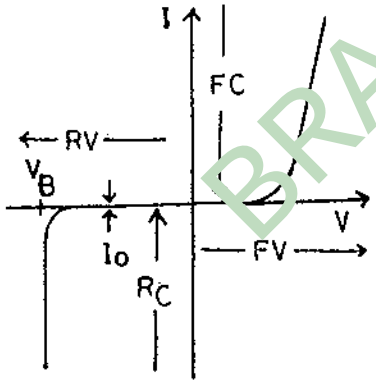


Fig.24.3 Typical diode characteristics
FC - Forward current, FV - Forward volgage, RC - Reverse current, RV - Reverse voltage.

The basic shape of any diode voltage current characteristics is shown in Fig.24.3 for both reverse bias and forward bias. In the forward direction even small voltages result in appreciably large currents, while in the reverse direction the current, is almost negligible until a certain voltage, called breakdown voltage, V_B on the characteristics is reached. For voltages more positive than V_B the diode characteristic is given by the diode equation or rectifier equation.

$$I = I_o\left(e^{eV/KT} - 1\right) \qquad 24.1$$

Where I is the diode current resulting form an externally applied voltage V, I_o is a constant called saturation current whose value depends on the particular diode under test, k is Boltzmann constant and T is the absolute temperatuere. Equation (24.1) can be written in a slightly different fashion as

$$I = \left(e^{V/V_t} - 1\right) \qquad 24.2$$

Where V_t = KT/e is the voltage equivalent of temperature which equals 0.025 V at room temperature. The factors that govern the value of I_o at room temeprature for a particular diode are (a) and type of the material (germanium or silicon). (b) the doping levles of the p and n regions and (c) the geometry of the jucntion. The typical forward characteristics for silicon and germanium diodes are shown in Fig.24.4.

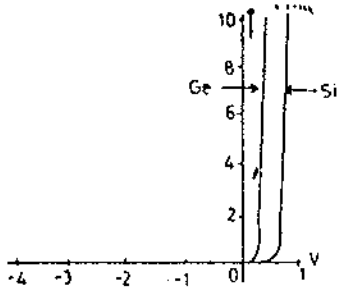


Fig.24.4 Comparision of germanium and silicon diode characteristics.

These experimental current-voltage characeristics of practical diodes are in good agreement with the rectifier equation (24.2). Because of the differences in energy gap and mobility of carriers in silicon and germanium semiconductors, diodes made of these materials exhibit different forward characteristics as well as the different reverse saturation currents. In general germanium diodes have reverse saturation currents in microampere (10^{-6} A) range whereas silicon diodes exhibit reverse saturation values in the nanoampere range 10^{-9}A.

24.4 THE TRANSISTOR

The transistor, abriviated BJT (Bipolar junction Transistor) has three terminals (electrodes). They are the collector, the base and the emitter. It consists of two parellel pn junctions juxtaposed in the same crystal. The emitter emits charge carriers into the base where control over the charge carrier can be exercised. These carriers are eventually, collected in the collector region. The base reigon lies in between the collector emitter regions which are doped with a same material. The base is always oppositely doped to that of collector and emitter. Two diodes are formed by the above process. Consequently we obtain two types of transistors. One kind has an n-type base and p-type collector and emitter. The other type has p-type base and both collector and emitter are n-type. These are termed as pnp and npn transistors respectively.

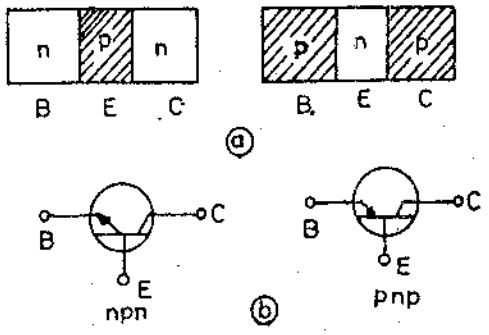


Fig.24.5 npn and pnp transistors
a) Block diagram, b) Circuit symbol. E - emitter B - Base, C - collector

Fig.24.5 shows the schematic representation of the two types of transistors viz. npn and pnp, along with their circuit symbols. The easiest way to remember the circuit symbol is to note the direction of the arrow on the emitter terminal. If the arrow points in, it indicates a pnp transistor. If the arrow points out' it indicates can npn transistor. Let us now examine the functioning of the transistor. A fairly large current is allowed to flow through the transistor, and the control over this current is exercised at the base either by increasing or decreasing the current.

Now we shall choose a pnp transistor as an example and very similar arguements can be extended to npn transistor.

In pnp transistor holes are the majority charge carriers in the emitter. In order to make these cross over the base junction and enter the base region, an external voltage is applied between the base and emitter terminals. The emitter and base regions constitute a pn junction diode which should be forward biased in order to cause the majority charge carriers (holes in the p-type emitter) to cross the junction. Once the holes enter the base (n-type), they diffuse in the base region. Some of them find their way to the base terminal and flow out and some of them reach them the collector base junction. In order to make the electrons to cross reverse baised voltage must be applied between the collector and base terminals. In this way the holes that started in the emitter and ended up in the collector constituting main current, while those holes that escaped out of the base terminal make up the small controlling current. In addition to these currents we must also consider the other currents due to minority charge carriers. As a result of the forward bias on the basae-emitter junction, we would expect not only the ejection of holes into the base but also electrons to enter from the base to the emiter. We ought have taken this also into consideration. Although electrons do enter their effect is quite neglible, in comparison with the hole current, on the net emitter current flow in the transistor because the base is very lightly doped whereas the emitter is heavily doped. Effectively when the juction is forward biased, the heavaily doped p-type emitter produces large number of holes for conduction, but the lightly doped n-type base has only a small number of electrons to offer. Therefore the current across the base-emitter junction in pnp transistor is essentially the result of hole flow.

The reverse bias at the collector-base junction not only transports the holes from the base into the collector, but also causes another current to flow across the collector base junction. This current is consititued by the minority charge in both collector and base regions namely holes from the n type base and electrons from the p type collector. The current component resulting in this way, from monority charge carriers across the collector-base junciton is quite small. It is called the collector-base reverse saturation current or collector cut-off current I_{CBO} and as a saturation current, it is very sensitive to temperature. The holes supplied by the emitter flowing towards the base correspond to a conventional current flow into the emitter. Let us designate this current as emitter current I_E Similarly the holes flowing out of the base and collector correspond to a net temirinal currents I_B and I_C respectively. These currents are dipicted in Fig.24.6.

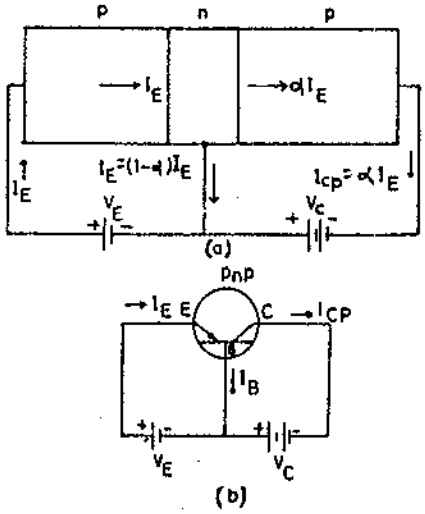


Fig.24.6 a) Currents in pnp transistors, b) Bias for normal operation of pnp transistor showing actual directions or terminal currents.

As a result of the externally applied voltage the depletion region at the collector-base junction widens and the depletion region at the base-emitter junction narrows down as shown in Fig.24.7

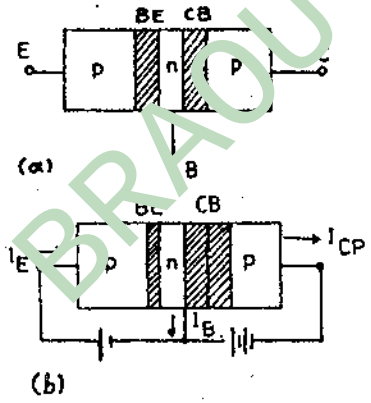


Fig.24.7 Depletion regions inside pnp transistor. a) No bias condition, b) With normal bias

24.5 THE TRANSISTOR PARAMETERS

We can see that the net current levaing the transistor in Fig.24.6 is the sum of the currents I_B and I_C and the net current entering the transistor is I_E. Therefore

$$I_B = I_B + I_C \qquad 24.3$$

The collector current is made up of the current in the collector resulting from holes that started in the emitter, labled I_{PC} together with the reverse satuaration current I_{CBO} Thus

$$I_C = I_{PC} + I_{CBO} \qquad 24.4$$

The ratio of the hole current in the collector I_{PC} to the total current I_E in the emitter, designated α (alpha) is an important parameter for the transistor α is defined as

$$\alpha = \frac{I_{PC}}{I_E} \qquad 24.5$$

We have seen already that some of the holes that started in the emitter are lost in the base, hence I_{PC} is always less than I_B so α is always less than unity. However it is close to 1, typically varying between 0.98 to 0.995. α is called the current gain factor in common-base configuration.

If we make use of Eqn. (24.4) in defining the Eqn. (24.5) for α we have

$$\alpha = \frac{I_C - I_{CEO}}{I_E} \qquad 24.6$$

Solving for I_C give us an important current relation for the transistor.

$$\alpha I_E = I_C - I_{CBO}$$

or $$I_C = \alpha I_E + I_{CBO} \qquad 24.7$$

The operation of an npn transistor is similar to that pnp. But in this case electrons from the n-type emitter are injected into the p-type base by forward biasing the base-emitter junction. Electrons are further transported through the base and some of them are gathered by the collector with the aid of a reverse bias on the collector-base junction. In fact the remaining discussion runs on similar lines as that of pnp transistor except with appropriate substitutions (n instead of p, p instead of n, and electron instead of hole) are made. There are some aspects common to both npn and pnp transistors. The BE junction diode is forward biased and the CB junction diode is reverse biased irrespective of the type. Eqn (24.3) shows that both I_B and I_C are in a direction opposite to that of I_E. For example the arrow on an npn transistor points out of the transistor. Therefore I_E flows out of the transistors.

Whereas I_C and I_B both flow into the transistor.

Using Eqns. (24.3) and (24.7) we can write

$$I_C = \alpha(I_C + I_B) + I_{CBO} \qquad 24.8$$

Solving for I_C we get

$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO} \qquad 24.9$$

We now define β (Beta), the dc short circuit current gain in the common-emitter configaration.

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha} \qquad 24.10$$

Now the equation (24.9) can be written as

$$I_C = \beta I_B + (\beta + 1) I_{CBO} \qquad 24.11$$

Typical values of β range from 20 to 1000 in practical transistors. Correspond to the current gain or amplification.

24.6 THE TRANSIATOR CHARACTERISTICS

The perforanance of transistors may be determined from the characteristic curves of their voltage and current relations. A circuit to obtain the common emitter static characteristics of an npn transistor is shown in figure 24.8.

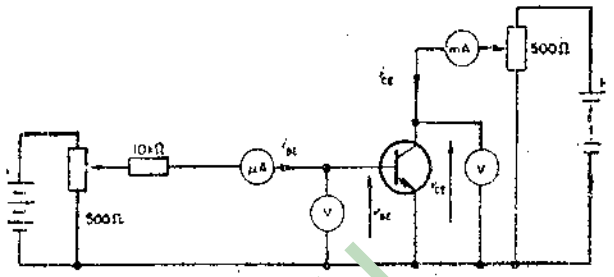


Fig 24.8

Variable DC power supplies B_1 and B_2 are used in the circuit to vary the base and collector supplies.

Fig.24.9 shows different types of characteristic curve which can be obtained for all types of transistors with slight differences.

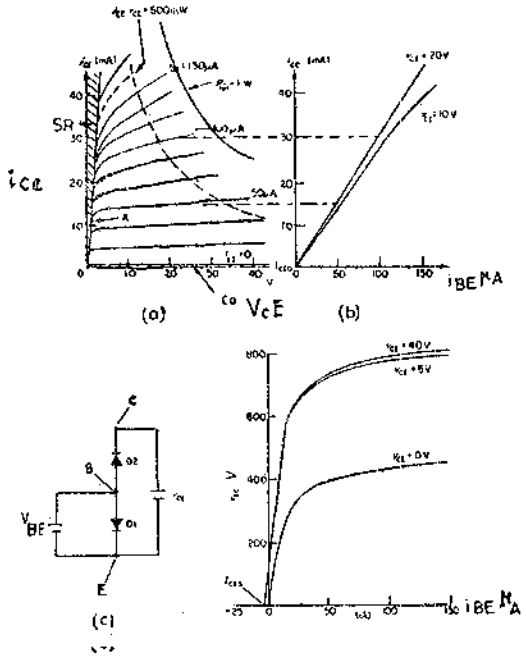


Fig.24.9 (a) Input characteristics (b) Static Transfer characteristies (c) output characteristics (d) Equivalent circuit of an npn trainstor.
CO - cut off region, SR - satuated region, I_{CES} - collector leakage current.

Dr. B.R. AMBEDKAR OPEN UNIVERSITY

(Undergraduate Programme)

3rd Year Syllabus

Physics Course - 3

MODERN PHYSICS

Block – I : ELEMENTS OF QUANTITATIVE THEORY

Unit-1 : Black Body Radiation and planks law
Unit-2 : Photo Electric effect, Einstein explanation, Compton effect in X rays
Unit-3 : Matter Wave Davisson and Germer experiment on electron diffraction G P Thomson experiment.
Unit-4 : Group and Phase Velocity
Unit-5 : Uncertainity Principle and its consequences
Unit-6 : Schrodinger Wave Equation (Time dependent and independent) Eigen values, eigen functions
Unit-7 : Significance of Wave Function (Million, Schrondinger and Born interpretation)
Unit-8 : Applications of Schlondinger's Wave Equation particle in a box (one dimentional), potential barrier.

Block – II : SPECTROSCOPY

Unit-9 : Line Spectra, Bohr's Theory, somm[illegible] theory, Energy levels of Hydrogen atom, vector model of atom (Space quantization and spin of electron) Frank - Hertz experiment
Unit-10 : Quantum Numbers Pauli exclusion principle, - Molecular, Vibiational spectra.
Unit-11 : Molecular spectra
Laser and its applications, spontaneous and stimulated emission of radiation, Einstein's coefficents Principle of a laser, He-Ne Lesers

Block – III : NUCLEAS STRUCTURE AND PROPERTIES OF NUCLEI

Unit-12 : Natural Radioactivity, Radioactive Disintegration, Radio active equillibrium, age of earth
Unit-13 : Transmutation of Elements Rutherfords experiment Chad-wick's discovery of Neutron
Unit-14 : Radiation Detectors, G.M. Counter, Cloud chamber, Bubble chamber, Scintillation detectors
Unit-15 : Nuclear binding energy, Artificial Radio activity, Nuclear reactions - Transuranic elements
Unit-16 : Nuclear Models, shell structure, liquid drop model, magic number
Unit-17 : Nuclear Fission and Fusion, Nuclear Reactors

Block – IV : ELEMENTARY PARTICLES

Unit-18 : Classification
Unit-19 : Conservation Laws
Unit-20 : Quark Model

Block –V : MAGNETIC PROPERTIES OF MATTER

Unit-21 : Para, Dia and Ferromagnetism
Unit-22 : Nuclear Magnetism

Block –VI : SEMICONDUCTOR PHYSICS

Unit-23 : Classification of solids - Band theory of solids, Semiconductors (Intrisic, Extrinsic) expression for carrier density, determination of energy gap
Unit-24 : Semiconductors devices, PN Junction diode, NPN, PNP Transistors and characteristics

Now let us try to understand the curves. In these group of curves let us know how to draw the input characteristic curve (a) To draw this curve, collector to emitter voltage V_{CE} is held constant and the variation of base to emitter voltage V_{BE} is measured for different values of base to emitter current I_{BE}. At very low base currents. a large variation of base voltage is observed. After certain value of base current, for a small charge in base voltage large variation of base current is observed. That is small emitter voltage variation is sufficient to produce a large variation of emitter current. I_{CES} is the collector leakage current with the base short circuited to the emitter. To draw the static characteristic curve (b) variations of collector to emitter current I_{CE} is measured as a function of base to emitter current I_{BE} at a constant value of collector to emitter voltage V_{CE}. From circuit design point of view using transistors, output characeristic curve (c) is useful. These curves are drawn as a variation of collector to emitter current I_{CE} for different values of collector to emitter voltages V_{CE} keeping base to emitter current I_{BE} constant. Different curves are dawn for different values of base current. These curves are known as output characteristics - when the base circuit is disconnected, the collector current is equal to the intrinsic leakage current I_{CEO} in the common emitter mode. It is important to note that almost entire variation in the collector current takes place at very low values of the collector voltage. When the collector voltages raised above a certain value it collects all the charge carriers that diffuse through the base to the collector to base junction and the collector current becomes independent of the collector voltage known as saturation current. From these curves we can understand that, for a transistor in the common emitter mode, the input resistance is very low and the output resistance is very high.

For a better understanding of the saturation region consider the equivalent circuit of the transistor Fig. d. Her D_1 is the base emitter junction diode and D_2 is the collector base junction diode. When base voltage is greater than the collector voltage both the diodes are forward biased and the current flows to the emitter and collector regions from the base region. That is both the diodes are saturated with current carrier and hence it is known as saturation operation. When the collector voltage is greater than the base voltage the collector junction becomes reverse biased and the transistor is said to be operated in unsaturated region. This operations is generally used in amplifiers.

The slopes of different curves yield different parameters.

1. Input resistance = $\frac{V_{BE}}{I_{BE}} = h_{ie}$ (at constant V_{CE})

2. Forward current gain = $\frac{I_{CE}}{I_{BE}} = h_{fe}$ (at constant V_{CE})

3. Reverse voltage ratio = $\frac{V_{BE}}{V_{CE}} = h_{re}$ (at constant I_{BE})

4. Output admittance = $\frac{I_{CE}}{V_{CE}} = h_{oe}$ (at contant I_{BE})

24.7 SUMMARY

The diffusion of holes into the n-region and the diffusion of electrons into the p-region continues until and equilibrium state is established. The diffused charge carriers on either side of the pn junction forms a region called the space charge region or depletion region.

The pn junction offers high resistance in the reverse direction and allows the curent flow in the forward direction, Hence it acts as a rectifier.

Two closely sandwitched pn juctions in juxtaposition in a single crystal of a semiconductor form a transistor.

A transistor is three terminal device. They are the collector, the base and the emitter.

The base region is always oppositely doped to that of collector and emitter. Consequently we obtain two kinds of transistors, viz. pnp and npn transistors. In one type of transistor n-type base is located in between p-type collector and emitter. This is known as pnp transistorf. The other type has p- type base and both collector and emitter are n-type. This is called npn transistor.

There is one aspect common to both pnp and npn transistors. The base-emitter junction is forward biased and the collector-base junction is reverse biased irrespective of the type of the transistor.

The two important parameters of a transistor are (alpha) the current gain factor in common-base configuration and β (Beta) the current gain factor in common-emitter configuration.

24.8 MODEL ANSWERS

Check Your Progress - I

1. Depletion region is the region on either side of the junction where the stationary charges appear.

24.9 MODEL EXAMINATION QUESTIONS

I. Answer the following questions in detial.

1. Deduce the condition $I_C = \beta I_B + (\beta + 1) I_{CBO}$ in a transistor and explain the significance of α and β
2. Explain the functioning of pnp and npn transistors
3. How a pn junction is formed of pnp and npn transistors
4. How a pn junction is formed and discuss its properties.

I. Answer the following questions briefly.

1. Describe the dynamics of formation of the space charge region.
2. Explain why semiconductor material have a negative temperature coefficient.

Dr. B. R. AMBEDKAR OPEN UNIVERSITY
FACULTY OF SCIENCE
B.Sc.3rd Year (3. Y.D.C)

Time : 3 Hours] | Model Question Paper | *[Max. Marks : 70*
Physics Course - 3 | *[Min. Marks : 25*
Modern Physics

SECTION - A
[Marks : 3 x 15 = 45]

Instructions to the candidates :

1) ***Answer any three of the following questions in about 30 lines each.***
2) ***Each question carries 15 marks.***

1) Discuss Wein's law for the distribution of energy in the radiation spectrum of a black body. Give an account of its experimental validity.

2) Derive the time dependent Schrodinger's wave equation.

3) Discuss the constructional details and working of a Geiger Muller Counter.

4) Describe the Various functionaries of a Nuclear reactor.

5) Explain Stem - Gerlach experiment. State the principle of the method and its significance.

6) Discuss the classification of materials based on the band theory of solids.

SECTION - B
[Marks : 5 x 5 = 25]

Instructions to the candidates :

1) ***Answer any five questions in about 10 lines each.***
2) ***Each question carries five marks.***

7) Write about the significance of wave function.

8) Explain the Bohr's correspondence principle in brief.

9) Discuss the main features of the curve "binding energy per nucleon" versus mass number"

10) Write the Bohr's postulates

11) Compare the properties of α, β and γ rays.

12) Explain the chain reaction.

13) Write about the origin of Mesons.

14) Write about the population inversion in He-Ne Laser.

15) Distinguish between the intrinsic and extrinsic semiconductors.

16) Write briefly about the Quark model.

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