PHYSICS

COURSE - II

Electromagnetism
&
Thermodynamics

"We may forgo material benefits of civilization, but we cannot forgo our right and opportunity to reap the benefits of the highest education to the fullest extent..."

-Dr.B.R.Ambedkar

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PREFACE

This book deals with the topics in Electromagnetism and Thermodynamics in the syllabus for the second year of the B.Sc., Course offered by Dr.B.R.Ambedkar Open University. These topics cover the core area of the subject to be studied in the second year of the three year Degree Course in Science. The syllabus is for the sake of convenience divided into Blocks, each of which comprises a number of units. Each unit generally covers a specific area of the subject. The units are prepared by specialists in accordance with a format so designed as to enable the student read and understand them without much difficulty. Each unit begins with the objectives to be achieved after going through the unit. Generally technical terms with which the student may not be familiar are given at the end of each block under the head Glossary.

Blocks 1 to 5 of the book deal with Electromagnetism. Blocks 6 deals with the branch of physics called laws of Thermodynamics. The university hopes that this course material will help the students to get acquainted with the concepts and principles of Electromagnetism and Thermodynamics.

CONTENTS

UNIT 1: VECTORS

Contents

1.1. OBJECTIVES

This unit introduces the fundamentals of vectors and scalars & the Concept of 'Del operator', emf and divergence of a vector. Also aims at explaining line, surface & volume integrals and also some important theorems pertaining to vector integration.

At the end of this unit one would be understanding the meaning of vectors and their various applications in physical problems.

1.2. INTRODUCTION

As it is evident that we know of certain physical quantities such as force, velocity momentum, speed, work & so on like this innumerable physical quantities are classified as vectors and scalars. Depending on the physical quantity whose magnitude or along with magnitude its direction is also specified, based on these they are classified as above two categories. When after classifying, vectors especially follow certain rules of addition, multiplication, commutation etc. Not only this, a vector its components and its integral are based on these certain theorems are enumerated which are mostly applicable in static electric fields, gravitational and also fluid mechanism.

When a vector physical quantity is expressed from point the region of space by a continuous vector function A (x, y, z) then the region is a vector field. Examples are gravitational magnetic, electric intensity etc.

We are familiar with the flux lines of a bar magnet the tangent at any point o a flux line give the direction of magnetic intensity at that point. More over the number of lines passing through unit area of the surface perpendicular to their direction is called as magnetic flux. When the flow lines are parallel to each other, then the vector field is called as stationary vector field.

The concept of vector field can easily be understood with the help of a couple of examples.

(i) Consider a body, which is rotating about at an axis. The velocity of the body is different at points i.e. the velocity is a function of position of the point. Hence the velocity is a vector field.

(ii) Consider the heat flow through a block of material whose different faces are maintained at different temperatures. There will be flow of heat from hotter face to colder face. The heat will be flowing in different directions in different parts of the block. This heat flow is a vector field.

(iii) $V = I\,ax + j\,ay + k\,az$ etc

Like this many more examples coming across in physical problems can be taken into consideration

BLOCK – 1: VECTORS AND ELECTROSTATICS

The vector sum is associative (A+B)+C=A+(B+C) we draw the conclusion that the sum of Vectors A, B, C is independent of the order in which they are added.

(b) **Subtraction of Vectors:** Here we shall subtract one vector from the other i.e., 'B' from vector 'A'. The subtraction of the vector 'B' from 'A' is equivalent to the addition of (-B) with 'A' i.e. A-B=A+(-B). The negative sign means a vector of equal magnitude but opposite direction. This procedure is illustrated in Fig 1.4 below:

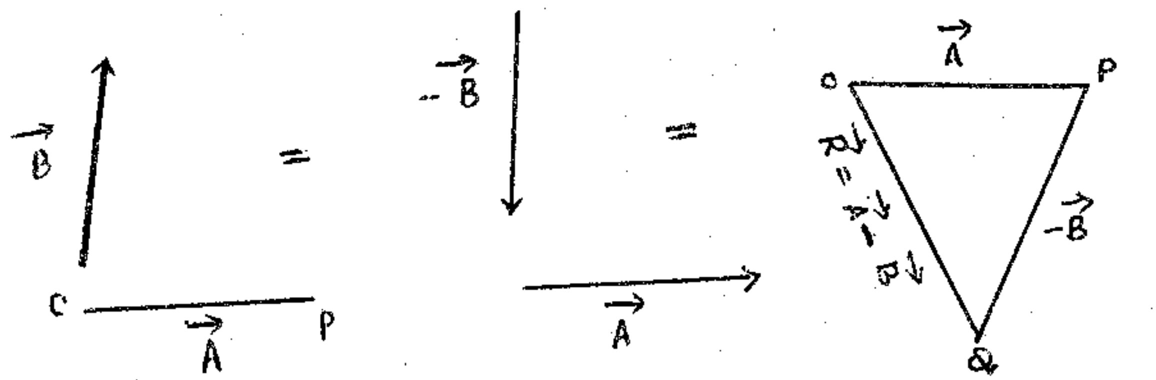


Fig 1.4

Here we draw a line 'OP' to represent the vector 'A' and then from the head 'A' another line 'PQ' is drawn to represent vector (B). Further the tail of 'A' is joined to head of (-B). Now the line 'PQ' represents a vector 'R' which is difference of two vectors A & B.

When A &B are in the same direction, (A-B) is a vector whose length is (A-B) and the direction is along the longer vector. If the two vectors A & B have the same length and direction then their difference is a null vector. Further

$$(A - B) = -(B - A). \qquad \ldots(1.2)$$

(c) **Multiplication of a vector by a scalar:** When a vector 'A' is multiplied by a scalar or a pure number 'n' then the resultant vector may be written as

$$R = nA \qquad \ldots(1.2(a))$$

The magnitude of vector 'R' being 'n' times of vector 'A' and the direction is the same as that of 'A' when 'n' is positive but opposite when 'n' is negative.

When the scalar 'n' is a physical quantity with a unit, then the unit R will be different from 'A'

Example: 1) when mass (scalar) is multiplied by velocity (vector) then the product (mv) represents the momentum, which is a different quantity i.e.is a (vector)

2) Similarly when 'mass' multiplied by acceleration which are scalar and vector quantities respectively results in 'Force' which is a vector quantity (F=ma)

Thus, we draw conclusion that when a vector is multiplied by a scalar quantity it gives result to a vector quantity, but this is not always true.

1.3.4 (a) Scalar Product of Two Vectors

The scalar product of two vectors 'A' & 'B' is defined as the product of the magnitudes of vectors (i.e. 'A' & 'B') and the *cosine* of the angle between them.

$\mathbf{A} \bullet \mathbf{B} = AB \cos\theta$ ...(1.3)

Where = Angle between the 2 vectors

If incase (i) $\theta = 0$, A.B=AB (i.e. they are parallel to each other)

(ii) $\theta = \pi/2$, A. B = AB=0 (i.e. Means the 2 vectors are perpendicular to each other)

(iii)The scalar product of unit orthogonal vector, i., j, & k has the following relations.

$i \cdot i = j \cdot j = k \cdot k = 1$ ... (1.4)

$i \cdot j = j \cdot k = k \cdot I = 0$ ...(1.4 (a))

(iv) The scalar product of 2 vectors is equal to the sum of the products of their corresponding x,y & z components.

If $A = i\,Ax + j\,Ay + k\,A_z$ ...(1.5)

$B = i\,Bx + j\,By + k\,Bz$ ...(1.5 (a))

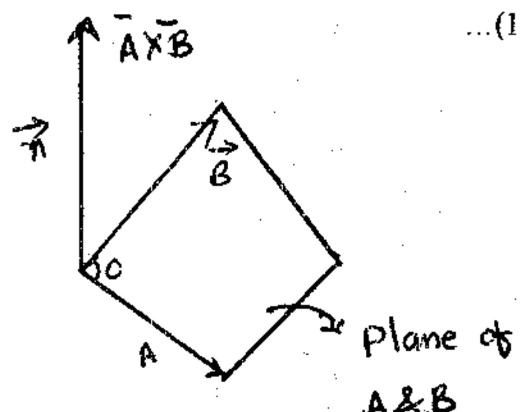
Then $A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$

Example: An example of dot product is work done by a force . Let a force 'F' produce a displacement 'd' when it acts on a body. The work done = F . d (It is a scalar quantity).

(b) Vector product or Cross product of 2 vectors: The vector or cross product of two vectors is defined as a vector having magnitude equal to the product of the magnitude of 2 vectors & the sine of the angle between them. The direction being perpendicular to the plane containing the two vectors.

If A & B are 2 vectors, then the vector product is A x B which can be written as

$A \times B = AB \sin\theta\ \hat{n}$ ...(1.6)



Where A & B are magnitudes of A & B.

θ is the angle between them, $\hat{n}$ is a unit vector Perpendicular to the plane of A & B. The direction Of A & B is given by the right hand rule.

Fig. 1.5

(i) The magnitude of the vector product of 2 vectors mutually perpendicular i.e. at right angles to each other is equal to the product of the magnitude of the vectors.

$A \times B = A \sin\theta .\hat{n} = AB \sin 90^\circ .\hat{n} = AB\,\hat{n}$ ... (1.6 (a))

(ii) The vector product of two parallel vectors is a null vector (Zero)

1.3. BASICS OF VECTORS

In the past, Astrologers in order to establish the place of planet from other planet, it seems they have for the first time introduced vectors. They have established the fact that any line joining two planets is a vector. That is why a scalar has no direction but only magnitude such as density, volume, length etc, whereas a vector has got both magnitude and direction such as force velocity, acceleration etc.

1.3.1Representation of a vector:

Let 'O' be an arbitrary fixed point in space and 'P' be any other point in it. Then the straight line 'OP' has magnitude as well as direction. Therefore the directed line segment 'OP' is capable of representing a vector quantity. We denote the vector as $\vec{OP}$ or simply by OP and read it as 'OP' o ↗ p.

Example: A body has a magnitude of 20 mts/sec and traveling with a velocity as that from east to west then the direction is as indicated in the arrow. Whereas speed if we take it, is having only magnitude but no direction. Thus we say speed is a scalar quantity, whereas velocity is a vector quantity. In general terminology we on so many occasions velocity and speed are being mixed up but it is not one and the same. Direction must be specified in order to define a vector.

Different kinds of vectors: (Concept & Details)

(a) Zero or Null vector: The zero or null vector is a vector whose modulus is zero and whose direction is intermediate. The null vector is represented by the symbol, O. In case of a null vector the initial and terminal points coincide. Thus $\vec{AA}$, OO are null vectors.

(b) UNIT VECTOR: A vector whose modulus or magnitude is unity is called a unit vector.

LIKE & UNLIKE VECTORS: Two vectors having the same directions are called like vectors and those having opposite directions are called unlike vectors. If a is a vector having magnitude $|a|$ as a result the Unit vector a has same direction $\hat{a} \Rightarrow a\,|a|$ $\quad \therefore a = |a|\,\hat{a}$

(c) Collinear or Parallel Vectors: Vectors having the same line of action or having the lines of action parallel to one another are called collinear parallel vectors.

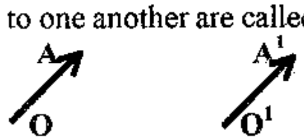


Fig. 1.1

(D) Equal Vectors: The vectors are said to be equal if and only if they are parallel and have the same sense of direction and the same magnitude.

The starting point of the Vectors are immaterial. It is the direction and magnitude which is of importance. To denote equal vectors, the sign (=) is used, Thus if a & b are equal vectors, we write $\vec{a} = \vec{b}$

AB=CD=EFGH

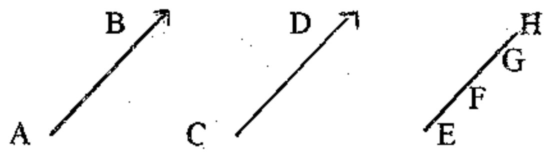


Fig 1.2

The equality of two vectors, as discussed above does not mean that the two quantities represented by a & b are equivalent in all respects. For example: If two equal forces (Acting in the same direction) are applied at different points of a rigid body, then they may have different mechanical effects.

1.3.3 Some Simple Properties of Vectors:

Addition, associative, subtraction, commutative and resolution are some properties of a vector and they are discussed here in this chapter.

(a) Addition of Vectors: We shall illustrate the method of addition of two vectors. A & B of magnitude A & B (See Fig 1.3)

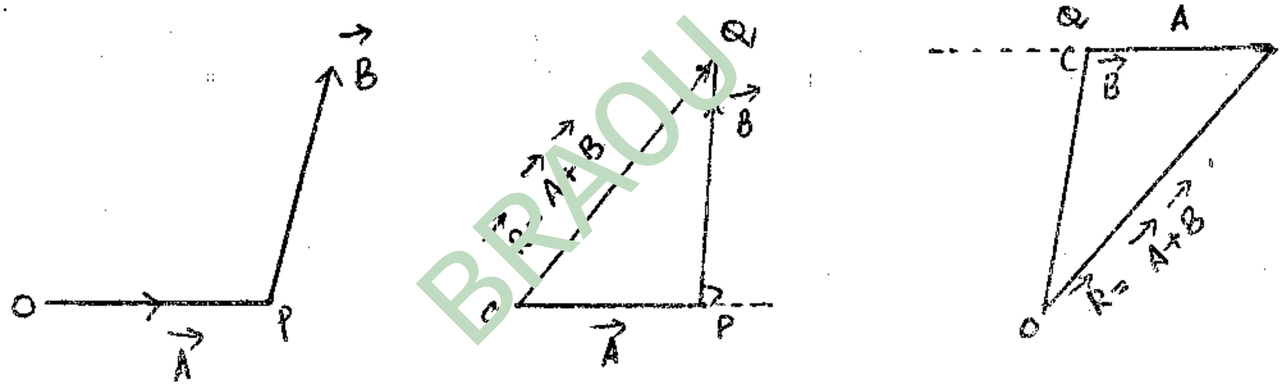


Fig 1.3

First of all we draw a line 'OP' of magnitude 'A' and put an arrow at 'P'. Now 'OP' represents a vector A.

From the head of 'A' i.e. from 'P', we draw a vector 'B' we join 'OQ' i.e. line is drawn from tail 'A' to head of 'B'. The vector 'OQ' represents the resultant of two vectors. Symbolically the addition of two vectors can be represented as $\vec{R} = \vec{A} + \vec{B}$

We can also draw vector 'B' first and then from that head of vector B. We draw vector A. Now by joining of the tail of 'B' to the head of vector A, We get the sum of 2 vectors, in this case.

$$\vec{R} = \vec{B} + \vec{A} \qquad \ldots(1.1)$$

This way we see that vector sum is commutative

A + B = B + A

$$A \times B = A \sin\theta\, n = AB \sin\theta\, n = 0 \qquad \ldots(1.6\,(b))$$

(iii) The vector of unit orthogonal vectors i, j & k have the following relations.

$$i \times i = j \times j = k \times k = o$$

$$i \times j = -j \times i = k$$

$$k \times i = -k \times j = I$$

$$k \times i = -I \times k = j \qquad \ldots(1.7)$$

(iv) The vector product of 2 vectors in terms of their x, y & z components can be expressed in the form of determinant

If A= i Ax+ j Ay + k Az

B= i Bx + j By + k Bz

$$\text{Then } A \times B = \begin{vmatrix} i & j & k \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix} \qquad \ldots(1.8)$$

1.3.5. Scalar and vector fields

It is a known fact that a physical Quantity can be expressed as a continuous function of position of a point in the region of space.

For example, when a rod is heated at one end, there is a variation of temperature along the length of the rod. The physical quantity temperature at any point (x, y, z) can be expressed by a continuous functions T (x, y, z). Such a function is termed as a joint function or function of position. The region specifying the physical quantity the field may be a scalar or vector. About these two quantities it is explained in the following articles.

1.3.5 (a) Scalar field: The scalar field in three dimensions can be represented by a scalar point function ϕ (x,y,z). For example the electric potential due to a single +ve charge 'q' coulombs depends upon the position of the point from the charge. Then V_0 (x_0, y_0, z_0) and V_1 (x_1, y_1, z_1) are the scalar point functions at (x_0, y_0, z_0) & (x_1, y_1, z_1). Now the region will be a scalar field.

The concept of a scalar field can easily be understood with the help of the following example.

(i) Consider a block of material whose faces are maintained at deferent temperatures. Now the temperature of the body will vary from point to point i.e. temperature will be a function of position co-ordinates (x, y, z) in a rectangular co-ordinate system. Hence the conclusion is drawn that temperature is a scalar field.

(ii) The density at any point inside a body occupying given region is a scalar field.

(iii) $\phi = 2xyz \neq x^2y$ defines a scalar field.

1.3.5 (b) Vector Field: When a physical quantity such as magnetic or any such quantity is expressed from point to point in the region of space by a continuous function A (x, y, z) then the region is known as a vector field. The best examples of such field are gravitational, magnetic, electrical intensity. Both magnetic & direction change continually from point to point throughout the field region. Suppose if we start from a point in the field and proceeding throughout infinitesimal distances from the point in the direction of field. We obtain a curved line, so the field can be mapped, by curved lines of force known as flux lines or lines of flow. The tangent at any point to any line of force gives the direction of 'A' at that point. Here 'A' is that particular point. The magnitude at any point in the field is given by number of lines crossing unit perpendicular to their direction drawn at that area point. The number of lines passing through unit area of the surface perpendicular to their direction is called as magnetic flux. The magnetic flux depends on the difference of the point from the magnetic pole.

In this way scalar and vector fields can be discussed and analyzed.

1.4 GRAD, DEL AND DIVERGENCE OF A VECTOR FIELD

1.4.1 Divergence of a vector field

The operator ∇ can be involved in the multiplication with a vector. The scalar or dot product of operator ∇ with a vector A (i.e. & A) is called divergence.

The divergence of vector field at any point is defined as the amount of flux per unit volume diverging from that point. The divergence is a scalar quantity because it represents simply the amount of flux.

If 'A' is a vector function differentiable at each point (x, y, z) in a region of space. Divergence of ' A' is given by

$$\nabla \cdot A = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)(i\,Ax + j\,Ay + k\,Az)$$

$$= \frac{dAx}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz} \qquad \ldots(1.9)$$

$$\therefore \text{div } A = \nabla \cdot A = \frac{dAx}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz} \qquad \ldots(1.9(a))$$

This is the expression of divergence in Cartesian co-ordinates. This quantity has important application in hydrodynamics and electricity.

1.4.2. Derivation of the Expression

Consider a small rectangular parallelopiped in a vector field as shown in Fig 1.6

Let dx, dy and dz be the lengths of the sides of parallelopiped parallel to the co-ordinates x, y & z respectively. Let a vector 'A; represent the velocity of a fluid at center 'C' with component - Ax, Ay and Az along the three axes respectively. Let d Ax/dx represent the rate of charge of Ax along the axis. Similarly d Ay/dy & d Az/dx will be the rates of charge of Ay and A z along the y & z axes respectively.

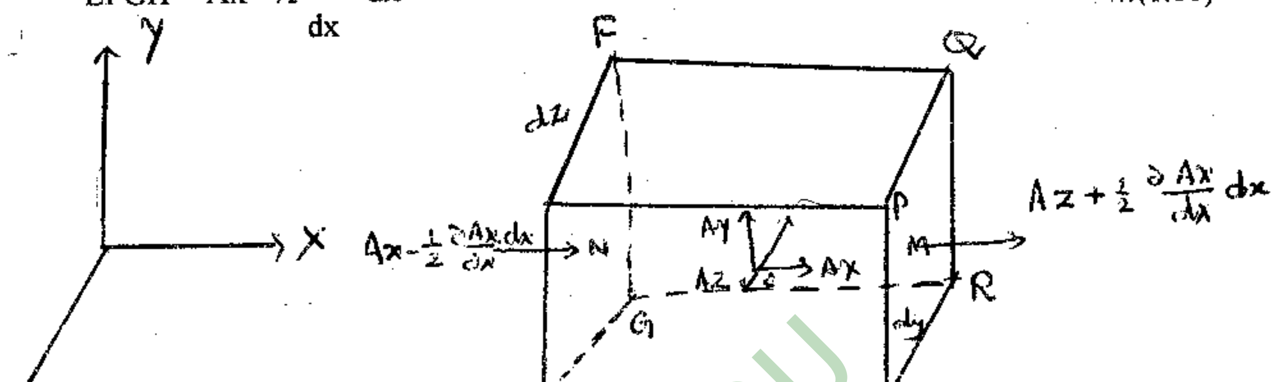
The volume of Ax at the center M of face PQRS

= Volume of Ax at center C + increase in magnitude from C to M
= Volume of Ax of centre + rate of change X distance

$$= Ax + \frac{dAx}{dx} \times \frac{Ax}{2} = Ax + \frac{1}{2}\frac{dAx}{dx}dx \qquad \ldots(1.10)$$

Similarly the magnitude at the centre N of face

$$EFGH = Ax - ½\frac{dAx}{dx}dx \qquad \ldots(1.11)$$



The negative sign is taken because N is towards left of C.

Fig 1.6

We know that the volume of fluid flowing per unit time through a faces equal to the product of the area of face and normal component of vector upon it. This is known as flux through the face. Hence flux entering the face

$$EFGH = \left(Ax - ½\frac{dAx}{dx}\right)dy\,dz$$

Where dy dx is the area of face EFGH and flux leaving the parallelopiped over that entering in X-direction is given by

$$\left(Ax - ½\frac{dAx}{dx}dx\right)dy.\,dz - \left(Ax - ½\frac{dAx}{dx}.dx\right)dy.\,dz$$

$$= \frac{dAx}{da}.dx.dy.dz$$

Similarly the net flux leaving the parallolopiped in Y and Z direction are

$$\frac{dAy}{dx}.dx.dy.dz + \frac{dAy}{dz}dx.dy.dz$$

∴ Total flux leaving or diverging from parallopiped

$$= \frac{dAy}{dx}.dx.dy.dz + \frac{dAy}{dy}dx.dy.dz + \frac{dAz}{dz}dx.dy.dz$$

$$= \left(\frac{dAy}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz}\right)dx.\,dy.\,dz$$

Because dx dy dz is the volume of the elementary parallelopiped
Hence the amount of flux diverging per unit volume

$$= \left(\frac{dAy}{dx} + \frac{dAy}{dy} + \frac{dAz}{dx} \right)$$

This is defined as divergence of A

$$\therefore \quad \text{div A} = \frac{dAx}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz} \qquad \text{...(1.12)}$$

1.4.3. Significance of divergence

If 'A' represents the velocity of a fluid (liquid or gas), then (div A) gives the rate of flow of the fluid at that point per unit volume. If div A is positive at that point then either the fluid is undergoing expansion or the point it self is a source of fluid. Similarly a negative value of divergence means that either the fluid is undergoing contraction or the point serves as a sink. If however div A =0, then the fluid entering and leaving element is the same i.e., there is no change in density of fluid (or fluid is incompressible). The vector satisfying this condition is called solenoidal

Ex: If $A = iy + j(x^2 + y^2) + k(yz + zx)$ the.

Find div 'A' at point (1,-2,3)

We know that div A =∇. A & $i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz} = \nabla$

$$\text{div A} = A \,\&\left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz} \right)\{ iy + j(x^2+y^2) + k(yz + zx)\}$$

= 0+2y+(y + x)= 3y +x at point (1, -2,3)

div A= 3x – 2 + 1 = -5

1.4.4. Gradient is a Scalar field

The gradient is differential operator by means of which we can associate a vector field with that of a scalar.

Also the gradient of scalar function ϕ is a vector whose magnitude at any point is equal to the maximum rate of change of scalar function ϕ with represent to space visible and has got the direction of that change.

For example, the intensity of electric field E, (a vector quantity) is the gradient of potential V (a scalar quantity) with the negative sign i.e.,

E = - grad V ...1.13

The negative sign indicates that the direction of field intensity is opposite to the direction of increase of potential.

Let S (x, y, z) be any continuously differentiable scalar function depending on the three Cartesian co-ordinates in space. Suppose ds/dx, ds/dy, ds/dz be the partial derivatives along the three perpendicular axes respectively. Now the gradient of the scalar function S can be defined as

$$\text{Grad } S = i\frac{ds}{dx} + j\frac{ds}{dy} + k\frac{ds}{dz} \qquad \ldots(1.14)$$

or grad S = ∇ S where $\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}$ generally called del or del operator·

1.4.5. Physical Interpretation of grad S

The scalar field can be mapped out by a series of level surfaces. Consider two such surfaces, very close to each other all shown in figure 1.7

These surfaces are specified by constant scalar functions S and S +ds respectively Consider the two points P & R on the level surfaces S1 and S2 respectively.

Let 'r' and (r + dr) be the position co-ordinates of P& R respectively with respect at any arbitrary origin: then

PR= dr

If co-ordinates of P are (x, y, z) and of R are (x + dx), (y + dy), (z + dy) then

$$dr = idx + jdy + kdz \qquad \ldots(1.15)$$

As the continuous scalar function at P (x, y, z) has the value S and at R (x + dx+ y + dy, z + dz) has the value S + ds; we have

$$ds = \frac{ds}{dx}\cdot dx + \frac{ds}{dy}\cdot dy + \frac{ds}{dz}\cdot dx \qquad \ldots(1.16)$$

This equation may also be written as

$$ds = i\left(\frac{ds}{dx} + j\frac{ds}{dy} + k\frac{ds}{dz}\right)(idx+jdy+kdz)$$

$$= (\vec{\nabla}s).\, dr \qquad \ldots(1.17)$$

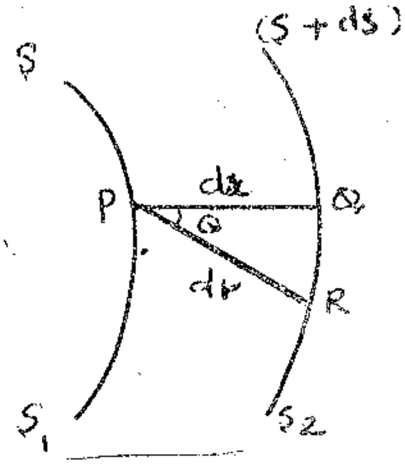


Fig 7

In particular if we consider that dr (i.e. point R) lies in the level surface S1, then

$$ds=0, \text{ i.e., } (\nabla s).dr=0 \qquad \ldots(1.18)$$

Showing here that the vector $\vec{\nabla}S$ is normal to the surface S_1 (i.e. the surface S = constant). If dx denotes the divergence along the normal from point 'P' to the surface S_2, we may write

$$dx = PQ = ds\, cos\, \theta = \hat{n}\ .\ dr \qquad \ldots(1.19)$$

Where $\hat{n}$ is a unit vector normal to the surface S_1 at P (i.e. along PQ)

As the value of the scalar function increases by ds, as we proceed from P to Q along PQ we can write.

$$ds = \frac{ds}{dx}.dx = \frac{ds}{dx}\hat{n}.dr \qquad ...(1.20)$$

Combining equations 1.16 and 1.20 we may write

$$ds = (\nabla.S)\,dr = ds/dx.\,\hat{n}.dr \qquad ...(1.21)$$

as dx is an arbitrary vector, equation (1.20) gives

$$\nabla.S = \frac{ds}{dx}.\hat{n} \qquad ...(1.22)$$

$$\text{Thus grad } S = \nabla S = \frac{ds}{Dx}.\hat{n} \qquad ...(1.23)$$

Definition of grad S:

Conclusion that we draw is that gradient of a scalar field S is a vector whose magnitude at any point is equal to the rate of change of S with distance along the normal to the level surface and whose direction is along the normal to the level surface at that point.

It S represents that potential in an electrical field, then the intensity of the field at any point is in the direction of the greatest rate of fall of potential which is normal to the equipotential surface and is equal to this rate, thus

$$E = \frac{ds}{dx}.\hat{n} = -\text{grad } S \qquad ..(1.24)$$

1.5. CURL OF A VECTOR FIELD

It can be shown that a vector field can as a scalar gradient field, that in that field in closed enclosure the line integral will be zero. But it is not possible to show every time that a Vector field to be represented of scalar gradient field, in terms which case $\oint U.\overrightarrow{ds}$ is not equal to zero. It ∇A is the area bounding the circuits then curl 'U' can be written as follows

$$\oint U.\,ds = (\text{curl } U).\,\overrightarrow{\Delta A} \qquad ...(1.25)$$

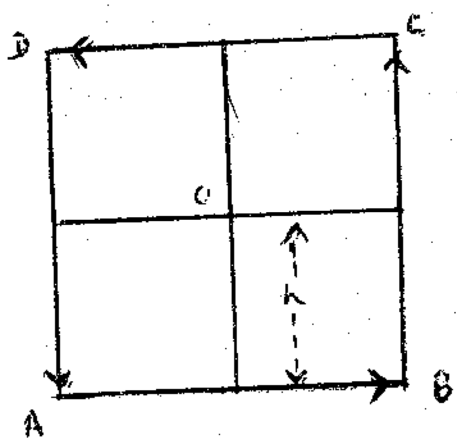


Fig 1.8

In order to deduce the expression for the curl we shall follow the figure as shown in fig 1.8

In order to find the component of Curl, in figure xy, yz, zx planes to be considered as closed circuits. ABCD is a circuit having side 2h components of V are Vx, Vy, Vz. Along ABCD, $\oint U.ds$ is time integral. In this $\overrightarrow{ds} \rightarrow$ AB, CD along y-axis like wise BC, DA along Z-axis.

Therefore along AB, CD we consider Uy, along BC, DA we consider Ux that is enough Uxo, Uyo, Uzo, are the co-ordinates of 'O' & the co-ordinates of U.

Along AB, Uy value:

$$Uyo - \left(\frac{dUy}{dz}\right) x - \left(\frac{dUy}{dy}\right) y$$

Along CD, $Uyo + \left(\frac{dUy}{dz}\right) x + \left(\frac{dUy}{dy}\right) y$

Along Ab the value of line integral value

$$\int_A^B U.ds = \int_{-h}^{h} \left[Uyo - \left(\frac{dUy}{dz}\right)h + \left(\frac{dUy}{dy}\right) y\right] dy$$

$$2h\,(Uyo) - 2h^2\left(\frac{dUy}{dz}\right) \qquad \text{..................(1.26)}$$

Similarly for side CD,

$$\int_c^d U.ds = \int_{-h}^{h} \left[Uyo - \left(\frac{dUy}{dz}\right) h + \left(\frac{dUy}{dy}\right) y\right] dy \qquad \text{...(1.27)}$$

$$2h\,(Uyo) - 2h^2\left(\frac{dUy}{dz}\right)$$

Sum of equation 1.26 & 1.27

$$\int_A^B \vec{U}.ds = \int_C^D U.ds = 4h^2 \left(\frac{dUy}{dz}\right)$$

Same way if the vertical sides are takes.

$$\int_B^C U.ds + \int_D^A U.ds = 4h^2 \left(\frac{dUy}{dz}\right)$$

$\therefore 4h^2$ is the area of ABCD

$$\therefore (\text{Curl } U)\, x = \frac{dUy}{dy} - \frac{dUz}{dz} \qquad ...(1.28)$$

$$\text{and } (\text{Curl } U)x = \frac{dUx}{dz} - \frac{dUy}{dx} \qquad ...(1.29)$$

$$\text{also } (\text{Curl } U)\, z = \frac{dUy}{dx} - \frac{dUy}{dy} \qquad ...(1.30)$$

From the above 3 equations

$$\text{Curl } U = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ Ux & Uy & Uz \end{vmatrix} \qquad ...(1.31)$$

Using equation (1.31) the physical meaning of the curl can be explained.

Along curves $\oint U.\, ds$ U is circulation & its limits gives the circulation at that point.

Example: Consider the velocity of a liquid in a field with constant velocity with angular velocity 'w'.

The velocity field u can be written as

$$\text{As } u = w \times r = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ Wx & Wy & Wz \\ x & y & z \end{vmatrix} \qquad ...(1.32)$$

$$= \hat{I}\,(Wyz - Wxy) + \hat{J}\,(Wzx - Wxz) + \hat{K}\,(Wxy - Wyz)$$

Using equation (1.27)

$$\text{Curl } U = (Wx\hat{i} + Wy\hat{j} + Wz\hat{k}) = 2\,W \qquad ...(1.33)$$

If curl U = 0 then the field will be vertex free. If curl $\neq 0$ then the field will be non-vertex free. This is how it can be analyzed.

Some examples are given where the curl is of use.

Example (1): Consider the velocity field of water flowing in a river, if there is a deep pot in the bed of river, then the velocity of water flowing has rotational component around that point. Consequently the water whirls rapidly if a swimmer gets into this region, he starts rotating rapidly and it becomes very difficult for him to get out of the region.
Example (2): When a rigid body is in motion, then the curl of its linear velocity at any point gives twice its angular velocity in magnitude and direction.

$$\text{Curl } U = \overrightarrow{2W} \qquad \ldots(1.34)$$

Example (3): When a current is passed through a conductor, then magnetic field is developed around it. At any nearby point the curl of the magnetic field represents the current per unit area passing through that point. So curl B is also known as magneto motive force.

1.5.1. Different types of vector fields

1) Field where, having no divergence and no curl
In this case $\nabla . A = 0$ & $\nabla \times A = 0$

The examples of such fields are: steady state flow, electrostatic field, gravitational field, irrotational motion of incompressible ideal fluid etc.

2) Where in field having divergence and not curl
In this case, $\nabla . A \neq 0$ but $\nabla \times A = 0$

The examples of such field are: gravitational field inside a mass, electric field with in a volume distribution of charge, time independent schrodinger's equation etc.

3). In a field having curl and no divergence:
In this case, $\nabla \times A \neq 0$ but $\nabla . A = 0$

Examples of such fields are: Magnetic field due to steady current, incompressible fluid with velocity etc.

4) Field having curl and divergence:
In this case, $\nabla \times A \neq 0$ & $\nabla . A = 0$

The Maxwell's equations within matter are examples of such field.

1.6 LINE, SURFACE & VOLUME INTEGRALS (INTEGRATION OF VECTORS)

1.6.1 . Line Integral

The integration of a vector along a curve is called its line integral.
As shown in figure 1.9 let 'AB' be curve drawn between two points A & B in a vector field. Let 'dl' be an element of length along the curve 'AB' at R.

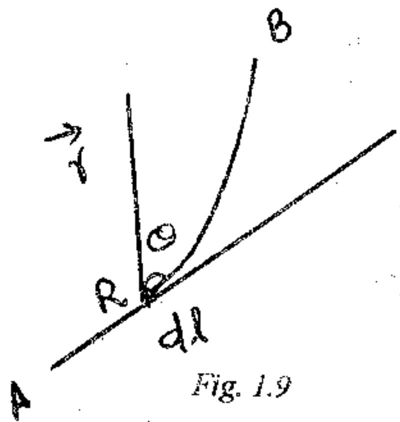


Fig. 1.9

Suppose r is a vector at $\vec{R}$ making an angle θ with the direction of dl then

$$r.dl = r.dl\ \text{Cos}\ \theta = dl\ (r \cos\theta) \qquad ...(1.35)$$

Equation 1.35 shows that the volume or r. dl at any pint of the curve is equal to the product of small element dl and the component (r cos) of r along the direction of dl.

The value of (r.dl) for the complete curve AB can be obtained by integrating equation 1.35

$$\text{Hence} \int_A^B r.dl = \int_A^B dl\ (v \cos\theta) \qquad ...(1.36)$$

Integral $\int_A^B r.dl$ is defined as the line integral of A along the curve AB

1.6.2. Calculation of work done by a varying force on a body using the line integral.

Work done by a force on an object that under goes an infinitesimal vector displacement 'dr' can be written as

$$dw = F.dr$$

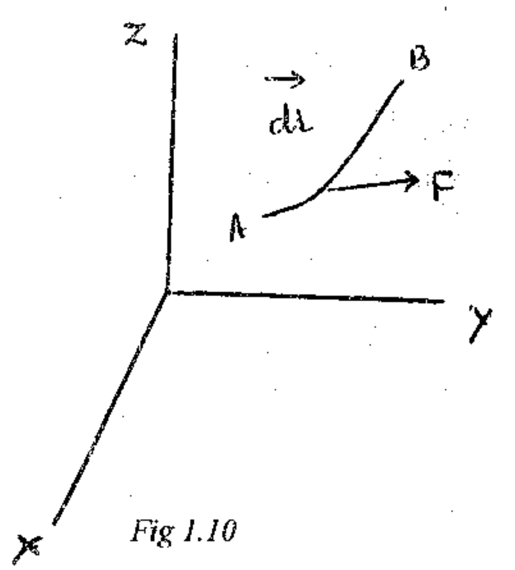


Fig 1.10

In general, the forces F acting on the object varies from point to point. For example, the force on a changed particle in an electric field would be function of x, y. z. However, along a curve, x, y, z are related by the equation of the curve. Since along a curve there is only one independent variable we can write F & $dr = i\,dx = j\,dy + k\,dz$ as functions. of a single variable. The integral of $dw = F.\,dr$ along the given curve is then reduced to one ordinary integral of a function of one variable and the total work done by F in moving an object say from A to B, can be determined as shown in fig 1.10. This integral is also used in conservative field.

Another good example would be if 'r' represents the electric field intensity at any point, then the integral represents the potential difference between A & B.

1.6.3. Surface Integral

If in a surface area of 'S' an infinitesimal area 'ds' in considered and if A is a vector at the middle of the element 'ds' in a direction making an angle θ with the unit positive (outward drawn) normal to ds fig (1.11)

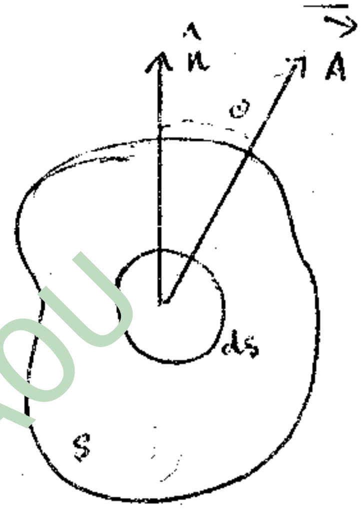


Fig 1.11

Then the integral over the surface

$$\iint A.\,ds \Rightarrow \iint A.\,n\,.\,ds \Rightarrow \iint A\cos\theta\,ds \qquad \ldots(1.37)$$

is defined as the surface integral or total flux of ' A' through the whole surface S.

As an example if A represents the velocity of a moving fluid in which a fixed surface 'S' is drawn then $\iint_s A.ds$ represents the rate of flow of fluid through the surface S.

1.6.4. Volume Integral

Let, us consider a closed surface in space enclosing a volume V. If A is vector function inside it and dv is a small element of volume then.

$$\iiint A.\,dv \Rightarrow \int_x\int_y\int_z (\hat{i}\,Ax + \hat{j}\,Ay + \hat{k}\,Az)\,dx\,dy\,dz$$

is called the volume integral of 'A' over the volume V. (dv = dx dy dz).

The above three integrals are also useful in solving many of the physical problems.

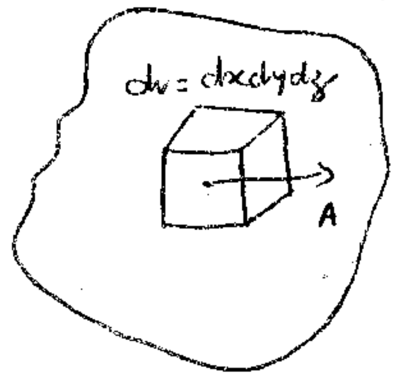


Fig 1.12

Check your progress-1

1. Is divergence a vector or scalar quantity ?
2. Define divergence of a vector field ?
3. Curl A=-----------
4. What is the method provided to reduce triple integral to double integral?
5. What do you understand by line integral and surface integral?

Note: a) Space is provided for your answer.
b) Compare your answers with those given at the end of the unit.

....

....

....

1.7. STOKE'S THEOREM

Statement: This theorem states that the line integral of vector field 'A' around a closed curve is equal to the surface integral of the curl of that field taken over the surface S surrounded by the closed curve i.e.

$$\int \text{Curl } A.\, ds \Rightarrow \int A.\, dv \qquad ...(1.39)$$

Here ds = is a very small element of area of surface S

And dv = is a very small element of the bounding C of the surfaces.

Obviously this is a theorem by means of which the surface integral of the curl of the vector field 'A' is converted to the line integral of the vector field and vice-versa.

1.7.1 Proof stoke's theorem

Let us consider a surface 'S' with C as its bounding & is enclosed in a vector field A.

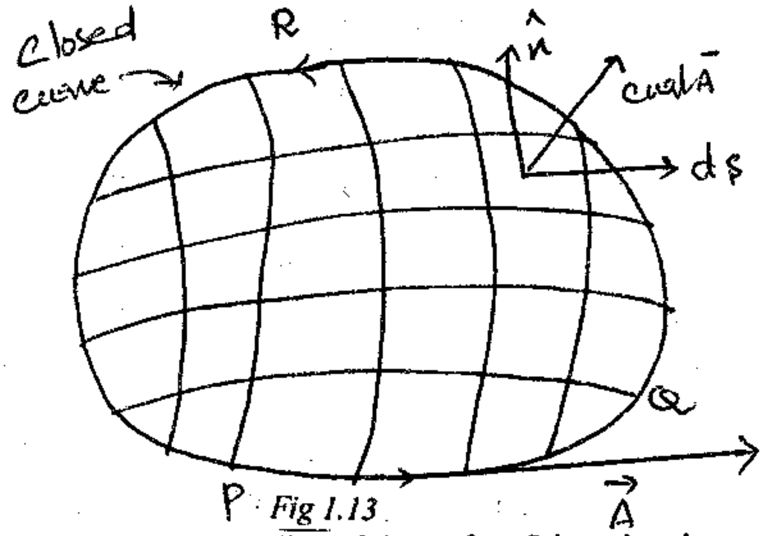


Fig 1.13

As shown in figure the bounding of the surface S is a closed curve 'PQR'. The line integral A around the curve. PQR traced counter clockwise is

$$\oint A.dr \qquad \ldots(1.40)$$

Let the entire surface be divided into a large number of square loops. Let the area enclosed by each infinite small loop be ds. Suppose $\hat{n}$ be a unit positive outward normal to ds. The vector area of the element is

$$\hat{n} \, . \, ds = ds \qquad ..(1.41)$$

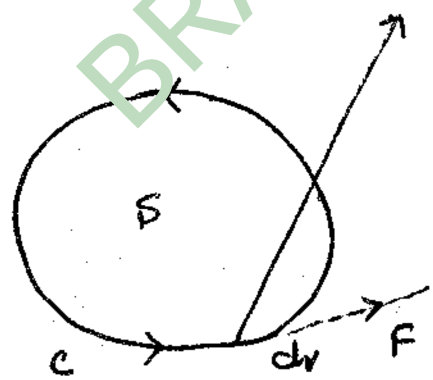


Fig 1.14

We know that curl of a vector field at any point is the maximum line integral of the vector computed per unit area along the boundary of an infinitesimal area at that point. So the lineintegral of A around the boundary of the area ds is

$$\iint_S \text{Curl } A. \, ds \qquad \ldots(1.41)$$

This is the surface integral of A. from the fig 1.14 it is clear that the line integral along the common sides of the continuous element, mutually cancel because they traverse in opposite directions. Now the sides of the elements which lie on the periphery of the surface (i.e. in the closed curve) contribute to the line integral. The sum of the line integrals on the boundary line of the curve is given by equations above. Hence

$$\oint A.\, dr \Rightarrow \iint_S \text{Curl } A.\, ds \Rightarrow \iint_S (\nabla \times A).ds \qquad ...(1.43)$$

This is known as Stoke's theorem

1.8. GAUSS DIVERGENCE THEOREM

A method of reducing triple integral to double integral is provided by the Divergence theorem of Gauss

It states that the surface integral of the normal component of vector 'A' taken over a closed surface 'S' is equal to the volume integral of the divergence of vector A over the volume V endorsed by the surface 'S' i.e.,

$$\iint A.\, \overrightarrow{ds} \Rightarrow \iiint_V \text{div } A.\, dv \Rightarrow \iiint_V (\nabla \times A).\, dv = \int \oint A.\, \overrightarrow{ds} \qquad ...(1.44)$$

This theorem provides a method for connecting volume integral (triple integrals) to surface integrals (double integrals)

1.8.1. Proof of Gauss theorem

Consider a closed surface 'S of any arbitrary shape drawn in a vector field A as shown in Fig 1.15

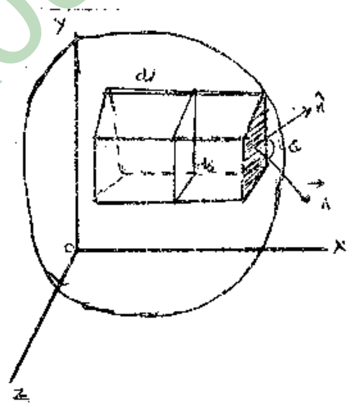


Fig 1.15

Let the surface encloses a volume V. Now the whole volume may be assumed to be divided into a very large number of cubical volume elements adjoining to each other. Consider a small cubical volume element as shown in figure 1.14

The amount of flux (div A) diverging per unit volume and hence the flux diverging from the element of volume dv is known to be (div A dv). So the total flux coming out from the entire volume is given by

$$\iiint_V \text{div } A\, dv \qquad ..(1.45)$$

Now we consider a small element of area as on the surface 'S' as shown in figure 1.15

Let $\hat{n}$ represent the unit vector drawn perpendicular to ds. In convention the outward drawn normal is taken as +ve. If the field vector A & outward normal $\hat{n}$ are at an angle θ then the component of A along n is.

$$A \cos\theta \Rightarrow A.\, n \qquad ...(1.46)$$

The flux of A through the surface element ds is given by $(A.\, \hat{n})\, ds \Rightarrow A.\, ds$ ...(1.17)

($\because$ Flux is defined, as the product of normal component of vector is surface area)

So the total flux through the entire surface S is given by $\iint_S A.\, ds$ ...(1.48)

This must be equal to the total flux diverging from the whole volume V enclosed by the surface S.

Hence from equation 1.45 & 1.48 we get

$$\iint_S A.\, ds \Rightarrow \iiint_V \text{div } A.\, dv \qquad ...(1.49)$$

This is Gauss theorem of divergence. Gauss theorem may also be written as

$$\iint_S (A.\, n)\, ds \Rightarrow \iiint_V (\nabla . A)\, dv \qquad ...(1.50)$$

1.9. SUMMARY

In fields especially in general, physical quantities have different values at different points in space. Thus for example, the temperature in a room varies from place to another, place, being higher near a fire place and lower near an open window. Similarly the electric field near a point charge is larger that at points farther from it. Similarly the magnetic field or intensity near a current carrying conductor is more at nearer to conductor and less away from it. The expression field is used to imply both the region and the value of the physical quantity in the region (electric field, gravitational field or magnetic field etc.)

If the physical quantity is of the scalar category (for ex temperature) then we are only concerned with scalar field. However if the quantity is that of vector type (for ex electric field, magnetic field due to a steady current, incompressible fluid with velocity etc.

The general meaning of curl is rotation when curl A is zero, it means that no rotation is attached with vector A where as curl A is non zero, it means that rotation is attached with vector A.

To make it clearer, consider the flow of a liquid. Let a friction less paddle is placed in the path of the liquid flow. In a hypothetical case, if we assume that all the liquid layers are moving with the same velocity which in the present context paddle will not rotate. This shows that the curl of velocity vector is zero. If we consider that the different layers are moving with different velocities (as in the actual case), then the paddle will rotate. The rotation is due to non zero value of curl of velocity vector. Like the one above many such similar problems can be dealt with the aid of vectors and vector theorems and rules.

Check your progress: Answers.

1. Divergence is a Scalar quantity, because it represents simply the amount of flux.

2. The divergence of a vector field at any point is defined as the amount of flux/unit volume diverging from that point.

3.

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ax & Ay & Az \end{vmatrix}$$

4. A method of reducing triple integrals to double integrals is provided by the Divergence is theorem of Gauss.

5. Line Integral: The integration of a vector along a curve is called its line integral and surface integral is called its line integral and surface integral is the integral over a surface "S". i.e. $\int\int_S A.\,ds \Rightarrow \int\int_S A.\overline{n}\,ds \Rightarrow \int\int_S A\cos\theta\,ds$ is defined as the surface integral or it is also the total flux of 'A" through the whole surface S.

1.10. WORKED OUT EXAMPLES

Examples: If $A = iy + j(x^2 + y^2) + k(yz + zx)$ then find div A at point (1, -2, 3)

Solution: we know that div $A = \nabla . A$

$$\text{Where } \nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}$$

$$\therefore \text{div } A = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)\{iy + j(x^2 + y^2) + K(yz + zx)\}$$

$$= \frac{d}{dx}y + \frac{d}{dy}(x^2 + y^2) + \frac{d}{dz}(yz + zx)$$

$$= 0 + 2y + (y + x) = (3y + x)$$

At point (1, -2. 3), div A = 3 (-2) + 1 = -5

(2) It 'r' is the position vector of a point, them show that

a) div r = 3 & b $\nabla(\nu.\,A) \Rightarrow A$

Solution: a) the position 'r' is written as

$$r = ix + jy + kz$$

$$\therefore\ r^2 = x^2 + y^2 + z^2$$

$$\text{Now div } r = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)(ix + jy + kz) = \nabla . r$$

$$= \frac{dx}{dx} + \frac{dx}{dy} + \frac{dx}{dy} = 1 + 1 + 1 = 3$$

(8) Using stokes theorem, prove that

$\oint r.dl = 0$, where r is position vector.

Solution: By stokes theorem we know

$\oint_C A.\,dl = \iint curl\ r.\ ds$

Replacing the vector A by the position vector r, we get

$\oint r.\,dl \Rightarrow \iint curl\ r.\ ds \Rightarrow \iint_S o.ds$ (curl r = 0)

Hence $\oint_c r.dl \Rightarrow 0$

(a) Using stokes theorem, prove that curl grad $\phi = 0$

Solution: According to stoke's theorem

We know $\oint. A.\,dl = \iint_S Curl\ A.ds$

Let A = grad ϕ, then

$$\oint_C Grad\ \phi\ dl = \iint_S curl\ grad\ \phi\ \hat{n}\ .\ ds$$

But grad ϕ. dl $\Rightarrow \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)(i\,dx + j\,dy + k\,dz)$

$$\Rightarrow \frac{d\phi}{dx} + \frac{d\phi}{dy} + \frac{d\phi}{dz} = d\phi$$

$\oint_c d\phi \Rightarrow [\phi]A$ where 'A' is any point on C $\Rightarrow 0$

Hence $\iint curl\ grad\ \phi\ .\hat{n}.\ ds = 0$

This is true for all surface elements 'S' i.e. curl grad $\phi = 0$

1.11. SAMPLE EXAMINATION QUESTIONS

1. a) What do you understand by the gradient of a scalar field? Explain the physical significance.

   b). Obtain an expression for the gradient of scalar function in rectangular Co-ordinates.

2. a). Explain the physical significance of divergence of a vector field.

   b). Derive an expression for div A in terms of Cartesian Components.

3. a). Explain Curl of a vector field. Give its physical significance also meaning of curl.

   b). Derive an expression for curl of a vector field show that Curl A = $\nabla \times A$

4. Explain the terms divergence & curl of a vector field? Give examples from physics where these are used.
5. State and prove Gauss's theorem of divergence.
6. Write short notes in a) Line, Surface & Volume integrals
7. State and prove stokes theorem.
8. What are scalar vector fields? Explain the gradient of a scalar field & divergence of a vector field.

II. Solve the following problems.

1. If ϕ (x,y,z) = 3x2y – y3z3, find the value of grad ϕ at a point (1, -2, 1)

(Ans: -i12 – j9 – k 16)

Hint: Grad ϕ $\nabla\phi = i\left(\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)(3x^2y - y^3z^2)$

$= i\,6xy + j\,(3x^2 -, y^2z^2) - k\,2y^3z^2$

(Now substitute x = 1, y= -2, & z = -1)

2. If $A = I\,3x^2 + j\,5xy^2 + k\,xyz^2$ fine the value of dive A at point (1, 2, 3). (Ans:38)

Hint : div A = $d\frac{Ax}{dx} + d\frac{Ay}{dy} + d\frac{Az}{dz}$

$= (3x^2)\frac{d}{dx} + ((5xy^2)\frac{d}{dy}\ (xyz^2)\frac{d}{dz}$

= 6x + 10 xy + 2 xyz

(Substitute x = 1, y= 2, z = 3)

3. Prove that 1) $\nabla \times (\phi A) = \phi(\nabla \times A) + \nabla\phi \times A$ where ϕ b scalar

2) $\nabla \times (A + B) = \nabla \times A + \nabla \times B$.

4. Evaluate 1) div grads 2) Curl grads 3) grad div A and 4) div curl A & 5) Curl

Ans :(i) $\nabla^2 s$ (ii) 0

(3) $i\frac{d}{dx}\left(d\frac{Ax}{dx} + d\frac{Ay}{dy}\right) + j\frac{d}{dy}\left(d\frac{Ax}{dx} + d\frac{Ay}{dy}\right) + k\frac{d}{dz}\left(\frac{dAx}{dx} + \frac{dAy}{dy} + \frac{dAz}{dz}\right)$

(4) 0 & (5) grad div A - $\nabla^2 A$

b) $r . A \Rightarrow (ix + jy + kz) . (iAx + jAy + kAz)$

$$\Rightarrow xAx + yAy + zAz$$

$$\therefore \Delta (r.A) = (i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz}) (XAx + YAy + ZAz)$$

$$= i Ax + jAy + kAz = A$$

(3) If the gravitational potential at any point is(- GM / r), where 'r' is the position vector of the point, find the intensity of gravitational field at the point

Solution: Here V = ix + jy + kz

Also $r = (x^2 + y^2 + z^2)^{1/2}$

Intensity $= \nabla (-GM/r) = - GM \nabla (1/2) = - GM \nabla (x^2 + y^2 + z^2)^{-1/2}$

$$\Rightarrow -GM \{ i \frac{d}{dx}(x^2 + y^2 + z^2) + j \frac{d}{dy}(x^2 + y^2 + z^2)^{-1/2} k \frac{d}{dz}(x^2 + y^2 + z^2)^{-1/2}$$

$$\Rightarrow -GM \{ i (-1/2) (x^2 + y^2 + z^2)^{-3/2} . 2x + j (-1/2) (x^2 + y^2 + z^2)^{-3/2} . 2y +$$

$$k (-1/2) (x^2 + y^2 + z^2)^{-3/2} . 2z\}$$

$$\Rightarrow -GM \frac{ix + jy + kz}{(x2+y2+z2)^{3/2}} \Rightarrow GM \frac{r}{r^3} = \frac{GM}{r^2}$$

(4) If $A = iy + j (x^2 + y^2) + k (yz + zx)$, then find curl A at point (2, 2, -2)

Solution: We know that curl A = ∇ . A

$$\therefore \text{Curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ Ax & Ay & Az \end{vmatrix} = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y & x^2+y^2 & yz+zx \end{vmatrix}$$

$$\Rightarrow i \{ \frac{d}{dy}(yz + zx) - \frac{d}{dz}(x^2 + y^2) \} + j \{ \frac{d}{dz} y \frac{d}{dx}(yz + zx)\} + k \left[\frac{d}{dx}(x^2 + y^2) - \frac{d}{dy} y \right]$$

Curl A= i (z – 0) + j (0 – z) + k (2 x – l)

$$\Rightarrow iz - jz + k (2x - 1)$$

Substituting the values we get

Curl A = -2i + 2j + 3k

(5) Find the constants a, b, c so that

Vector A = i (x + 2y + az) + j (bx – zy – z) + (4x + cy + 2z) is irrational.

Solution: The vector is irrational when curl A = 0

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ (x+2y+az) & bx-3y & 4x+cy+2z \end{vmatrix}$$

$$= i\left\{\left(\frac{d}{dy}\right)(4x+cy+2z)-\frac{d}{dz}(3x-3y-z)\right\}+j\left[\left\{\left(\frac{d}{dx}\right)(x+2y+az)-\frac{d}{dx}(3x-3y-z)\right\}\right]+$$

$$k\left\{\frac{d}{dz}(3x\ d\ 3y-z)-\frac{d}{dy}(x+2y+az)\right\}$$

$$\Rightarrow i(c+1)+j(a-4)+k(b-2)$$

The value of this is zero where C = -1, a=4 & b=2

(6) Find the value of div (grad S)

Solution: div (grad s) = ∇.∇S

$$= \left(i\frac{d}{dx}+j\frac{d}{dy}+k\frac{d}{dz}\right)\left(i\frac{ds}{dx}+j\frac{ds}{dy}+k\frac{ds}{dz}\right)$$

$$= \frac{d}{dx}\left(\frac{ds}{dx}\right)+\frac{d}{dy}\left(\frac{ds}{dy}\right)+\frac{d}{dz}\left(\frac{ds}{dz}\right)$$

$$= \frac{d^2s}{dx^2}+\frac{d^2s}{dy^2}+\frac{d^2s}{dz^2} = \nabla^2 S$$

$$\text{Where } \nabla^2 = \frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$$

(7) Prove that div (ϕ A) . ∇ (ϕ A)

$$\Rightarrow \left(i\frac{d}{dx}+j\frac{d}{dy}+k\frac{d}{dz}\right)(i\phi Ax+j\phi Ay+k\phi Az)$$

$$\Rightarrow \frac{d}{dx}(\phi Ax)+\frac{d}{dy}(\phi Ay)+\frac{d}{dz}(\phi Az)$$

$$\Rightarrow \phi d\frac{Ax}{dx}+\phi d\frac{Ay}{dy}+\phi d\frac{Az}{dz}$$

$$\Rightarrow \left(\phi\frac{dAx}{dx}+Ax\frac{d\phi}{dx}\right)+\left(\phi\frac{dAy}{dy}+Ay\frac{d\phi}{dy}\right)+\left(\phi\frac{dAz}{dz}+Az\frac{d\phi}{dz}\right)$$

$$\Rightarrow \left(\phi\ \frac{dAx}{dx}+\frac{dAx}{dy}+\frac{dAx}{dz}\right)+\left(Ax\frac{d\phi}{dx}+Ay\frac{d\phi}{dy}+Az\frac{d\phi}{dz}\right)$$

$$\Rightarrow (\phi\ \text{div } A + \{(iAx+jAy+kAz).\left(i\frac{d\phi}{dx}+j\frac{d\phi}{dy}+k\frac{d\phi}{dz}\right)$$

$$\therefore\ \text{div } (\nabla A) \Rightarrow \phi\ \text{div } A + A\ .\ \text{grd } \phi$$

UNIT –2: ELECTRIC FIELDS AND GAUSS THEOREM

Contents

2.1 OBJECTIVES

This unit introduces you to the concept of electric field and its intensity. To help you understand the concepts the unit explains

1. The electric field and electric field intensity
2. Lines of forces
3. Gauss Theorem

By going through this unit you will understand what is an electric field and lines of forces and what Gauss theorem means. You will also understand how the lines of forces are used in explaining the electrical induction and how Gauss theorem is used for various electrical problems.

2.2 INTRODUCTION

An apple falling under the action of gravity was observed by Newton in 16th Century. This is explained as due to the force of attraction on apple by earth. Or the apple is said to be in the gravitational field of the earth. The gravitational field over a limited surface is fairly constant.

The flow of water in a river is under the action of flow field. The water would not flow into a river had there been no influence of flow gradient. The filed is said to be constant over a limited space.

Similarly the rod rubbed with fur will not be repelled by another rod rubbed with fur, if there are no charges on the rods. The region surrounding the charged material within which the influence of electric charges is felt on conductors is referred to as electric field the electric field as well as gravitational and flow fields, are elective over certain regions surrounding them. The fields are said to be stationary if the fields do not vary with time.

Earlier to faraday, the force acting between charged particles was thought of as a direct and instantaneous one. This action-at-distance view was held for magnetic and gravitational forces also. But to-day forces are preferable interpreted in terms of electric fields as follows:

If only we are interested in the forces between stationary charges, the field and action-at-a distance points of view would be perfectly equivalent. But how as acceleration of one charge q_1 is communicated to its neighbor? The field concept says that the distances in the field surrounding the accelerated charges are communicated to q_2 at a velocity equal to that of light. The action-at-a-distance view requires two instantaneous communications of acceleration from q_1 and q_2. But the second concept is found to be wrong. For example, the accelerated electrons in an antenna: (of a radio transmitter) influence electrons in a receiving antenna only after a time l/c, where l and c are the separation of antennae and velocity of light respectively.

The forces experienced by a charge in a field varies from point to point-both in intensity and direction. If there is no variation in both, the field is said to be uniform. It is a vector quantity.

2.3 INTENSITY OF ELECTRIC FIELD

The force experienced by a unit electrostatic charge, placed at a point within the electric field is a measure of the electric field intensity, strength of the field or the electric field vector at that point.

The presence of unit charge does not in anyway vary the intensity of electric field appreciable.

By inverse square law, the force on a charge q_2 in the, presence of q_1 at a distance r (from q_1) was given by coulomb as,

$$F = \frac{q_1 q_2}{Kr^2} \qquad \ldots(2.1)$$

K is the dielectric constant of the medium where in both q_1 and q_2 are situated. But the force, acting on unit charge (i.e. $q_2=1$) placed at a distance r front q_1 is by definition, so the electric field intensity

$$E = q_1/kr^2,$$

Hence $F = q_2E$

$$\vec{E} = \frac{q_1\hat{r}}{4\pi\Sigma_0 Kr^2} \text{ N/Coulomb}$$

Above eqn. gives us a new definition of electric field intensity **E.** We can define **E** as,

$$= \frac{q_1\hat{r}}{4\pi\,\Sigma_0 r^2} \quad ; \text{ if } K = 1$$

$$= 9 \times 10^9 \frac{(q_1)}{r^2} \text{ N/Coulomb}$$

In CGS units $E = q/Kr^2$ dynes/esu or esu/cm

2.4 INTENSITY OF FIELD ON THE SURFACE OF A CHARGED CONDUCTOR COULOMB'S THEORM

Consider a tube of force originating from small area ds on the surface of a charged conductor. This has a surface density σ and is placed in a medium of dielectric constant k

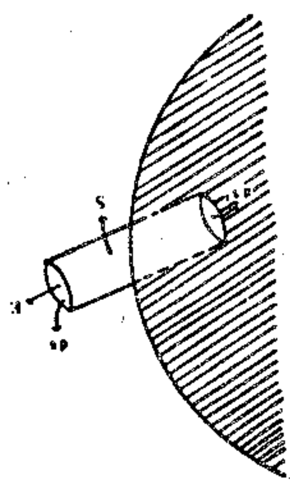


Fig 2.1 Coulomb's theorem

The direction of E for points close to the surface is at right angle to the surface pointing away from the surface if the charge is positive. If E were not normal to the surface, it will have components lying in the surface. This component would act on the charge carries in the conductor and set up currents. The absence of such currents under the assumed electrostatic conditions. sets in the condition that E should be normal to the surface.

Let the tube extend a short distance within the conductor, so as to form a cylinder. Let the planes ds and ds be drawn infinitesimally close to s one outside and the other inside at equal distances. The normal electrical induction through side, or perpendicular to the length of cylinder formed by ds and ds as faces zero, as they form Gaussian surfaces. Since the charge resides on the surface of the conductor, there is no charge on ds and the normal electrical induction through it is zero.

Total normal electrical induction through ds = K Eds

$$= \sigma ds \varepsilon_0$$

(Since σ ds is the charge on s), by Gauss' Theorem

$$\text{So } E = \frac{\sigma}{\Sigma_0 K} \qquad \ldots(2.2)$$

Since every portion of the conductor can be treated in the same fashion the electric intensity at any point close to a charged conducting surface is given by

$\frac{\sigma}{\Sigma_0 K}$ This is exactly coulomb's Theorem.

The electric Displacement D = Σ_0K = 4Πr

Here σ is in Coulomb/m^2

In CGS system $E = \frac{4\pi}{K}$ Volts/m

It is to be noted that E is the intensity due to the whole of the charge on the conductor.

From Eqn. (2.2), the electric intensity near the surface of a charged conductor is twice as great near a plane sheet of charge. In fig 2.2. all the electric flux emerges from the outside surface while in Fig 2.2 half the flux Plane sheet of charge $\frac{\sigma}{2K\varepsilon_0}$ emerges from one side & half from the other.

Examples – 1 : Show that, of the total intensity $\frac{\sigma}{2\varepsilon_0}$ at a point A at a distance of half an inch from the plane, one half is due to the charges at points with in an inch of A.

Solution : The charge on an elemental area at R is ds, and if R is ds, and if r is its distance from A, the electric intensity it produces at A is $\frac{\sigma ds}{4\pi\varepsilon_0 r^2}$ along RA, its component along the normal AN is $\frac{\sigma ds \cos\theta}{4\pi\varepsilon_0 r^2}$

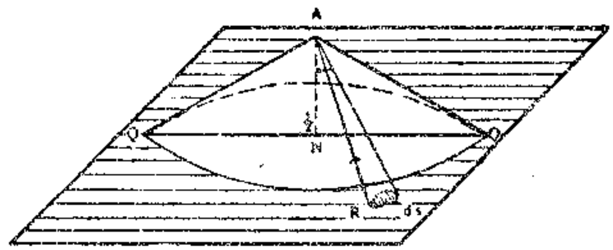


Fig 2.2. Electric field intensity of a right circular cone

Or $\frac{d\omega}{4\pi\varepsilon_0}$ where $d\omega$ is the solid angle subtended at A by ds.

The points of the plane which lie within an inch of A, lie with in a circle of center N and redius NQ, where AQ=1. The charge on this circular area, thus contributes to the total intensity at A an amount.

$$\frac{d\omega}{4\pi\varepsilon_0} = \frac{\sigma 2\pi (1-\cos 60^0)}{4\pi\varepsilon_0} = \frac{\sigma}{4\varepsilon_0}$$

Hence, half of the total intensity is due to the charge which lie within an inch of A (which are embraced in a right circular cone of semi-vertical angle 60o

2.5 LINES OF FORCE

A line of force is defined as a curve indicating the direction in which a unit positive charge would travel. The tangent at any point of this curve gives the direction of the electrical force at that point.

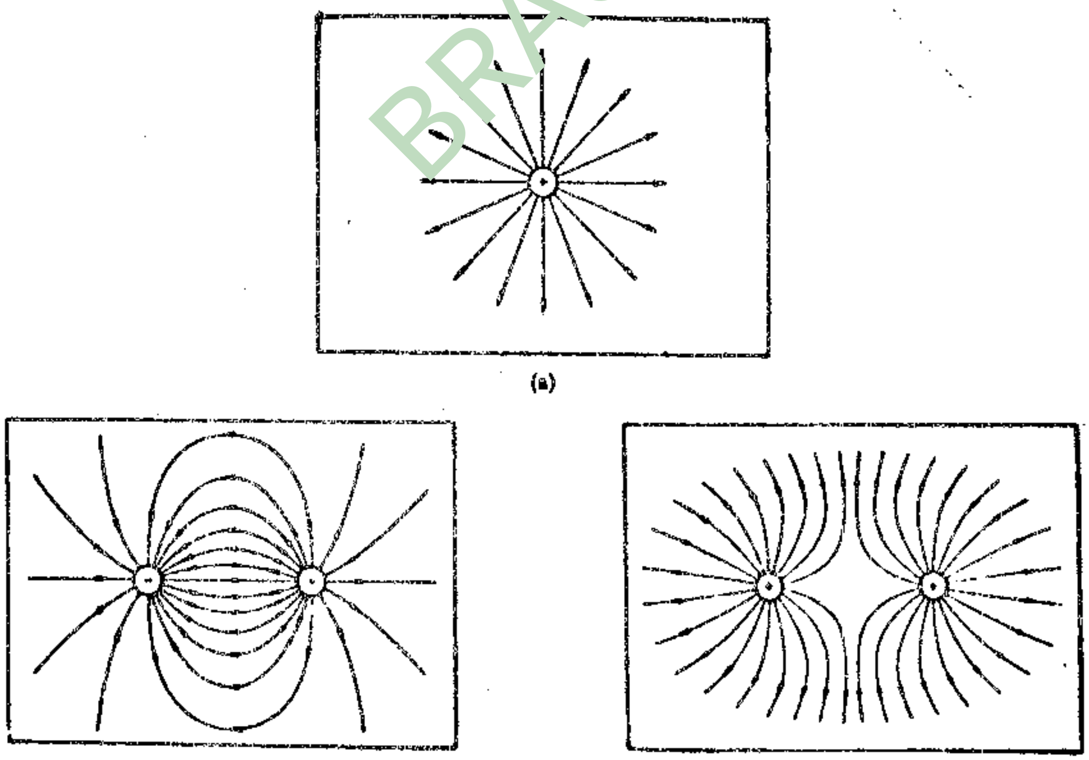
The concept of lines of force had been used by Michael Faraday to represent E. This will help visualization of electric field patterns easily. This gives a vivid picture of the way that the electric field varies through a region of space. The number of lines per unit

Cross-sectional area is proportional to the magnitude of electric field $\vec{E}$. When $\vec{E}$ is large, the lines of forces are close together and they are far apart when $\vec{E}$ is less.
The existence of lines of force can be shown by placing a highly charged body below horizontal glass plate and sprinkling some sawdust particles or powder or gypsum salt on the plate. Then these particles acquire charges and place themselves along definite directions. These lines of force enclose a tube of force.

The field lines begin and end on charges. Electric field is detected by stationary charges and is produced by charges which may be stationary or moving. They are continuous in an isotropic dielectric but end on opposite charges. One end is positive while the other end is negative. The lines of force leave or end on a conductor when the charges are in equilibrium. No charge moves across a charged conductor.

According to Faraday, the forces of attraction and repulsion are explained by assuming that there exists a longitudinal tension along the, tubes of force and lateral pressure at right angles to these tubes of forces. They produce strain in the medium where ever they lie.

When a tube of force from a positively charged body comes into contact with an unchanged body, it will then induce a negative charge on the latter, since a tube is subjected to axial tension it tends to shorten along its length. Thereby opposite charges come nearer. This results in attraction. The repulsion due to the positive charge on the far side is small. In the case of two like charges, the lines of forces repel each other. Direction of lines of force in a few cases are shown in Fig (2.3)



(a)

(b)

(c)

Fig: 2.3 Lines of force

2.6 ELECTRIC INDUCTION

Faraday gave a quantitative significance for the lines of force and tubes of force. The number of lines per unit area represent the intensity of the electrostatic field at a point in e.s units.

If 4π lines of force emanate from a unit charge, these lines are then called unit lines of induction. The number of lines of induction per unit area of a spherical surface of radius **r** is called the flux density. The flux density is given by

$$\frac{4\pi q}{4\pi r^2} = \frac{q}{r^2} = \kappa \frac{q}{kr^2} = \kappa E$$

Lines representing **E** are lines of forces, but lines representing, κ E are lines of induction Total number of lines of induction that cut through a surface normally is called total normal electric induction or electric flux.

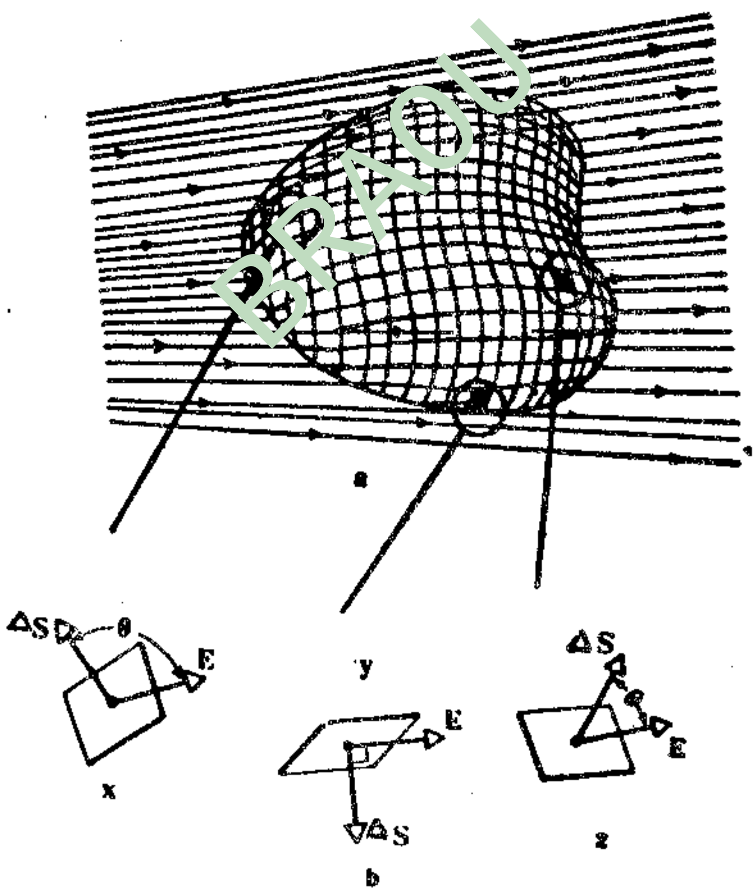


Fig 2.4 Electric flux over different surface elements

The total flux $= \int\int_s \vec{E} ds$

$\int\int_s$ is the surface integral. If **E** varies from point to point and the surface element ds is inclined at an angle θ to the lines of force. Then the electric flux ϕ is given by

$$\phi = \int\int_s E \cos\theta ds$$

For closed surfacesθ, is positive if the lines of force point outwards and negative if they act inwards.

Flux is a property of any vector field. It refers to hypothetical surface in the field, which may be closed or open. For example, for stream line flow, flux is measured by number of stream lines cutting through such a surface. For magnetic field, flux is measured by number of magnetic lines of force cutting through such a surface.

2.7 ELECTRIC DISPLACEMENT

Since a tube of induction starts on positive charge and ends on a negative charge, the positive charge may be considered as a displacement in one direction at one end of the tube, and negative charge is displacement in the opposite direction at the other end of the tube.

The charge displaced per unit area perpendicular to the field gives the amount of displacement usually represented by D.D is called the electric displacement and is measured by the product of electric intensity E and permitivity K of the medium and the primitivity of vacuum ε_0 $\varepsilon_0 \kappa E$.

If κE is the flux density and S is the surface area, κ.E.s is the total normal electric induction and is constant over a surface

$$D = \varepsilon_0 \kappa E = \varepsilon_0 \kappa \frac{q.1}{Kr^2} = \frac{\varepsilon_0 q}{r^2}$$

So D is independent of κ

$$\text{D in MKS units} = \frac{\varepsilon_0 K q}{4\pi\varepsilon_0 K r^2} = \frac{q}{4\pi r^2}$$

$$= \frac{q}{4\pi r^2} \text{ Coulomb/m}^2$$

While electric field **E** is similar to stress and electric displacement, **D** is similar to strain in the medium. The existence of strain in the medium surrounding a charged body lead Maxwell to formulate electromagnetic theory of radiation.

2.8. GAUSS THEOREM

Karl Friedrich Gauss (1777-1855), director of Gottingen observatory made outstanding contributions to astronomy, mathematics, electricity and magnetism. Gauss' theorem will enable us to determine the intensity of electric field at any point on a closed surface, if the charge inside the surface is known. Similarly the charges producing the field can be determined, if the charge inside the surface is known. Similarly the charges producing the field can be determined, if the intensity is known. Gauss' theorem states that. "*the total normal electric induction (or flux) over a closed surface of any shape drawn in an electric field is 4π times the total charge (or the algebraic sum of the charges) within the surface.*

Proof of the Theorem

A point charge of q coulombs is placed in a uniform isotropic medium of dielectric constant κ (Fig 2.5). Let E be the electric intensity at a point **P** directed outwards on the Closed Surface drawn around the charge q. If **r** is the distance of **P** directed from q.

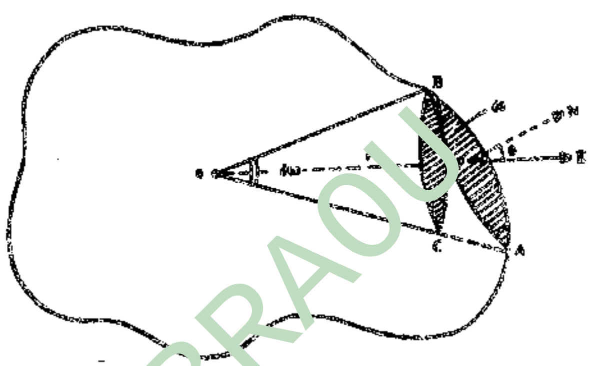


Fig 2.5 Gauss Theorem

$$\text{Then } E = \frac{q}{4\varepsilon_0 \kappa r^2}$$

Then, small elemental area 'ds'. Is surrounding the point P. The normal drawn to the surface the small elemental. area 'ds'., is surrounding the point P. the normal drawn to the surface at P is making an angle 0 with PE. E is the direction of electric field. So the normal component of E along PN is E cos θ

The normal electrical induction over elemental area 'ds' is

$$= \varepsilon_0 \kappa E \cos\theta ds \quad = \varepsilon_0 \kappa \frac{q}{4\pi\varepsilon_0 \kappa r^2} \cos\theta ds = \frac{q ds \cos\theta}{4\pi r^2}$$

But $\frac{ds \cos\theta}{r^2}$ is usually called the solid angle subtended by ds at q (please see appendix – I) .This is represented by $d\omega$

Normal electrical induction over $\vec{ds} = \frac{q d\omega}{4\pi}$

The total normal electrical induction over the whole closed

Surface = $\int_0^{4\pi} \frac{q\,d\omega}{4\pi}$ = q Coul, since the solid angle subtended at a point in space is *equal to* 4π

If q is positive. the induction is directed outwards. When q is Negative it is directed inwards. If there are more than one charge within the surface, each charge contributes an amount of q. So total normal electrical induction due to all charges is Q

$$= \sum_{I=1}^{n} q_i$$

If the charge is continuously distributed over a volume then $q = \iiint_v \rho\, d\upsilon$

were ρ is charge density (charge per unit volume measured in coul/m^3 over the elemental volume 'dv').

Then the total normal induction = $\iiint_v \rho\, d\upsilon$

2.9 APPLICATION OF GAUSS' LAW TO FIELD

The total normal electrical induction = $\iint_s D_n.ds.D_n$ is the normal component of displacement at an elemental area ds. But D_n is related to E_n the normal component of electrical intensity as $D_n = \varepsilon_0 \kappa E_n$.

$D_n = \varepsilon E_n$ (since $\varepsilon = \varepsilon_0 \kappa$)

So, $\iint_2 D_n.ds = \iint_2 \kappa E_n.ds = \iiint \rho.dv$

$$\text{Or } \iint_2 E_n\, ds = \frac{1}{\varepsilon_0 \kappa} \iiint_v \rho.dv \qquad \ldots(2.4)$$

The charge outside the closed surface do not contribute to any field or normal flux. The closed surface drawn in an electric field is called Gaussian surface because Gauss' Law holds good in this surface.

Coulomb's Law Derived from Gauss' Law

A second charge q_2 is put at a point **P** where E is to be calculated. The first charge qi is enclosed in the closed surface containing the point P. Then

$$\vec{F} = q_2 \vec{E}$$

$$\text{But } E = \frac{q_1 \hat{r}}{\kappa r^2}\text{ ; So, } F = \frac{q_1 q_2}{\kappa r^2}\hat{r} \qquad \ldots(2.5)$$

Eqn. (2.3) is nothing but Coulomb's law.

It may thus be noted that Gauss's Law and Coulomb's law are the same though expressed in a slightly different manner. Gauss' Law does not hold good if the law of force were inverse cube. Gauss' law gives a connection between field and its sources (charges). When Coulomb's law tells how to derive the electric field if the charges are known, Gauss' law gives a method of knowing the amount of charges present in the region, if the eletric field is known.

Gauss' law is one of the fundamental equations of electromagnetic theory of Maxwell, Coulomb's law is not listed in that series of equations as it can be derived from Gauss' law.

2.9.1 Applications of Gauss' Law Intensity of Field of Uniformly Charged Sphere

(a) Consider a point P near, but outside a uniformly charged sphere A with a magnitude of charge q Coulombs. Let the radius of this sphere be R. Let the surrounding medium have a dielectric constant κ

Draw a concentric spherical Gaussian surface B about the charged sphere A so that it passes through P as shown in Fig 2.6

Now we have to find out the electric field intensity at the point P. The total surface area of this sphere = $4\pi r^2$ where **r** is the distance of P from the center of the charged sphere. Let E be the intensity of electric field at any point on this sphere. This will be the normal to the surface at every point on the surface.

The total normal electric induction = $\varepsilon_o \kappa E 4\pi r^2$ But the total normal electric induction. According to Gauss' Theorem is q

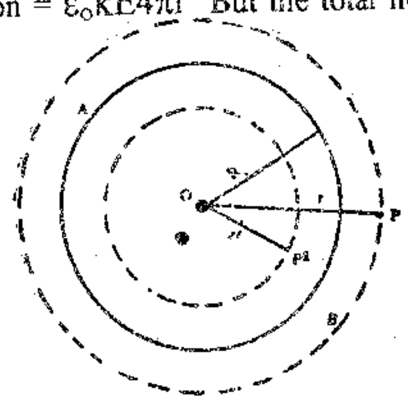


Fig 2.6 Intensity near a spherical conductor

So, $\varepsilon_0 \kappa E.4\pi r^2 = q$

or $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ N/Coulombor Volts/m for free space ...(2.6)

$= \dfrac{q}{4\pi\varepsilon_0 R^2}$ N/Coulomb in a medium of dielectric Constant K (=εκ)

If the charge is continuous, then $q = 4\pi r^2\sigma$ where σ is the surface density of charge

Then $E = \dfrac{4\pi r^2 \sigma}{4\pi\varepsilon_0 \kappa\, r^2} = \dfrac{\sigma}{\varepsilon_0 \kappa}$ Volts/m ...(2.7)

When σ is in Coul/m^2

The field becomes weaker as the lines of force spread out from the surface.
If r = R, i.e., on the surface of the uniformly charged sphere

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R^2}$ ...(2.8)

Eqn. (2.6) shows that the intensity of electric field at an external point of a spherically charged conductor is the same as if the charge were concentrated at the center. If q is in Coulombs, E will be in N/Coulomb. In CGS system

$E = \dfrac{q}{\kappa r^2}$ esu/cm or dynes/esu of charge

Check your Progress:

1. The intensity of field of a uniformly charged sphere is ...... where, q is charge on the sphere and R is the Radius of sphere.
2. Intensity of electric field at a distance 'r' from the charge is given by.......
3. What is a line of force?

Note: a. Space is given below for your answer.
b. Compare your answers with the one given at the end of the unit.

....
....
....
....

2.9.2 When the point P lies Inside the Sphere (i.e., r<R)

Draw another concentric spherical surface with point P1 lying on the spherical surface. The electric flux over the sphere = $4\pi r^2 \kappa E$. But according to Gauss' Theorem, the electric charge inside the closed surface is zero.

Therefore E = O

So the electric field inside the conducting sphere is zero

$$E = \frac{q}{4\pi r^2} \quad \text{when } r > a$$

(ii) Let r < a, then the 'Gaussian surface' encloses only ρ 4/3 πr^2 units of charge, where Gauss' flux theorem gives.

$$\varepsilon \iint_s E.ds = \rho\, 4/3\, r3\, \pi = \frac{Qr^3}{a^3}$$

$$\text{or } \varepsilon E 4\pi r^2 = \frac{qr^3}{a^3}\text{ ; So, } E = \frac{qr^3}{4\pi\varepsilon a^3}$$

2.10. SUMMARY

The region surrounding the charged material within which the influence of electric charges is felt on conductors is referred to as electric field. Electric field is a vector quantity and it is analogus to gravitational field. The distribution of electric field is indicated by the lines of force. The total normal electric induction, over a closed surface, in an electric field is 4π times the total charge within the surface.

Check your progress: Answers

1. $E = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2}$

2. $E = q/Kr^2$ esu/cm.
3. Line of force is a curve indicating the direction in which a unit position charge would travel.

2.1 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail

1. 'State Gauss' Theorem? Discuss the application of Gauss' law to find the intensity of the electric fielduniformly charged conducting sphere.
2. Derive coulomb's law from Gauss'law. Discuss the application of uniformly charged non-conducting sphere.

II Answer the following questions in briefly.

3 Is the Gauss' law useful in calculating the field due to three equal charges located at the corners of an equilateral triangle? Explain.

4. Discuss the similarities between electric fields and gravitational fields. In what ways do they differ?

5. Justify the following statement: the electric field intensity on the surface of any charged conductor must be directed perpendicular to the surface.

III Solve the following problems

6. A charge of +2µ C place in an electric field experiences a force of * X 10'N. What is the magnitude of the electric field intensity?

[Ans : 400 N/Coul.]

7. What is the electric field intensity at 2m from the surface of sphere 20cm in diameter having a surface charge density of $+8 \dfrac{\text{nano coulomb}}{m^2}$?

[Ans : 2.05 N/Coul,]

3. A circular ring of radius 'a' meres has a uniform charge density σ Coulombs/m^2 calculate the field strength along the axis of the ring at a distance 'd' meters from center.

$$\text{Ans : } \left(\frac{ad}{2\,(a^2+d^2)^{3/2}} \right)$$

4.Is the electrical field necessarily be zero inside a charged rubber ballon if the ballon is (a) spherical and (b) sausage-shaped why

5 Calculate the field at a distance 'r' meters from the surface of a charged conducting sphere of radius 'a' meters, whose superficial charge density is σ Coulombs/meters^2

$$\left(\text{Ans: } E = \frac{A^2}{2\sigma(a^2+r^2)^2} \right)$$

6 Calculate the electric field strength at the nuclear surface of I) silver and ii) lead atoms.

7. From the electric field strength calculation for U^{235} can you discuss the stability of the nucleus.

An interesting special case of a spherically symmetric charge distribution in a uniform sphere of charge. This sphere is non – conducting. Such a uniform distribution of charge, occurs only in liquid or gaseous dielectrics.

The charge density $\rho = \frac{q}{4/3\ \pi^3}$ where q is the charge, r is the radius of the sphere.

This is constant for all points with in a sphere and would be zero for all points outside this sphere.

The total intensity E at a point P (Fig. 2.7) inside is obtained as if the charge were concentrated at the center.

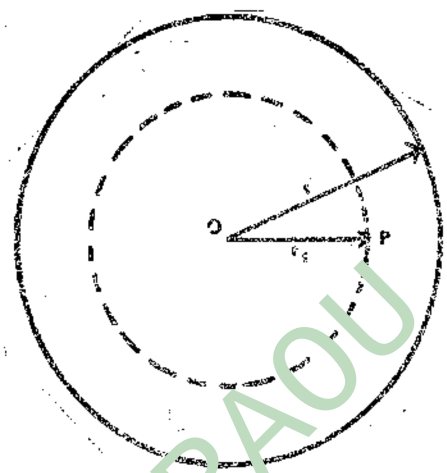


Fig.2.7 Concentric Gaussian surface

$$\varepsilon_0 \kappa E\, \pi r_1^2 = 4/3 \pi r_1^3 . \rho$$

$$\text{or } E = \frac{r_1 \rho}{3\kappa\varepsilon_0} = \frac{r_1}{3\kappa\varepsilon_0}\ \frac{q}{4/3\ \pi r3}$$

$$= \frac{q r_1}{4\ \pi\varepsilon_0 r^2 \kappa}$$

in CGS system, $E = \frac{qr1}{\kappa r^3}$

As the point P gets nearer to the center, the intensity of the field falls off to zero.

Worked Example –1: As an example, take Thomson atom model. The positive charges in the atom are assumed to be distributed uniformly through out a sphere of radius of about 1.0×10^{-10} m. Calculate the electric field intensity E, at the surface of gold atom (Z=79)

Solution: Then the electric field E at the surface of a gold atom (z=79), (neglecting the mutual repulsion of electrons) is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (\because \kappa = 1) \qquad \because q = Ze$$

$$= \frac{(9.0 \times 10^{9}\ N.m^2/Coul.^2)(79)(1.6 \times 10^{-19}\ coul.)}{(1.0 \times 10^{-10}\ m)^2}$$

$= 1.1 \times 10^{18}$ N/Coul.

But this is an erroneous concept. Later Rutherford refined the Thomson atom model and said that the positive charge of the atom is not spread but is concentrated in a small region at the center of the atom (called the nucleus). For gold, the radius of the nucleus is 6.9×10^{-15} m .Neglecting the effects, associated with the atomic electrons, the electric field strength at the nuclear surface, **E** is

$$E = \frac{9.0 \times 10^{9}\ N.m^2 / Coul.^2)(79)(1.6 \times 10^{-19}\ coul)}{(6.0 \times 10^{-15}\ em.)^2}$$

$= 2.3 \times 10^{21}$ N/Coul.

This field is 10^8 times as large as the field calculated by Thomson model.

Worked Example – 2 : If a total charge q were uniformly distributed through out the volume of the sphere of radius 'a' what would be the electric intensity at a distance r from the center of the sphere?

Solution: Since the charge g is uniformly distributed through out the volume $4/3\ \pi a^3$ of the sphere, the volume density of the charge shall be

Fig 2.8

Electrical intensity at a distance **r** from the center of the sphere.

$4/3\ \pi a^3 \rho = q$ or $\rho = 3q / 4\pi a^3$

(i) Let r > a, then by Gauss' flux Theorem

$$\varepsilon \iint_s \vec{E}.\vec{ds} = q \ or\ \varepsilon E \pi 4r^2 = q$$

So, $E = q / \varepsilon 4\pi r^2$

E is constant because of spherical symmetry and the outer sphere is a 'Gaussian surface'

UNIT – 3 ELECTRIC POTENTIAL

Contents

3.1 OBJECTIVES

This unit presents the concept of an electric potential. To help you understand the concept & the unit explains

1. Equipotential surfaces
2. Potential due to a point charge

After going through this unity you will be able to establish the potential due to a point charge, evaluate electric potential due to an electric dipole and describe the construction and working of vande graff generator.

3.2 INTRODUCTION

In this Unit we will discuss the concept of potential.

1. The concept about a dipole & torque as well
2. We will also learn about working principle of a Electrostatic generator.

3.2 ELECTRIC POTENTIAL

The electric field around a charged rod can be described not only by a (vector) electric field strength E, but also by a scalar quantity called electric potential These quantities are intimately related and often it is only a matter of convenience, which of them is used in a given problem.

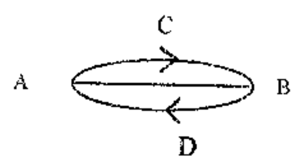
The *electrostatic potential* (or more simply, *electric potential*) at a point P (r) is the amount of work done, against the field, in carrying a unit positive charge from infinity to that point.

The electric potential determines the direction in which the electricity would flow if it is put in communication with another body. It is similar to the concept of gravitational level and temperature.

The objects, under free state, will always move in a direction from a point of maximum gravitational potential to a place of minimum potential. Heat flows from a body at higher temperature to another body of lower temperature. Similarly, electricity flows from higher potential to a lower potential. So it is a requisite condition that current (of electricity) will flow only when there is potential difference between two points.

A positive potential is defined as the electrical condition of the body, which would cause the electrons to flow on to the body from the ground. On the other hand, the body is said to, be at negative potential if the electrons flow from the body to the ground. The earth is so large that any gain or loss of electricity by it does not affect its electrical potential noticeably. Hence it is assumed to be at zero potential. Any object when connected to it will be at zero potential. Then the object is said to be earthed. The earthing is represented by the symbol. E

If a test charge no q_0. moves from A to B, then work is said to be done. The work W_{AB} is related to the potential difference by the relation.



$$V_B - B_A = \frac{W_{AB}}{q_0} \qquad \ldots(3.1)$$

Fig

The work done is a) positive b) negative or c) zero accordingly as V_B is (a) greater than, (b) less than or (c) equal to V_A

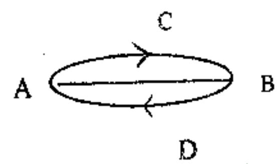
The unit in MKS system for electric potential (or potential difference) is Joules/Coulomb and is also represented by a special unit, 'Volt'. It is defined as 1 Volt = 1 Joule /coulomb. The corresponding unit in CGS system is 1 erg/esu of charge and is sometimes referred to as stat. Volt. Usually the point A is chosen to be at an infinitely large distance from all these charges and the electric potential VA is arbitrarily taken as zero. The Eqn. (3.1) becomes,

$$V = \frac{W}{q_o} \qquad \ldots(3.2)$$

The potential V_A being arbitrarily taken as equal to zero, is known as reference potential. The reference potential could be any value, say 10 volts. In many of the electrical circuit analysis problems, the earth or ground is taken as '*reference potential*'.

The potential near an isolated positive charge is positive because the work done (by an outside agent) to push the positive charge from infinity to the present position is positive. Similarly, the potential near an isolated negative charge is negative because an outside agent must exert a restraining force on a (positive) test charge as it comes in from infinity.

Electrical potential as given by Eqn. (3.2) is a scalar quantity because both work done and the charge are scalar quantities. So both W_{AB} and V_B-V_A in Eqn.(3.1) do not depend on the path followed in moving the test charge from A to B.



Fig

Check you Progress – I

1. what is positive and negative potential?
3. According to definition of Potential, it is work done when a charge q is brought from a point A to B ie the amount of work done

....

....

....

3.3 EQUIPOTENTIALS

As equipotential surface is the one drawn through all points of equal potential in an electric field. So if a charge is moved from one point on such a surface to another point on it, on work is said to be done since by Eqn. (1.3)

$$V_B - V_0 = \frac{W_{AB}}{q} = O \qquad \ldots(3.3)$$

If a charge is supposed to be concentrated at a point, all points equidistant from it should be at the same potential. The equipotential surface around a point charge are a series of concentric spheres having their centers at the point – charge. But the potentials differ from one spherical surface to the other (Fig.3.1). Te force acting on the surface of a changed conductor (i.e., the electric intensity) is perpendicular to the surface at the point. So it follows that the lines of force are those lines along which the force acts normal to the equipotential surfaces.

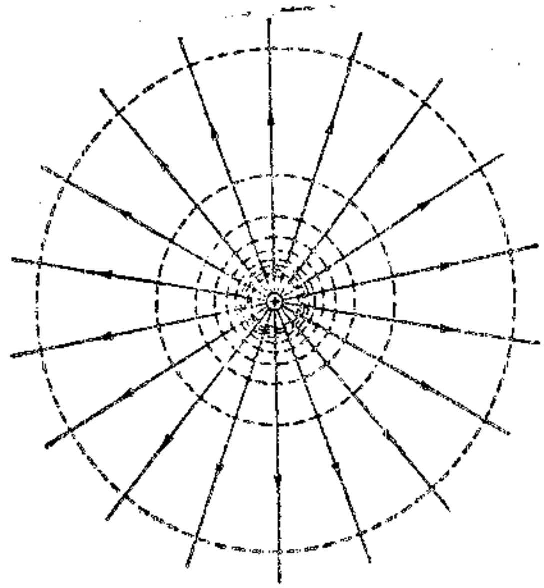


Fig 3.1 Equipotential surface

Lines of force always intersect equipotential surfaces perpendicularly, i.e the resultant electric intensity at any point is at right angels to the equipotential surface at the point(Fig 3.1) the existence of the lines of force and equipotential surface between charged bodies reveals a method of bringing a stream of moving charged particulars to a point focus or of making them divergent (Fig3.2)

A_1,A_2,A_3 are three hollow metallic cylinders lying side by side. Let the potentials on A_1, A_2 and A_3 be 500, 2000 and 500 V respectively.

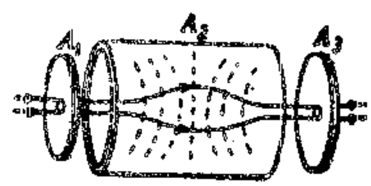


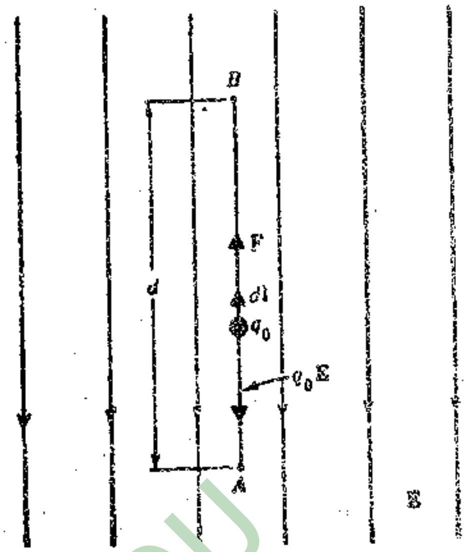
Fig 3.2 Electric lens

Then a beam of electron passing through A_1 will spread out in A_2 and then be directed though A_3 so as to move along curved lines of force. The dotted lines show the directions of equipotential surfaces. The electrons tend to move perpendicularly to the equipotential surfaces in the field in which they are traveling and this sort of arrangement of cylinders charged to different potentials forms an 'electric lens'/ By keeping A1 and A2 at same voltage and by changing the voltage on A2 the shape of the equipotential surfaces can be altered and the electrons can be brought to different "foci". The concept of an electric lens is very much used in the construction of cathode ray oscilloscope, electron diffraction unit, electron microscope etc.

3.5 POTENTIAL AND FIELD STRENGTH

Let A and B in Fig. 3.3 be two points in a uniform electric field E, set up by an arrangement of charges. Let A be a distance 'd' in the field direction from B. Assume that a positive test charge q_0 is moved without acceleration, by an external agent, from A to B along the straight line connecting them.

Fig 3.3. A test charge moving from A to B



The electric force on the charge is A_o E and it points downwards from the movement of charge in the fashion described above, this electric force must be counteracted by applying external force F of the same magnitude but directly opposite i.e., upwards. The work done W by the agent in supplying this force is.

$$W_{AB} = Fd = q_0Ed$$

But Eqn. (3.1) tells that $\frac{W_{AB}}{q_0} = V_B - B_A$ ...(3.4)

$$\text{So, } \frac{W_{AB}}{q_0} = (V_B - B_A) = Ed \qquad ...(3.5)$$

Eqn. (3.5) shows that the relation between the potential difference (pd) and the field strength for simple cases. Form Eqn. (3.5), we get another MKS Unit for electric field as Volts/meter. But this unit is identical with Newton/Coulomb, Fig3.3 could be caused to illustrate the act of lifting stone from A to B under the action of earth's gravitational field. This brings comparison between electrical field and gravitational field.

When the field is not uniform, and when the path of movement from A to B is not straight the work done can be computed over infinitesimally small line segments of the path dl and the total work done is obtained by integrating over the path length AB.

$$\text{Thus } W_{AB} = \int_A^B \vec{F}.d\vec{l} = -q_o \int_A^B \vec{E}.d\vec{l} \qquad ...(3.6)$$

This integral is called a line integral we have substituted - q_o in the place of F (otherwise the test charge q_o picks up acceleration) Thus

$$V_B - V_A = \int_A^B \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.\vec{dl} \qquad ...(3.7)$$

If A is taken to be a point at infinity where V_A is zero then

$$V = V_B = \int_{\infty}^{B} \vec{E}.\vec{dl} \qquad ...(3.8)$$

Eqns. (3.7) (3.8) allow us to calculate the potential difference between any two points if E is known at various points in the field.

3.6 POTENTIAL DUE TO A POINTCHARGE

Since the electric field due to a point charge q. is

$$E = \frac{1}{4\pi\varepsilon_o} \frac{q}{r^2}$$

and since a movement from A to B with increase in l (Fig 3.4) means decrease in r (r is measured from q as origin).

$dl = -dr$. Therefore.

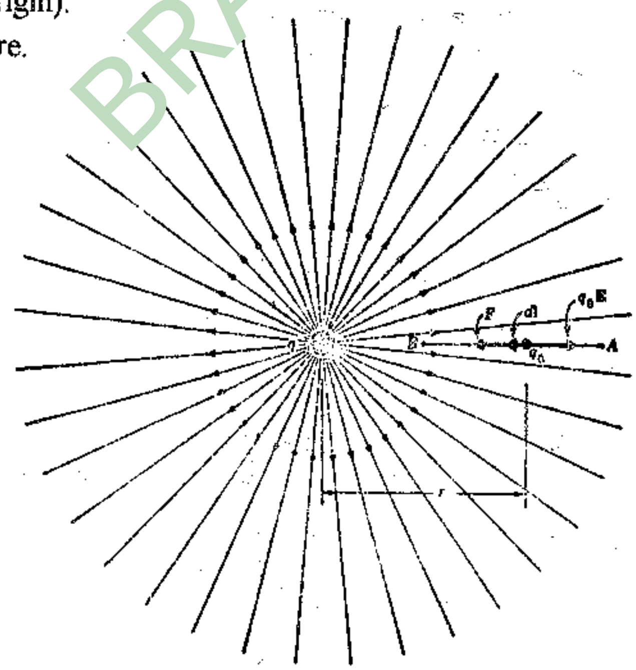


Fig. 3.4 Potential due to a point charge

$$-\int_A^B \vec{E}.\vec{dl} = -\int_{r_A}^{r_B} \vec{E}.\vec{dr}$$

We get $$V_B - V_A = -\frac{q}{4\pi\varepsilon_o} \int_{r_A}^{r_B} \frac{d}{r^2} \qquad ...(3.10)$$

$$(V_B - V_A) = \frac{Q}{4\pi\varepsilon_o} \left(\frac{1}{r_B} - \frac{1}{r_A} \right)_{ra \to \infty} \qquad ...(3.11)$$

If A is chosen as a point at infinity (Where r_A) $V_A = 0$

The $$V = \frac{1}{4\pi\varepsilon_o} \frac{a}{r} \qquad ...(3.12)$$

If there are group of point charges, the potential is obtained by calculating the potential due to each individual charge ignoring the presence of other charges and compounding the sum. By this, the mutual repulsion (or attraction) among charges is neglected.

Then $$V = \Sigma_n V_n = \frac{1}{4\pi\varepsilon_o} \Sigma_n \frac{q_n}{r_n} \qquad ...(3.13)$$

q_n, and r_n are the values of charge and distance of it from the point under consideration. The sum used to calculate V is the algebraic sum and vectorial sum. This is the computational advantage of potential over electric field strength.

Let the charge distribution be continuous. Then dq is the differential increase in charge and r is its distance from the point where V is calculated and dV is the potential it establishes at that point then.

$$V = \int d\upsilon = \frac{1}{4\pi\varepsilon_o} \int \frac{dq}{r} \qquad ...(3.14)$$

if P is charge density in the medium, then

dq – p dv (dv is the volume element).

Then $$V = \frac{1}{4\pi\varepsilon_o} \int \frac{\rho dv}{r} \qquad ...(3.15)$$

$$...(3.15a)$$

In CGS units $$V = \int \frac{pdv}{r}$$

Just as the electric intensity at a point in an electric field is the force per unit charge at that point, similarly the potential at a point is the potential energy per unit charge. Just as the energy is scalar quantity, potential is also a scalar quantity.

3.7 VECTOR FORM OF POTENTIAL

If the charge is supposed to be situated at the origin of the co-ordinate axes, we have

$$dV = -(E_x dx + E_y dy + E_z dz) \quad \text{... (3.16)}$$

Where Ex, Ey, Ez are the components of electric field intensity E along the x,y,x axex respectively.

$$\text{So } Ex = -\frac{dV}{dx} \text{ ; } Ey = -\frac{dV}{dy} \text{ : } Ez = -\frac{dV}{dz}$$

But vectorially, E is written as

$$E = iE_z + jE_y + kEz$$

$$= -i \frac{dV}{dx} - j \frac{dV}{dy} - k \frac{dV}{dz} = -\text{ gradient V} \quad \text{... (3.17)}$$

$$E = -\text{grad V} = -\nabla V \quad \text{...(3.18)}$$

'∇' here is called the differential operator and is equal to

$$\left(i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz} \right)$$

The Electron Volt

If a charge moves in an electric field work is done and is converted into kinetic energy since the charge of an electron = 4.77 x 10^{-10} esu. Or (1.602 x 10^{-19} Coul) the work done is in moving it through a potential difference (pd) of one Volt (-1/300esu) or the K.E.acquired by it is $\frac{4.77 \times 10^{-10}}{300} = 1.6 \times 10^{-19}$ Joules. This amount of energy is called an electron. Volt(ev).This unit of energy is used in the problems pertaining to nuclear physics, In higher energy acceleration like cyclotron, betatorn, the energy unit adopted is Me V, ie., million electron volts.

3.8 ELECTRIC DIPOLE

A system consisting of two equal charges of opposite signs separated by a fixed distance is called an electric dipole.

Let a dielectric, say a glass slab, be placed in between two equal and oppositely charged metallic plates. The atoms in the dielectric are subjected to an electrostatic field. Under such electrostatic field, the electrons (in the dielectric) are displaced towards the positively charged plate. The protons (or positive charges in the dielectric)are displaced towards the negatively charged plate. The amount of shift (or displacement) of these charges is proportional to the field strength between the plates. The strained atom then constitutes an electric dipole. The centers of gravity of the positive charges and negative charges, produced on account of the strain explained above, may coincide or may not. If they coincide the molecule is said to be non-polar: if the points are at a short distance apart, then the molecule is called a polar molecule, Water is a polar molecule while benzene is non -- polar. The product of charge and distance between the positive and negative charges is called the dipole moment.

3.9 ELECTRIC FIELD INTENSITY AND POTENTIAL DUE TO A DIPOLE

Consider a dipole whose charges are + q and – q units separated by a distance AB = 2l (l is of atomic dimensions) (fig. 3.5) The potential V at a point P, at a distance r from O, is given by

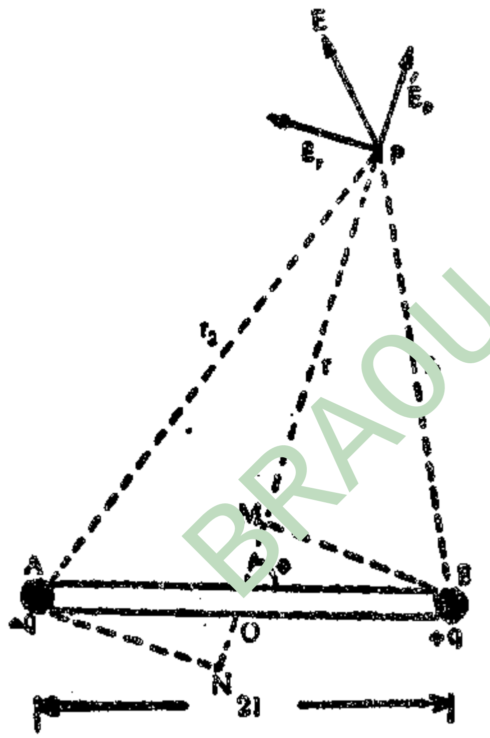


Fig 3.5 Electric dipole

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) \quad \text{... (3.19)}$$

$$\frac{1}{4\pi\varepsilon o}\left(\frac{q}{r - l\cos\theta} - \frac{q}{r+\cos\theta}\right) \quad \text{...(3.20)}$$

$$\frac{2ql\cos\theta}{4\pi\varepsilon_o(r^2-l^2\cos^2\theta)} = \frac{2ql\cos\theta}{4\pi\varepsilon_o r^2} \quad \text{...(} \because l << r)$$

The quantity = 2ql is, according to the definition, is electric dipole moment and is designated as P.

$$\therefore V = \frac{P\cos\theta}{4\pi\varepsilon_o r^2} \quad \text{... (3.21)}$$

the electrical intensity E at P in the direction OP due to the dipole is

$$Er = -\frac{dV}{Dr} = -\frac{2P\cos\theta}{4\pi\varepsilon o r^2} \quad \text{...(3.22)}$$

And E_P, in the direction perpendicular to OP is

$$E_p = -\frac{1}{r}\frac{dv}{d\theta} = -\frac{1}{r}\frac{d}{d\theta}\left[\frac{P\cos\theta}{4\pi\varepsilon_o r^2}\right] = \frac{P\sin\theta}{4\pi\varepsilon_o r^3} \quad \text{...(3.23)}$$

The resultant field intensity is E given by

$$|E| \quad \sqrt{E^2_r + E^2_p} = \frac{1}{4\pi\varepsilon_o} \sqrt{\left(\frac{2P\cos\theta^2}{r^3}\right) + \left(\frac{P\sin\theta^2}{r^3}\right)}$$

$$E = \frac{1}{4\pi\varepsilon_o r^3} P\sqrt{1 + 3\cos^2\theta}$$

$$= \left(\frac{1}{4\pi\varepsilon_o r^3}\sqrt{1 + 3\text{Cos}^2\theta}\right)P \quad \text{...(3.24)}$$

$$\text{if } \theta = O, V = \frac{P}{4\pi\varepsilon_o r^3} \text{ and } E = \frac{P}{2\pi\varepsilon_o r^3} \quad \text{... (3.25)}$$

$$\text{if } \theta = \pi/2, V = O, Er = O \text{ and } E = (P/4\pi\varepsilon_o r^2) \quad \text{... (3.26)}$$

Thus the right bisector of AB is the equipotential surface.

outer surface of the sphere, which would therefore be smooth and by applying a few hundred volts between A and earth. It can be raised to a few million volts.
A similar arrangement, with A connected to the negative end of the battery, of which the positive end is connected to earth, will enable the sphere to be charged negatively.

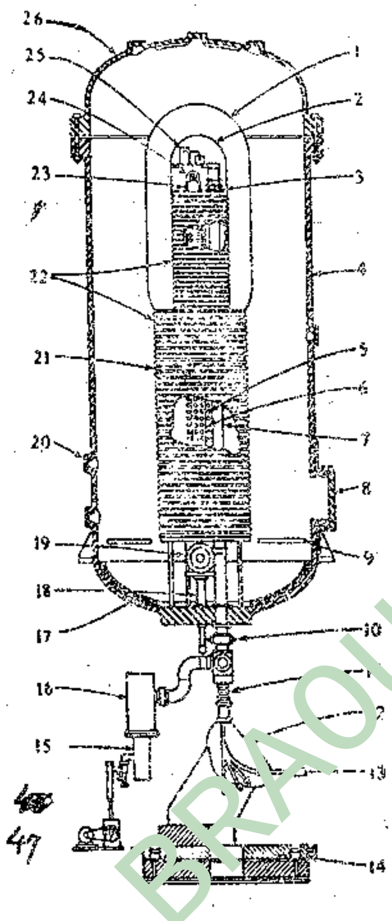


Fig 3.8 Van De Graff Generator

1. Equipotential shied 2. High voltage terminal 3. Positive iron source 4. Steel pressure tank 5. Field control rods. 6. Insulating belt 7. Positive ion accelerating Tube 8.Manhole 9. Movable platform 10. Main value 11. Flexible coupling 12. Analyzing magnet 13. beam axis 14. Adjustable magnet base 15. Pumping system 16. Dry ice trap 17. Lead shielding 18. Belt Tension Adjustment 19. 1,8000 run motors 20. Windows. 21. Insulating column 22. Equipotential planes 23. Charge collector 24. Built – in Kw power supply 25. Electronic circuits 26. Removable tanks over.

Air surrounding the charged sphere is unable to with stand high potential; leakage starts when the air is at ordinary pressure. In order that there may not be any leakage , the generator is surrounded by a big metallic enclosure 9which is earthed), is provided with two taps to allow air at high pressure 3.5 Kg to 7.0 Kg. Per sq. cm to be introduced into the space between the sphere and the belt and metallic tank. In a Van de Graff generator, constructed in 1947 at the Carnegie Institute, Washington the metallic tank has average diameter Institute. Washington the metallic tank has average diameter of about 12 meters and this generator generates potential of 5 Mev

An electrostatic generator of Van de Graff used in MIT, Massachusetts (USA) generating (9 Me.. protons is shown in Fig 3.8 In India at present , We are having Van de Graff generator in Saha Institute of Physics, Calcutta, and Andhra University, Waltair and BARC, Bombay.

Example – 1:

Uniform electric fields obtained by applying a potential of 500 V. to metal plates which are 0.01 meters apart. Find the force on a positive charge of 5×10^{-9} Coul. When placed in this field.

Solution:

$$Q = 5 \times 10^{-9} \text{ Coul.} \quad E = \frac{500}{0.015} \text{ N/Coul.}$$

$$\therefore \text{ Force } = \frac{500}{0.015} \times 5 \times 10^{-9} = 16.67 \times 10^{-5} \text{ N.}$$

Example – 2:

A soap buddle has a radius of 1.5×10^{-2} m. To what potential should it be raised so that the pressure inside and outside is the same. Surface tension of soap buddle is 24×10^{-3} N./m

Solution

Pressure acting inward due to surface tension $= \frac{4T}{r}$

Mechanical force acting outward $= \frac{\sigma}{2\varepsilon_0}$

If q is the total charge on the sphere, its potential is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$\text{So, } \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{r}$$

$$\frac{1}{2\pi\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{4T}{r}$$

$$q = \frac{2 \times 8.85 \times 10^{-12} \times 16\pi^2 (1.5 \times 10^{-2})^4 \times 4 \times 24 \times 10^{-3}}{(1.5 \times 10^{-2})}$$

$= 3.009 \times 10^{-3}$ Coul.

3.10 TORQUE EXPERIENCED BY A DIPOLE

When an electric dipole (e.g. a hydrogen atom) is acted on by a uniform electric field of intensity E, it experiences a couple or torque given by tan T = PE sin θ.

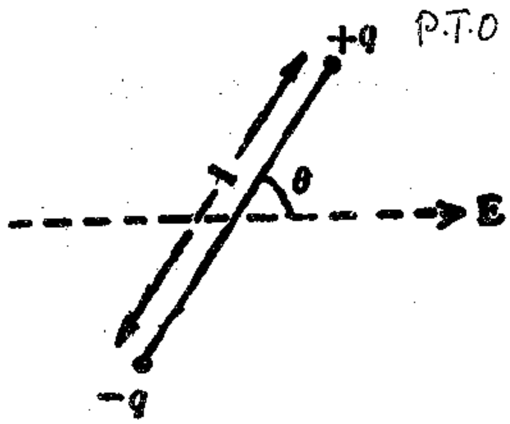


Fig 3.6 Torque due to a dipole

Where θ is the angle between the dipole axis and E [See Fig. 3.6)]
The potential energy of the dipole is given by
P. E = - PE Cos θ = - q E cos θ

In vector notation, couple $= \overline{P} \times \overline{E}$ ...(3.27)

So far, we assumed that the electric field will not get distorted in the presence of unchanged conductor. In actual practice. There is distortion of the field produced because the amount of induced charge is not uniform. The distribution of induced charge on an uncharged conductor may readily be found by the method of electrical images devised by Lord Kelvin. This method also enables us to fined the intensity on and potential of , a conductor when placed in an electric field.

In attempting to find the electrical image, the conditions to be satisfied are :

(i) The potential of the conductor must remain the same.

(ii) The potential at infinitely distant point must remain the same.

(iii) The total normal induction over any closed surface in the original field must remain the same.

Check your progress – II

1. Torque experienced by a dipole is given by the expression........
2. what is the Similarity between intensity at a point in an electric field & Potential at a point. They are both......... quantities

Note: a. Space is given below for your answer
b. Compare your answers with those given at the end of the unit.

....
....
....

3.11 THE ELECTROSTATIC GENERATOR

The electrostatic generator was conceived by Lord Kelvin in 1890 and put into useful practice in modern form by Van de Graff in 1931. It is a device to produce electric potential difference of a few million volts. Its chief application in physics is to use the potential difference to accelerate charged particles high energies. Beams of energetic particles made in this way can be used in many different 'atom-smashing' experiments.

The essential parts of such a generator are known in fig 3.7 it consists of a hollow large metal sphere S of about 4.5 meters diameter supported on an insulated column. A belt of some insulting material, such as silk of rubber, is mounted on the two pulleys P_1 (the idling pulley at the center of the sphere) and P_2 connected

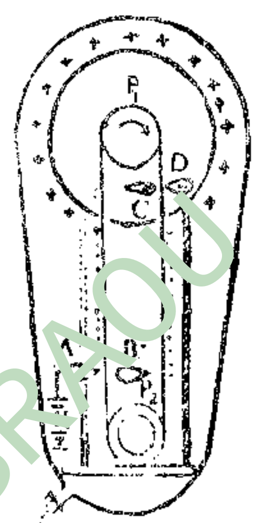


fig 3.7 van de Graff Generator

to the electric motor. The belt is made to run between two columns. A, B, C and D are four pointed conductors with their pointed ends as shown in the diagram. A is given slight positive charge by connecting it to the positive end of a battery the negative end of the battery is being connected to the earth. The pulley P_2 is also connected to the earth. D is connected to S when the pulley runs in the direction shown is fig 3.7 the positive charge on A is discharged on to the ascending belt by the action of the pointed end (Corona discharge).

The ascending positive charge is discharged at C making it positively charged. This, in turn, induces a negative charge on D and positive charge on the sphere. The negative charges on D is sprayed on to the belt (while descending) and is discharged to B, thus enhancing positive charge on A. The negative charge on the belt is discharged to B and the earth through P_2 The sphere gains more and more of the charge, which distributes itself on the

$$\text{Potential} = V = \frac{q}{4\pi\varepsilon_0 r}$$

$$\frac{3.009 \times 10^{-8}}{4\pi \times 8.85 \times 10^{-12} \times 1.5 \times 10^{-2}}$$

1.803756×10^{4} Volts.

18037.56 volts

3.12 SUMMARY

Electrical potential at a point is the amount of work done against the field, in carrying a unit positive charge from infinity to the point. Electrical potential is similar to gravitational potential. The electric potential is a scalar quantity.

The electric intensity at a point in an electric field is the force per unit charge at that point. A pair of equal and opposite point charges separated by a distance is called a dipole.

Check your progress: Answers

I. 1. $V_B - V_A = W_{AB}/q$

II. 2. Tan λ =P E sinθ.

3. Just as the electric intensity at a point in an electric field is the force /unit charge at that point. Similarly the Potential at appoint is the PE/unit charge. Just as the energy is scalar quantity. Potential is also a scalar quantity.

1.Check your Progress: Answers

Positive potential is the potential near an isolated positive charge. It is positive because the work done to push the positive charge from infinity to the present position.

2.Check your progress: Answers

Electron volt is the amount of work done in moving it through a potential difference of one volt.

3.13 SAMPLE EXAMINATION QUESTIONS

1.Answer the following questions in detail

1 Show that the potential difference between two points is the line integral of the electric field between the two points.

2 Find the electric potential at a point, at a distance 'r', from a point charge q.

3 Show the $E = -\nabla v$

4 What is an electic dipole? Calculate the electric potential at a point due to a dipole. There by find the value of the electric field. What is the value of the field a point. (i) on the axis of the dipole (ii) on the normal to the axis

II SOLVE THE FOLLOWING PROBLEMS

1. A charge of 2.5×10^{-7} coul is located between two parallel plates separated by a distance of 0.04 meters What voltage must be applied to the plated which will exert a force of 10^{-3} N on the charge?

(Ans: 160 Volt)

2. Two metal spheres of 3 cm radii carry charges of + 30 and – 90 esu and are 200 cm apart. Find the potential at a point (i) midway in between them and (ii) the potential of each sphere.

(Ans 0.6 esu; 9.7 and 30 esu)

3. Find the potential through which an electron must fall to attain the speed of light (according to Newtonian mechanics) (Hing:Use $K.E = 1/2mV^2 = eV$)

(Ans: 2.5×10^5 Volt)

4. (a) Calculate the potential at a point A which is 30 cm from a charge of $-2\ \mu C$.(b) What is the potential energy if a + 4nC charge is placed at A?

(Hint : $1\ \mu C = 10^{-6}$ coul ; $1nc = 10^{-9}$ Coul).

(Ans : (a) -6×10^{-4} volts
(b) -24×10^{-5} Joules)

UNIT–4: CAPACITANCE

Contents

4.1 OBJECTIVES

This unit discusses the concept of capacitance and its relation to storage of electrical charges and voltage. To help you understand the concepts, this unit explains the conditions required for the storage of energy in a charged condenser.

After going through this unit you will be able to (1) calculate the effective capacitance in the series and parallel connections of capacitances (2) the capacitance of isolated sphere. and(3) concentric spheres and cylindrical condensers.

4.2 INTRODUCTION

In this Unit we will discuss the concept of capacitance and its relation to storage of electrical charges and voltage. We will also study the effective capacitance when they are connected in series and parallel.

4.3 CAPACITANCE

The potential of charged conducting sphere V has been shown to be equal to

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{...(4.1)}$$

Where q is the charge on the sphere of radius 'r'. The conducting sphere has been assumed to be completely isolated from the surroundings.

What happens when another conducting sphere of radius ''s charge to '-q' Coulombs is brought near the above sphere? Let us discuss this aspect here in detail. The potential due to the second conducting sphere is

$$V^{I} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{..(4.2)}$$

We assume that the effect of second sphere, in the surroundings of the first one, does not affect the spherical symmetry of the charge distribution. This is true as long as the separation between the two charged spheres is larger when compared to their radii. But when they come nearer, the spherical symmetry is affected. A positive charge when brought near the isolated object serves to raise the potential of that object. A negative charge serves to lower it. Thus the potential of the positively charged sphere will be lowered by the presence of the negatively charged sphere. Similarly the potential of the negatively charged sphere will be raised by the presence of the positively charges sphere. Thus although the charges on the conducting spheres remain the same, the potential difference between them will decrease as the distance between them is reduced. The ratio of the charge to the potential difference to which it is raised is referred to as capacitance or capacity of the conductor.

$$Q/V = C \qquad \text{...(4.3)}$$

If 'V^{I}' is the potential at a distance d^{I} and the capacitance is C and if V^{II} is the potential at d^{II} and capacitance C^{II} then

$$q = C^{I}V^{I} = C^{II}V^{II} \qquad \text{...(4.4)}$$

The capacity or capacitance of a conductor is defined as the charge required to raise the potential of the conductor by unity.

In MKS system the unit of capacitance is Coulombs/Volt. A special unit, the FARAD is used to represent it.

It is named in honor of Michael Faraday, who among other contributors developed the concept of capacitance.

Thus 1 Farad = 1 Coul./Volt

$$1\ \text{Farad} = \frac{3 \times 10^{9}\ \text{e.s units of charge}}{1/300\ \text{e.s. units of potential}}$$

$= 9 \times 10^{11}$ e.s units of capacitance.

Since the Farad is too big a unit, a microfarad 1 μF = 10^{-6} / Farads) and a Pico farad or micro farad (1 μμF = 1 pF = 10^{-12} Farads) are used in practic

The e.s *units of capacitance* is however defined as te capacity of a body whose potential is raised to 1 esu or by 1 e.s unit of charge.

In the medium of dielectric constant K, the potential V of a charge body becomes V^1/K So the capacitance. Will, then be KC^1. (V^1 and C^1 refer to vacuum).

An analogy can be made between a capacitor carying charge q and a rigid container of volume V containing μ moles of ideal gas

According to ideal gas law $\mu = \left(\frac{V}{RT}\right) P$

For a capacitor q = (C) V ...(4.5)

Thus the capacitance is analogous to the volume of the container at a particular temperature. Just as any amount of charge can be put on capacitor, so any amount of gas can be filled in a container (upto certain limits). If the charge exceeds a critical value, breakdown of capacitance occurs. If the mass of gas exceeds a critical limit, rupture of container walls occurs.

If two conductors (of equal and opposite charges) of any shape are brought near a distance apart, that assembly of conductors is called a apacitor condenser. The conductors are called the plates. The capacitance of a caacitor depends on (a) the geometry of each plate, (b) their spatial relationship with resect to each other and (c) the medium in which they are immersed. The capacitors are enerally represented by the symbol, "—| |—"

The capacitors are very useful devices, and are of greate interest to physicists and engineers. For example, (1) the capacitors are used to defle electron beams. (2) They are used to store electrical energy since they can confine stong electric fields to small volumes. (3) There could not have been a progress in the fielc of electronics and modern communication engineering without the capacitors. They ae used in (a) filtering the electrical fluctuations. (b) transmitting pulsed signals and (c) generating or detecting radio frequency waves and so many other functions.

4.4 ENERGY STORED IN THE FIELD OF A CHARGEICAPACITOR

The energy of a charged capacitor is equal to the work dne in charging it. Suppose the potential difference between the plates at any instant of time is V, the work done (dW) in bringing a small charge, dq to the capacitor, when it potential is V, is given by dW = V. dq

The total work done i charging it with a charge q is

$$W = \int Vdq ergs \qquad \ldots(4.6)$$

This can not be integrted directly as V is not a constant but is proportional to q. (i.e.) V = q/C , hence

$$W = \int_0^q \frac{q}{C} dq = \frac{q}{2C} = 1/2CV^2 \qquad \ldots(4.7)$$

Eqn. (4.6) represent the energy stored in a charged condenser, when q is in Coulombs, C. in farads and V in volts, W will be in Joules.

This energy is store up in the dielectric charging a condenser at constant potential. If the distance beveen the plates is decreased, the capacity increases and the charge stores will also increse. This increase in charge is supplied by the battery. This process is called charging at constant potential. If V is the voltage of the battery and ΔC is the increase in capacity he work done in charging.

$$W = \int Vdq = \int V^2 dC \text{ .ules}$$

$$W = CV^2 \text{ Joules}$$

4.5 COMBINATION OF CAPACITORS

To find the single cpacitance C equivalent to three capacitors connected in parallel, we proceed in the following way. Equivalent capacitor means that the parallel combination and the single capacitor are indistinguishable by their electrical measurements, when connecd to terminals 'a' and 'b'

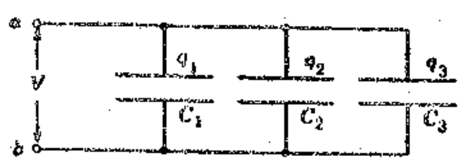


Fig. 4.1Three capacitors in parallel

The potential differece across each capacitor in Fig. 4.1 will be the same. This follows because all of te upper plates are connected together to terminal 'a' while all the lower plates are connected to 'b'. Applying the relation q = CV to each capacitor.

$$q_1 = C_1V; \; q_2 = C_2V \text{ ad } q_3 = C_3V \qquad \ldots(4.8)$$

The total charge on the combination is

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)\ V$$

The equivalent capacitance C is = $\frac{q}{V} = (C_1 + C_2 + C_3)$

Thus $C_{eff} = C_1 + C_2 + C_3$. ...(4.9)

This result is extendable to any number of capacitors connected in parallel.

4.5.2 Capacitor in Series

Fig. 4.2 shows three capacitors connected in series.

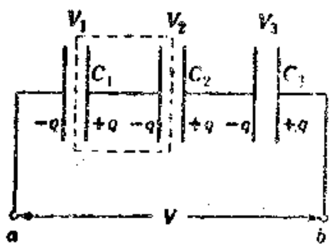


Fig. 4.2 Three capacitors connected in series.

To find the equivalent capacitance C, we proceed in the following way. In the connection shown fig 4.2, the charge q on each plate must be the same. This is because of the reason that the net charge present initially on these plates is zero. Connecting the plates to a battery will only produce a charge separation keeping the net charge on these plates zero. Assuming that neither C_1 nor C_2 'sparks over', no charge can enter from outside or leave the region enclosed by dashed line.

Since $q = CV$,

$$V_1 = \frac{q}{C^1};\ V_2 = \frac{q}{C^2} \text{ and } V_3 = \frac{q}{C^3} \qquad (4.10)$$

The potential difference for the combination series is

$$V = V_1 + V_2 + V_3$$

$$= q\ (1/C_1 + 1/C_2 + 1/C_3)$$

But $V = q/C$

The equivalent capacitance

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The equivalent capacitance is always less than the smallest capacitance in the chain. The result can be extended to any number of capacitors connected in series on similar lines.

4.6. CAPACITANCE OF A CONDUCTOR

4.6.1 An Isolated Sphere

The capacitance of sphere of radius r

$$= \frac{q}{q/4\pi\varepsilon_0 r} = 4\pi\varepsilon_0 r \text{ Coul./Volt}$$

In esu, if the sphere has a radius of 1cm., the capacity is 1 esu. Only a spherical conductor of radius 9 x 10^{11} cms will have a capacitance of 1 Farad. Its capacitance will be 1 μ F if its radius is 9 x 10^5 cms. A sphere of 0.9 cm. Radius has a capacity of 1 μμ F or 1 pF. The radius of earth is 6.4 x 10^8 cm. So the capacitance of earth ≈ 0.0007 Farads. If the medium is of dielectric constant, k, the potential of the sphere becomes $(q/4\pi\varepsilon_0 kr)$ and the capacitance = $4\pi\varepsilon_0 kr$. Thus the capacitance of a spherical conductor is more than surrounded by medium (of dielectric constant k) than when it is surrounded by air

Any arrangement by which the capacity of conductor is increased is called an electrical condenser or a capacitor. The condenser is a device used to store quantities of electricity just as a reservoir is a container for storing water

4.6.2. Two concentric Spheres

Consider two spheres arrange concentrically one into the other as shown in fig 4.3.

Let us discuss the capacitance of a condenser when the outer sphere is earthed:

Let **a** and **b** the radii of the two spheres A and B, B being earthed.

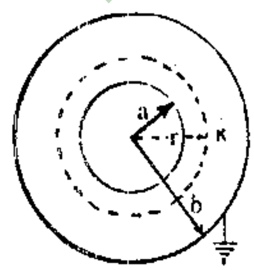


Fig 4.3 Two concentric spheres

The potential difference, V between A and B is

$$V = \int_b^a - \frac{q\,dr}{4\pi\varepsilon_0 k r^2} = \frac{q}{4\pi\varepsilon_0 k}\left[\frac{1}{a} - \frac{1}{b}\right] \quad \ldots(4.11)$$

$$\therefore C = q/V = \frac{q}{(q/4\pi\varepsilon_0)k\,(1/a - 1/b)} = \frac{4\pi\varepsilon_0 kab}{b-a} \text{ Farads} \quad \ldots(4.12)$$

If the two spheres are surrounded by air, $k = 1$

$$\varepsilon_0 = 8.85 \times 10^{12} \text{ Farad/m}$$

$$\text{In CGS units. } C = \frac{ab}{(b - a)} \text{ esu} \qquad \ldots(4.13)$$

By surrounding A with an earthed conductor, the capacity of A is increased. In CGS units Cac be written as

$$C = \frac{a}{(1 - a/b)} \text{ esu} \qquad \ldots(4.14)$$

If $b \rightarrow \infty$, then $C = a$ = radius of the inner sphere. Thus the charged sphere can be regarded as a condenser in which outer coating has been removed to an infinite distance.

In fact every charged conductor possesses some capacity. Ordinarily a wire has too little surface area to have much capacity. However, the wires of long telephone and power lines have sufficient capacity to act as condensers.

By bringing a charged conductor in the neighborhood of an earthed conductor, the potential of the former is lowered and therefore its capacity or capacitance increases. In order to maintain the potential on the charged conductor (near the earthed conductor), we must give extra charge to it.

A useful property of a condenser is that when it is placed in a direct (DC) circuit, it does not allow steady current of flow through it. A condenser used for such purpose is called a blocking condenser. Its behavior in an alternating current is altogether different.

4.6.3 Capacity of Cylindrical Condenser - Submarine Cable

A metal cylinder is placed coaxially inside a hollow metallic cylinder of large radius. The space between the cylinders is filled with a dielectric of dielectric constant K. Then we get a cylindrical condenser. A submarine cable is a practical example of such condenser. In a submarine cable, the inner conductor is a copper cable and the seawater is the outer-earthed cylinder. The insulating sheath (of polystyrene) forms the dielectric. Let **a** and **b** be the radii of the inner and outer cylinders (or the inner and outer edges of the dielectric) respectively. (K is the dielectric constant).

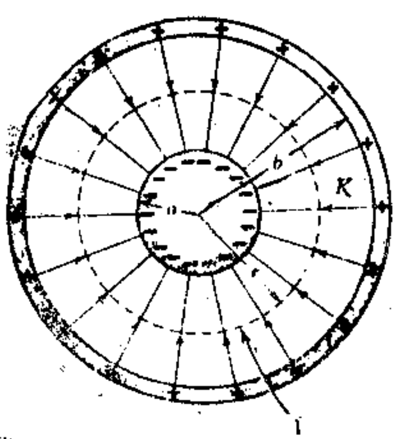
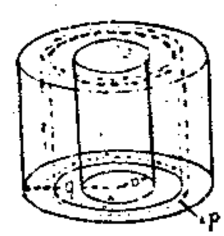


Fig 4.4 Cylindrical condenser



1. *Gaussian surface*

If the inner cylinder is charged to q Coulomb per unit length, then E, the electric intensity at a distance r from its axis is

$$= \frac{2q}{4\pi\varepsilon_0 kr} \quad \text{(Gauss theorem)} \qquad ...(4.15)$$

∴ The potential difference between outer and inners cylinder is

$$V = -\int_b^a \frac{1}{4\pi\varepsilon_0 k} \cdot \frac{2q}{r}\, dr \qquad ...(4.16)$$

$$= \frac{q}{2\pi\varepsilon_0 k} \text{ Loge } (b/a) \qquad ...(4.17)$$

Therefore, capacity per unit length, $C = \frac{q}{V}$

$$C = \frac{q}{\frac{q}{2\pi\varepsilon_0 k} \text{Log}_e\,(b/a)} = \frac{2\pi\varepsilon_0 k}{2.303\log_{10}\,(b/a)} \qquad ...(4.18)$$

Capacity for 1 cms of the cylindrical condenser,

$$C = \frac{2\pi\varepsilon_0 k}{\text{Loge}\,(b/a)} = \frac{2\pi\varepsilon_0 k}{4.606\log_{10}\,(b/a)} \qquad ...(4.19)$$

C will be in Farads when 1, b and a are in meters.

In CGS system $C = \frac{kl}{4.606\log_e\,(b/a)}$ ...(4.20)

Eqns. (4.19) and (4.20) are used in the expression for the determination of capacity and hence the potential difference in proportional counter.

4.6.4 Cylindrical Sliding Condenser

It is a variable condenser consisting of two metallic coaxial hollow cylinders of the same diameter separated by a small air gap.

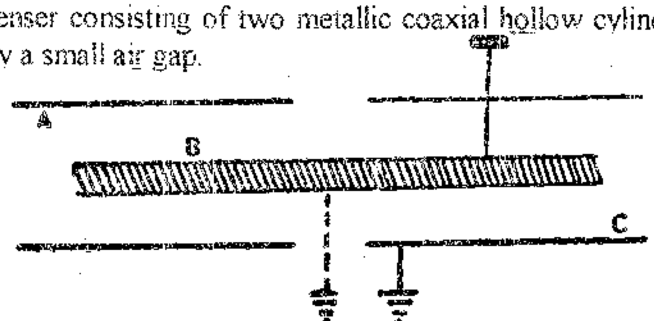


Fig 4.5 Cylindrical Sliding Condenser

Between them. One of these cylinders is earthed. C carriesan inner metallic cylinder B, which is also coaxial with the outer. It can be moved axiallyin and out of A by means of a micrometer screw fixed on C. This screw allows the ength of B inside A to be accurately measured.

The cylinder A is usually surrounded by another eathed cylinder to prevent the leakage of charge to outside bodies. B and C are first earthe. A is insulted and is given a charge. When B is moved into a by a distance l, the change ı capacity of A is given by

$$\frac{2\pi\varepsilon_0 kl}{2.303 \log_{10}(b/a)}$$ Where a and b are the radii of A and B respectively.

Thus if its capacity for a particular setting of ıcrometer is measured by comparison with a standard condenser, its charge in capacityis found with the help of the above expression. Then this condenser can be used for meauring the capacitance of any other unknown condenser.

Example –1:

The parallel plates of an air-filled capacitor are everywhere 1.5 mm apart. What must be the plate area if the capacitance were to be 1.5 farads

Solution:

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$

$$\text{or } A = \frac{Cd}{e_0} = \frac{1.5\times10^{-3}\ \text{m}\times1.5\text{Farad}}{8.9\times10^{-12}\ \text{Coul/J.m}^2}$$

$$A = 2.532\times10^{8}\text{m}^2$$

Example-2:

A capacitor C_1 is charged to a potential difference V_0. The charging battery is then removed and the capacitor is connected. As showr in figure to an uncharged capacitor C_2. Find the potential difference V across the combiation.

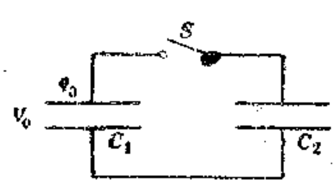


Fig 4.6

Solution:

This original charge q_0 is ow shared by capacitors C_1 and C_2. so, $q=q_1+q_2$

But = $q = CV$. So, $C_1V_1 = C_1V + C_2V$

or $V = \frac{C_1V_0}{C_1 + C_2}$

This suggests how to measure th unknown capacitance C_2 in terms of C_1

4.7 SUMMARY

The ratio of the charg Q to the potential difference V to which a conductor is raised is known as capacity C of conductor or $Q = CV$.

The unit of capacitance i Farad. The effective capacitance in series and parallel combinations of capacitors is gvn by

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \& \quad C_{eff} = C_1 + C_2 + C_3 \text{ respectively.}$$

The energy stored in a condensr is given by $W = ½ CV^2$ i.e., by equation (4.7)

4.8 SAMPLE EXAMINATION QUESTIONS

I. Answer the following qestions in detail.

1. Derive an expressio for the energy stored in a charge condenser.
2. Calculate the capacity of a condenser consisting to two concentric spheres of radii r and R separated by (1) air (2) a dielectric specific inductive capacity K
3. Find the expression for the capacity of a cylindrical condenser.

II. Answer the following quesions briefly.

1. Explain the concept of potential surface.
2. Explain a term similar to electrical dipole in magnetism.
3. ON what principle does Vande Graaff generator works.
4. What is an electrical image? Explain.

3. Energy stored in the field of a charged condenser; and also describe the construction, working and the uses of various types of condensers.
4. The effect of dielectric media on the capacity of a condenser.
5. The dielectric behavior from the atomic view point.

5.2 INTRODUCTION

In this unit we will evaluate the capacity of a parallel plate condenser and the dielectric Constant, and discuss the influence of dielectric media on the capacity of a condensers. Also study about the amount of energy stored in a condenser.
Study dielectricity from atomic point of view. Know about variable. Fixed and Guard –ring condensers, Also study about the amount of energy stored in a Condenser.

5.3 PARALLEL PLATE CONDENSER

The concept of capacitance and potential were discussed in previous units. We shall now discuss the capacity of a parallel plate condenser.

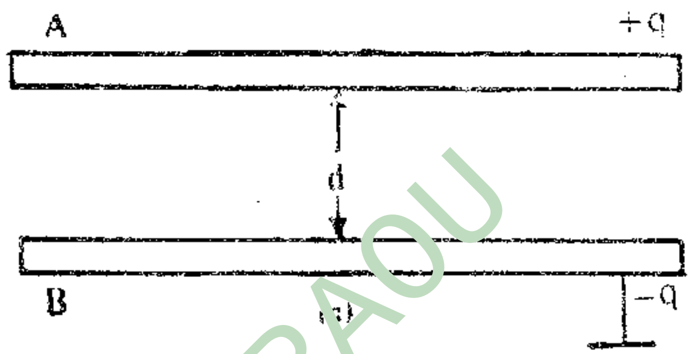


Fig. 5.1 (a) Parallel plate condenser

A parallel plate condenser consists of two metal plates usually in rectangular form separated by a Dielectric. If the plates A and B, shown in fig 5. 1. a, are further apart, the tubes of force at the end will be curved owing to lateral pressure and they will not be of constant cross sectional area nor are they equally spaced. However, as an approximation we assume that the plates are near enough so that the lines of forces are straight throughout the space A and B, E and electric field intensity between the plated is also assumed to be uniform.

The plates are of area A sq. m and given a charge of + q Coulombs. B is earthed. (earthing means that the plate is connected to earth and is maintained at zero potential. Usually earthing is indicated symbolically as shown above. d is the distance between the plates in meters and K is the dielectric constant of the medium, where in the condenser is placed.

The potential difference between A and B is V Joules /couls.

$$-V = \int_d^o \vec{E}.\,\vec{dr} = \int_d^o \frac{\sigma dr}{\varepsilon_0 K} = \frac{d\sigma}{\varepsilon_0 K} \qquad \ldots(5.1)$$

But σ is the surface density of charge = q/A

So, $$V = \frac{vd}{\varepsilon_0 A\kappa} \qquad \text{... (5.2)}$$

Hence. $$C = q/v = \frac{\varepsilon_0 A\kappa}{d} \text{ farad} \qquad \text{... (5.3)}$$

if κ= 1, $$C = \frac{\varepsilon_0 A}{d} \text{ farad}$$

in CGS system $$C = \frac{A}{4\pi d} \text{ esu}$$

To get a condenser of the same capacity (with K) as that of an air condenser, the thickness of the dielectric must be (κd)

Usually the dielectrics are often placed between the conducting plates very near to each other to permit a higher potential difference to be applied between them.

We can also derive the Eqn. 5.3 using Gauss theorem. It fig 5.1 b dotted lines show a Gaussian surface G enclosed by plane surfaces of area A of the same shape and size as the condenser plates.

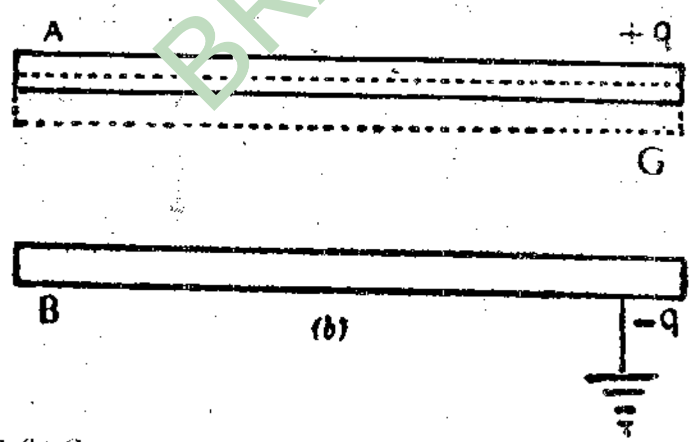


Fig 5.1 (b) Capacitance from Gauss law (Gaussian surface)

The normal induction over the sides is zero and it is also zero for the part of the surface which is inside A. The only induction which we have to consider is that leaving the face of the Gaussian surface lying between the plates. The normal induction here is EA.

Hence by Gauss' theorem

$$EA = \frac{q}{\varepsilon_0 \kappa} \text{ or } E = \frac{q}{\varepsilon_0 \kappa A} \qquad \text{...(5.4)}$$

III. Solve the following problems

1. A parallel plate condenser has circular plates of 8.0 cms radium and 1 mm separation. What quantity of charge will appear on the plated if a potential difference of 100 V is applied?
2. Find he capacity of a condenser consisting of sphere and a concentric spherical shell of radii 9cm and 10 cm respectively separated by air.

Find the potential of the sphere if it is given a charged of 13.3×10^{-9} Coulombs, while the outer shell (i) insulated (ii) earthed.

(Ans: 3×10^{-8} Coul; 1.48×10^{-9} Coul. And 0.48×10^{-9})

3. Sphere of radius 10 cms is charged to a potential of 3.33×10^{-9} Coulombs. One sq .mm of gold leaf spread on the surface of the sphere is removed to a point 20cms from the sphere. What is the work done?

(Ans 5.3×10^{-10} Joules)

4. A cable has a copper core of 4 mm radius. This is surrounded by one layer of insulting material and the inner layer has a thickness of 5 mm and dielectric constant 3.5. Find the capacity of 1 cm length of the cable.

(Ans: 100CGS units)

5. Two capacitor (2.0 μ and 4.0 μ F) are connected in parallel across a 3600 V potential difference. Calculate the total stored energy in the system.

(Ans:0.27 Joules)

6. Find the equivalent capacitance of the combination given in Fig II.1 and determine the charge of each capacitor.

Fig.II-1

(i) $C_1 = 10\ \mu F$, $C_2 = 5\ \mu F$, $C_3 = 4\ \mu F$ and $V = 100$ V.

(Ans: 7.33 μ F; $q_1 = q_2 = 333\ \mu C$; $q_3 = 100\ \mu C$)

(ii) $C_1 = 5\mu F$, $C_2 = 4\ \mu F$, $C = 1\ \mu F$ and $V = 100$ V.

(Ans: 3.22 μ F; $q_1 = q_2 = 222\ \mu C$; $q_3 = 100\ \mu C$)

UNIT – 5 PARALLEL PLATE CONDENSER

Contents

5.1 OBJECTIVES

This unit explains the equations for calculating the capacity of parallel plate condensers or those connected in series.

This unit discusses the dielectric constant, the way in which dielectric media influence the capacity of the condenser and presents the atomic view of dielectric behavior.

After going through this unit you will be able to calculate and explain.

1. The capacity of parallel plate condenser and
2. The force between the plates of a charged condenser

Is called 'stator. The second set is called 'rotor', Such type of condensers are used very much in various fields of electronics whenever the variation of capacity leading to change in frequency is needed. This is used in radio communications, televisions. Etc.

5.4.2 Fixed condensers

Fixed condensers of fixed capacity are however made in the form of parallel plate condensers, consisting of very thin layers of metal coated on to the surface of mica or paper, impregnated with paraffin, as the dielectric between them. The papers will then be rolled up to occupy less space. A number of condensers can be piled up in parallel, with the alternate foils fixed to one end each, to yield a large capacity. Such fixed condensers are sometimes arranged in boxes.

Ceramic materials are now-a-days used as low loss dielectric at all frequencies.

Electrolytic condensers are generally used to obtain large capacity although the dimensions of the condensers are small.

5.4.3 Guard – Ring Standard Condensers

The capacity of parallel condenser is given by $c = \left(\frac{\varepsilon_0 A}{d}\right)$ but it has been assumed that the electric intensity between the two parallel plates remains the same throughout the area of the plates. Since the field of force and hence the intensity at the edges is not uniform the above formula is only approximate.

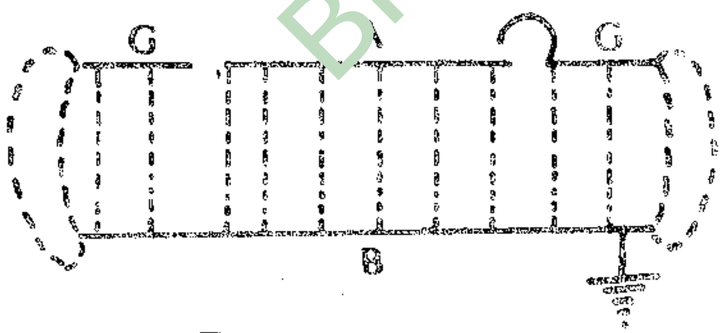


Fig 5.3 Guard ring condenser

The end effects were overcome by Lord Kelvin by using a circular plate surrounded by a ring G in the same place as the inner plate (Fig 5.3). The area of B = the area of A+ G together. To Start with. B is earthed. A and G are kept at same potential by means of conducting wire (between A and G) and the wire is then tilted over to G. Field between A and B is then uniform.

a) Effective area of a plate = A^1 = area of plate A = area of gap

Then $$C = \frac{A^1\varepsilon_0}{D} \quad \dots(5.14)$$

The distance between the plates can be altered by using micrometer fixed to B.

Example –1 :

A parallel plate condenser is made up of 15 metal strips each of 6 cms by 4 cms dimensions. They are separated by a sheet of mica of dielectric constant 6 and constant thickness 0.15 min. Calculate the capacity

Solution:

$C = [(n-1)\ \varepsilon_0 \kappa\ A./d)]$

$$= \frac{4\ \times 8.9 \times 10^{-12} (\text{Coul}^2\ / \text{N.m}^2) \times 6 \times\ (6 \times 10^{-2} \times 4 \times 10^{-2})}{1.5 \times 10^{-4}\ \text{m}}$$

$= \ 12000\ \mu\mu F$

$= 12000\ p\ F$

Example – 2:

A condenser is to have a capacity of 0.05μf. It is made of sheets of tin foil separated by sheets of mica. If there are 30 sheets of mica and the area of one side of each piece of tin foil is 10 cm^2 and if κ for mica is 6.5, find the approximate thickness of each mica sheet used.

Solution:

$$d = \frac{(n-1)\ \varepsilon_0 \kappa A}{C}$$

$$= \frac{29 \times 8.9 \times 10^{-12}\ \times 6.5\ \times 10 \times 10^{-4}}{0.5 \times 10^{-6}}$$

$$d\ = \frac{29 \times 8.9 \times 6.5}{0.05} \times\ 10^{-9} = 3.35 \times 10^{-5}\ m$$

5.5 FORCE BETWEEN PLATES OF A CHARGED CONDENSER

Neglecting the effects. the electric field intensity between two plates of a charged condenser is equivalent to the field o f an infinitely charged conducting plane. hence the expression obtained for the force on the surface of charged conductor can be applied in this case.

where q is the free charge.

But $V = -\int E.dr = Ed$ ... (5.5)

$$So,\ C = q/V = \frac{\varepsilon_0 \kappa AE}{Ed} \quad Farads \qquad ...(5.6)$$

$$In\ CGS\ system\ C = \frac{kA}{4\pi d}\ CGS\ units \qquad ...(5.7)$$

The effect of introducing an insulated uncharged conductor between the plates of an air condenser is merely to reduce the extent of the field between the plates of the condenser. The separation between the plates is shown in Fig 5.1c .There can be no field inside. C. Hence potential difference (pd)

$$= \frac{\sigma d_1}{\varepsilon_0} + \frac{\sigma(d-d_1-t)}{\varepsilon_0}$$

$$D = \frac{\sigma}{\varepsilon_0}(d-t) \qquad ...(5.8)$$

$$Capacity = \frac{\varepsilon_0 A\sigma}{\sigma(d-t)} = \frac{\varepsilon_0 A}{(d-t)} \qquad ...(5.9)$$

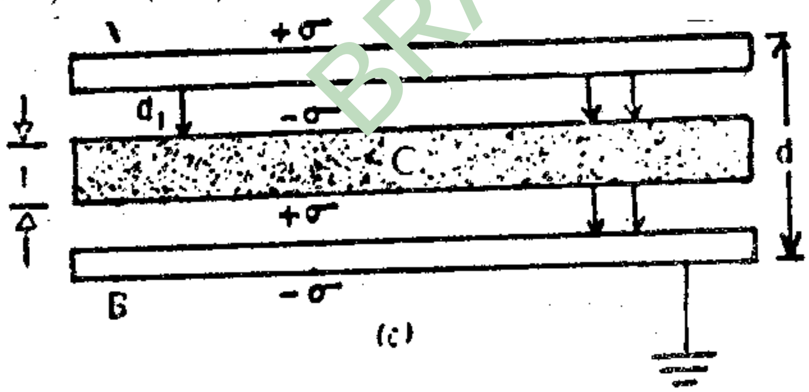


Fig 5.1 (c)Electric field in a parallel plate condenser

Charge in capacitance due to introduction of conductor

$$C = \frac{\varepsilon_0 A}{(d-t)} - \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 At}{d(d-t)} \qquad ...(5.10)$$

If the dimension of the plates and dielectric are given in cms, the capacity will be in esu which may be converted into micro farads by dividing by 9×10^5. Thus the capacity of a parallel plate condenser, with dielectric of constant K filling the space is

$$C = \frac{\frac{\kappa A}{4\pi d}\,10^{-5}}{9}\ \mu f \text{ (in CGS Units)}$$

Even if A,B are connected to either side of battery, (instead of one being, connected to battery and the other being earthed), the same expression is obtained for its capacity; the object or earthing the plates is to produce a charge equal and opposite to A and B.

If, instead of two plates, there are n similar plates at equal distances from each other, alternating plates being connected together, the capacitance of such an arrangement is given by

$$C = \frac{(n-1)\,\varepsilon_0 \kappa A}{d} \qquad \ldots(5.11)$$

$$\text{if } \kappa = 1,\ C^1 = \frac{(n-1)\,\varepsilon_0 A}{d} \qquad \ldots(5.12)$$

$$\text{In CGS system } C = \frac{(n-1)\kappa A}{4\pi d} \text{ and } C^1 = \frac{(n-1)\,A}{4\pi d} \qquad \ldots(5.13$$

Check your progress:

1. The capacity of a parallel plate condenser is given by…….
2. The capacity of a condenser how does effected by a dielectric?

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

5.4 TYPES OF CONDENSERS

5.4.1 Variable Condensers

The series of plates thus stacked to form a condenser is called variable condenser (Fig 5.2) In this, one set of plates will be fixed and

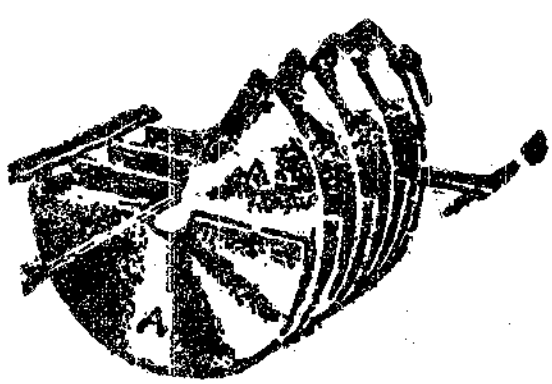


Fig 5.2 Variable Condenser

There are two cases:

(a) When the charges Remain the Same

Mechanical force or force of attraction per unit area between inside surface of each plate is equal to the outward electrical pressure over unit area of surface of A. It is given by

$$F = \frac{\sigma^2}{2\,\varepsilon_o\kappa} = \frac{d^2}{2\varepsilon_o\kappa A^2}\ \text{N/m}^2$$

Where σ is the surface density of charge on A. Therefore the force of attraction between A and B, each area a sq .m = $\frac{d^2}{2\varepsilon_o\kappa A}$ N: and the work done is separating the plates by a distance d is given by W = force x distance = $\frac{q^2 d}{2\varepsilon_o\kappa A}$ Joules.

$$\text{So, } W = \frac{q^2 d}{2\varepsilon_o\kappa A} \qquad \ldots(5.15)$$

If κ = 1, force of attraction = $\frac{q^2 d}{2\varepsilon_o a}$ N. Thus when the charge remains same, the force of attraction between the plates with a medium having 'K' as dielectric constant is (1/K) times the force with air as dielectric.

Since the generally measured parameter is the potential on the plates and not the charge, the expression for the force of attraction is more useful when expressed in terms of the potential difference between the plates instead of charge.

(b) When the potential difference between the plates remains the same.

When both plates are connected to the two ends of a battery, i.e., positive end to A and negative end to B. (Fig 5.1c)

Since q = CV = $\frac{\varepsilon_o\kappa A}{d}$ V and F = $\frac{q^2}{2\kappa\varepsilon_o A^2}$ the force of attraction per square meter.

$$F = \frac{(\kappa\varepsilon_o A.\ V/D)^2}{2\kappa\varepsilon_o A^2} = \frac{\kappa\varepsilon_o V^2}{2d^2}\ \text{N/m}^2 \qquad \ldots(5.16)$$

It shows that if V remains constant, the force of attraction is directly proportional to K, i.e., the force on a dielectric medium is K times that with air as dielectric

Example – 3:

A parallel plate consider of circular cross-section and radius 10 cms has its plates 1 mm apart and are separated by oil of dielectric constant 6. If the potential difference is 300 V, find the force of attraction between the plates.

Solution :

$$\text{Force of attraction} = \frac{\kappa\varepsilon_o A}{2d^2} = V^2$$

$$= \frac{6 \times 8.9 \times 10^{-12} \times \pi \times (10^{-1})^2 \cdot \times (300)^2}{2 \times (10^{-3})^2}$$

$= 7.54 \times 10^{-2}$ N

5.5.1 Energy Required in separating the plates

If the plates of a charged condenser at constant potential re insulated, and their separation is increased by a small distance x.

Work done = force X displacement

$$W = \frac{\kappa\varepsilon_o}{2d^2} V^2 x \qquad \ldots (5.17)$$

Further if E is the electric field intensity between the plates,

$$V = E.d$$

So work done $= W = \frac{\kappa\varepsilon_o}{2d^2} E^2 d^2 x = \frac{1}{2} \kappa\varepsilon_o E^2 x \qquad \ldots(5.18)$

Since how the volume swept out by 1 sq.com is x.1, work done per unit volume = energy Stored in unit volume

$$= \frac{1}{2} \kappa\varepsilon_o E^2 = (\text{Energydensity}) = \frac{\kappa}{2} DE \qquad \ldots (5.19)$$

Where D is the electric displacement = $\varepsilon_o E$

If the dielectric constant $\kappa = 1$,

Work done per unit volume $= \frac{1}{2} \varepsilon_o E^2 \qquad \ldots(5.20)$

In space of volume V, the energy stored is given by ½ $\int \vec{D}.\vec{E}dv$ (D is given in Coul/m^2 and E is in volts/m.

5.6.1 Energy Stored in the Field of a Charged Condenser

The energy stored in a Charged condenser is equal to the work done in charging it. Suppose the potential difference between the plates at any instant is V. The work done in bringing a small charge, dq, to the condenser (from infinity), when its potential is V, is given by dW = V.dq

Hence the total work done by charge q is

$$W = \int_{o}^{q} Vdq \, Joules \qquad \ldots (5.21)$$

But V is not constant. It is function of q.So,

$$W = \int_{o}^{q} q/c dq = \frac{q^2}{2c} = \frac{1}{2} qv = \frac{1}{2} CV^2 \qquad \ldots (5.22)$$

This eqn *5.22) represents the energy stored in charged condenser.
W will be inJoules if q is in coulombs, C in Farads and V in volts.

For a parallel plate condenser having the surface density σ, area of insulated plate A, and the distance between the plates, d, the energy is given by

$$\text{Energy} = \frac{1}{2} q^2 c = \frac{1}{2A^2\sigma^2} \frac{\delta}{\kappa A o} = \frac{\sigma^2 (Ad)}{2\kappa\varepsilon_0} \qquad \ldots (5.23)$$

$$\text{and energy density} = \frac{\sigma^2}{2\kappa\varepsilon_0} \qquad \ldots (5.24)$$

Thus the energy of a parallel plate condenser is the same as the mechanical work done in separating the plates. This energy resides in or is stored up in the dielectrics. This energy can easily be demonstrated with a Leyden jar of detachable parts. The energy so stored up is equal to $\frac{1}{2}\kappa\varepsilon_0 E^2$ Joules/m, of the dielectric. This is also the energy of a charged conductor, as this and the surrounding walls with air as dielectrics form a condenser.

If the potential (V) is the same for both conductors (one having r, as dielectric) constant and the other with air), the conductor with dielectric will have an energy k times that of the conductor surrounded by air. This is true for any conductor or condenser.

Let the plates be connected to the ends of a battery and charged to + q and –q respectively. If the battery is now removed so that the charge remains the same, and the dielectric (of dielectric constant) is introduced between the plates, the potential is reduced to 1/k and the energy $q^2/2c$ is also reduced in the same ratio (of 1/k) . the energy which has disappeared has been used in inserting the dielectric. If on the other hand, the battery connection is retained and the dielectric is introduced (so that the potential remains constant), the charge and hence the energy are increased K times to their initial values. The extra energy is supplied by battery. When the dielectric is introduced partially in between the plates of a condenser the energy tends to a position of minimum energy and hence the dielectric will be drawn or sucked into the plates.

Thus two similar conductors of different materials, charged to the same potential, will have their charges and energies directly proportional to their dielectric constants.

If the distance between the plates of a parallel plate condenser (of capacity C farads and connected to a battery of V volts), is decreased, the capacity increases. The charges stored will also increases. This increase in charge is supplied by battery. This process is called charging at constant potential The total work done by the battery in this process = CV^2 Joules.

The change in the energy of a parallel plate air condenser when the distance between the plates is altered is given under two cases. (i) when the charge remains the same, the change in energy $= \frac{q^2}{2\Sigma^0}(d-d^1)$. (ii) when the potential remains the same, the change in energy

$$= \frac{AV^2}{2\varepsilon_o}\left(\frac{d-d^1}{dd^1}\right)$$

Let there be a conductor (or the positively charged plate of a condenser) of capacity C_1 At potential (or pd)V_1. Let this he connected to another conductor (or positively charged plate of another condenser of capacity C_2 at potential (or pd) V_2, so that the charges are shared such that $V_1 > V_2$, it can be shown that the loss of energy.

$$\Delta\ E = \frac{C_1 C_2(V_1-V_2)^2}{2(C_1+C_2)} \qquad \ldots(5.25)$$

This loss of energy is always positive quantity unless $V_1 = V_2$ if V_1 # V_2, there is always a loss of energy on sharing the charges. Since no electricity can disappear, the total charge before contact is equal to the total charge after contact. The loss of energy appears in the form of a spark and hear in the conductor or the two conductors.

When a condenser is discharged by connecting to the earth, the entire charge is given to earth. This also happens when the plates of a charged condenser short circuit.

The discharge of condenser is used for taking flash photographs. The instrument consists of a very large capacity charged by a dry cell. This discharges through a discharge tube containing magnesium ribbon and gas at low pressure.

Example – 4:

A condenser A of 20 μ F charged to 1500 v is connected in parallel to condenser B of 10 μ F charged to 150 V. Find the total energy before contact, loss of energy after contact and common potential.

Solution:

Charge on A = $20 \times 10^{-6} \times 1500 = 0.03$ Coul

Charge on B = $10 \times 10^{-6} \times 150 = 0.0015$ Coul

Total Capacity = 30 μF

Total energy before contact = $\frac{1}{2}\, 20\,(10^{-6})\ (1500)^2 + \frac{1}{2}\, 10\,(10^{-6})(150)^2$

= 22.61 Joules

Energy after contact = $\frac{1}{2}\frac{q^2}{C} = \frac{(0.0315)^2}{2 \times 30 \times 10^{-6}}$

= 16.54 Joules

So loss of energy = 6.07 Joules

Common potential = $\frac{q^2}{C} = \frac{0.0315}{30 \times 10^{-6}} = 1050$ V

Example – 5 :

Assuming the earth of radius 6.4×10^6 m to be a charged sphere in free space and with an electric field or 300 V/m at its surface, find the energy of its charge, ad the heat generated if it were completely discharged.

Electric intensity = 300 V/m

$\therefore$ charge is given from Eqn. (5.4) for $E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}$

So, $300 = \frac{q}{4\pi (6.4 \times 10^6)\varepsilon_o}$

or $q = 4\pi (300)(6.4 \times 10^6)^2 \varepsilon_o$

$$= \frac{q^2}{2C} = \frac{(4\pi)^2 (300)^2 (6.4 \times 10^6)^4 \varepsilon^2}{2 \times 4\pi\varepsilon_o (6.4 \times 10^6)}$$

$= 2\pi (300)^2 (6.4 \text{ X } 10^6)\varepsilon_o$

$= 1.31 \times 10^1$ 5 joules

Heat produced = $\frac{1.31}{4.2} \times 1^{15} = 3.12 \times 10^{14}$ cal.

Example – 6:

The intensity of electric field due to a spherical conductor of diameter 4 cms at a distance of 20 cms from its center is 30 V/cm. Calculate the energy of the conductor.

SOLUTION:

E = 3000 V/m

$C = 4\pi\varepsilon_o (0.02)$; Vat 20 cm = $\frac{q}{4\pi\varepsilon_o (0.02)}$

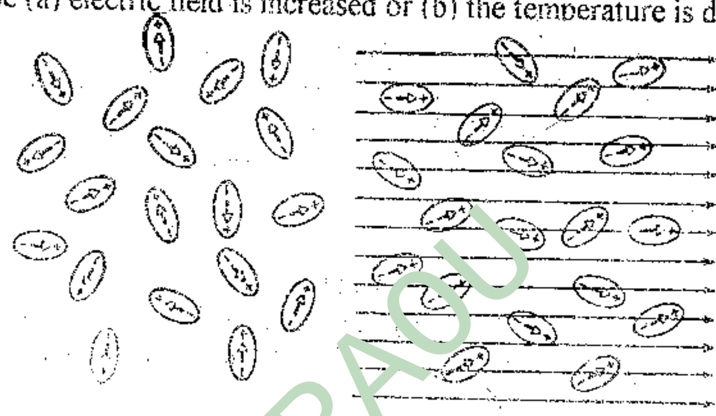
$V = E.d = \frac{q}{d}$ or$q = 4\pi\varepsilon_o (0.02) 3000 (0.02)$

$$\frac{q^2}{2C} = \frac{[4\pi\varepsilon_0 (0.02)\, 3000(0.02)]^2}{2 \times 4\pi\varepsilon_0}$$

$$4\pi\varepsilon_0 \frac{(3000)^2 (0.02)^3}{2} = 4.00 \times 10^{-13} \text{ Joules}$$

5.6 DIELECTRICITY – AN ATOMIC POINT OF VIEW

Let us now discuss what happens when a dielectric is placed in an electric field. The Atomic view will now be presented. The molecules of some dielectrics, like water, will have permanent electric dipole moments and they are called polar molecules. The permanent electric dipoles of dipole moment P tend to align themselves with the external field (Fig. 5.4) The degree of alignment of the polar molecules in the direction of electric field will not be complete due to thermal agitation. But the degree of alignment will increase if the (a) electric field is increased or (b) the temperature is decreased.



5.4 Polar molecules in an electric field [(a) without electric field (random orientation) (b) with electric field (partial alignment)

Irrespective of the nature of molecule whether a molecule is polar or non – polar, there will be some induction when they are placed in electric field. The external electric field will tend to separate the net negative and positive charge in the atom or molecule. This induced electric dipole moment is present as long as external electric field is there.

Let us use a parallel – plate condenser, carrying a fixed charge q_1 and not connected to a battery (Fig. 5.5) . This provides a uniform external electric field E_o. A dielectric, slab is placed in the field. The effect of field on the slab is that the center of positive charge of the entire slab is separated from the center of negative charge, however the slab as a whole, remains electrically neutral. So a pile of positive charges rests on one face of the slab (parallel to the face of the condenser plate) while the other face consists of negative charges and no charge remains with in the slab. The positive and negative charges must be equal since the atoms and molecules are electrically neutral. This displacement of electrons in the dielectric from their equilibrium position is less than the atomic diameters.

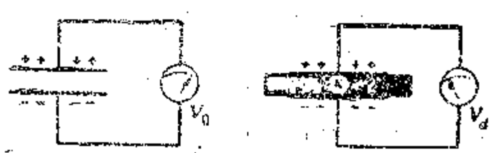


Fig 5.5 Parallel plate condenser with (a) air as medium (b) dielectric as medium

The induced surface charge will always appear in such a way that the electric field due to them E^1 opposes external electric field (E_o). The resultant field E is E_0+E^1.E^1 is in the same direction as E_o, but is smaller. Thus the induced surface charge in a dielectric due to external field. will always tend to weaken the original field within the dielectric. This weakening of the electric field reveals itself as a reduction in potential difference between the plates of a charged isolated capacitor when a dielectric is introduced between the plates. More specifically, if a dielectric slab is introduced into a charged parallel-plate condenser, then

$$\frac{E_0}{E} = \frac{V_o}{V_d} = k \qquad \ldots(5.26)$$

Induced electric surface charge is the explanation of the most elementary fact of static electricity. A dielectric body in a uniform electric field will not experience a net force

5.7 DIELECTRIC CONSTANT

The dielectric constant r. or relative permitivity depends upon the temperature, pressure and crystalline state. It plays an important role in electrostatic phenomenon. It has a much higher value for solids and liquids than for air. The dielectric constant of the material may also be defined as the ratio of the capacitance with dielectric (C^1) (inserted in between the plates of a parallel plate condenser) to that without the dielectric (C). Tables 5.1 give the properties of some dielectrics.

Variation of dielectric constant

The dielectric constant of the material depends not only on its purity but also upon factors such as temperature. Frequency of the applied voltage, humidity etc. Dielectric constant for solids increases with raise in temperature while for liquids, it decreases with increases in temperature.

Table 5.1 Properties of some dielectrics

Substance	Dielectric Constant k	Dielectric Strength * (kv/mm)	Substance	Dielecric Constant k	Dielectric Strenght* (kv/mm)
Flint	5.0		Vacuum	1.00.000	α
Crown glass	8.9		Paper	3.3	14
Sulphur	2.4		Porcelin	6.5	4
Ebonite	3.2		Fused quartz	3.8	8
Paraffin	4.0		Bakelite	4.8	12
Rubber	2.2		Polyethylene	2.3	50
Mica	4.6		Polystyrene	2.6	25
Acetone	21.0		Teflon(PTFE)	2.1	60
Ethylalacohol	26.0		Neoprene	6.9	12
Distilled water	70		TiO2	100	6

Air	1.00058	0.8	-	-	-
Hydrogen	1.0026				

(* the maximum potential gradient that can exist in the, dielectric without the dielectric breakdown)

5.8 EFFECT OF DIELECTRIC SLAB ON THE CAPACITY OF A PARALLEL PLATE CONDENSER

Michael Faraday, in 1837, first investigated the effect of filling the space between the plates with a dielectric faraday investigated this by constructing two identical capacitors. In one of which he placed a dielectric. The other contained air at normal pressure. When capacitors were charged to the same potential difference, he found by experiment that the charge on the one containing the dielectric was greater than that on the other. Since q is large for the same V, if a dielectric is present it follows from the relation c = q/V. that the capacitance of a capacitor increases if a dielectric is placed between the plates.

If the two capacitors are maintained at the same charge, instead of at same potential difference, the p.d. between the plates (have dielectric) V_d is related to the p.d.. the plates (without dielectric), V_o by the relation.

$$V_d = \frac{Vo}{K} \quad ...(5.257)$$

$$\text{Since } C = q/V,\ C_1 = \frac{q}{V_d} = \frac{q}{Vo/k} = K\frac{q}{Vo} \quad ...(5.28)$$

$$C^1 = kC \quad ...(5.29)$$

Thus the effect of introducing the dielectric in the space between the parallel plate condenser is to increase the capacitance by a factor 'κ' (κ being the dielectric constant). In general, the capacitance of any capacitor can be written as,

$$C = \kappa\varepsilon_0 L \quad ...(5.30)$$

Where L depends on the geometry and has the dimension of the length. For parallel plate condenser, L = (A/d); for a cylindrical condenser, $L = \frac{2\pi l}{Loc_e(b/a)}$

Check your Progress: I

1. The capacity of a Parallel with dielectric is..... and without dielectric is......
2. The induced surface charge will always appear in such a way that the electric field due to E opposes.
3. If C is the capacity A the condenser and 'q' is the charge on it then the energy of the condenser is given by.....

Note: a. Space is given below for your answer.

b. Compare your answer with those given at the end of the unit.

....

....

....

Where q represents only free charge, the induced surface charges being excluded. Table 5.2 gives the properties of electric vectors D, E and P.

Table 5.2 Three Electric Vectors

Name	Symbol	Associated with	Boundary Condition
Electric field strenght	E	All charges	Tangantial component continues
Electric displacement	D	Free charges only	Normal component Continues.
Polarization	P	Polarization charges Only	Vanishes in vaccumm
Defining equation For E	$\vec{F} = q\vec{E}$		
General relations amount the three vectors		$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$	
Gauss' law when dielectric media Are present		$\oint \vec{D}.\vec{ds} = q$ (q free charge only)	
Empirical relations for certain Dielectric materials		$\vec{D} = k\varepsilon_0 E$ $\vec{P} = (k-1)\varepsilon_0 E$	

Unoform dielectric

A description of a dielectric at an atomic level and the surface cahrges induced on account of incident external field is given in earlier section. Also the variation of potential and the electric field intensity are also discussed earlier. The variation in capacity as well as potential and other parameters such as electric displacement D, as a result of introducing a uniform dielectric medium like glass is also discussed earlier.

We shall now discuss how the capacity, charge, potential difference, electric field intensity and displacement will be affected when the space in between the parallel plate condenser is filled with compound dielectric.

5.10 CAPACITANCE OF PARALLEL PLATE CONDENSER WITH A COMPOUND DIELECTRIC

On the introduction of dielectric in the space between the plates of a condenser, there is a change in the potential difference between A and B (Fig 5.7) .For calculating the potential difference between A and B, we use the expression $V = \oint$ **E.dr** for air and for dielectric.

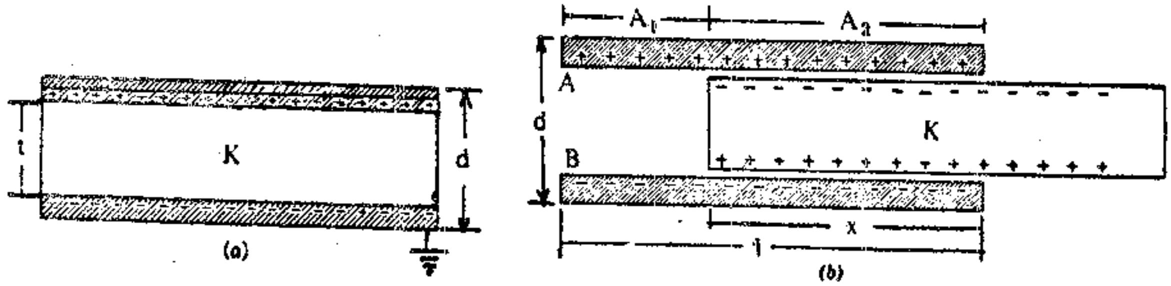


Fig. 5.7 Parallel plate condenser with compound dielectric partially inserted inside the plates (a) without the dielectric (b) partially filled with the dielectric.

When the plates are separated partly by a slab of dielectric constant k and thick ness t and partly by air of thickness (d-t), we have p.d. across the plates.

$$V = \int_{(d-t)}^{0} -\frac{\sigma}{\varepsilon_0}\,dt + \int_{0} -\frac{\sigma}{\kappa\varepsilon_0}\,dt \qquad \text{... (5.4.0)}$$

$$V = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma t}{\kappa\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{\kappa}\right) \qquad \text{... (5.4.1)}$$

But $\sigma = q/A$

So, capacity
$$C = q/V = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{\kappa}\right)} = \frac{A\varepsilon_0}{\left(d - t\frac{(k-1)}{k}\right)} \qquad \text{... (5.4.2)}$$

if $T = d$, $C = \frac{\kappa A\varepsilon_0}{d}$ ... (5.4.3)

Note that the original potential difference (when the dielectric is not introduced) is $\sigma d/\Sigma_0$ which is greater than after its introduction. But the capacitance increases after the dielectric is introduced.

It is improtant to note that the insertio of the dielectric between the plates does not alter the electric displacement, D since $k_1E_1 = k_2E_2$constant ... (5.44)

Since the potential falls on insertion of the dielectric we have to increase the distance between the plates, so as to have the same capacity as before. The distance to be increased is given by.

$$X = \frac{(k-1)\,t}{k} \qquad \text{...(5.45)}$$

(if besides air, there are two dielectric k_1 and k2 of thickness t1 and t_2 respectively, we have

$$p.d = \frac{\sigma}{\varepsilon O}\left[d - (t_1 + t2) + \left(\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2}\right)\right] \qquad \text{... (5.46)}$$

5.9 ELECTRIC DISPLACEMENT, ELECTRIC POLARIZATION AND ELECTRIC FIELD

For simple problems in electromagnetism such as rectangular slab placed at right angles to uniform electric field, the treatment of dielectric presented in earlier lession is sufficient. For treatment of more complex problems – such as finding E in a dielectric placed in a non uniform external electric field, a new formation presented below is necessary.

The induced surface charge per unit area of the surface is called electric polarisation.

$$P = (qo / A) \quad \ldots (5.31)$$

The name 'polarisation' is suitable because the induced surface charge q^1 appears when the dielectric is polarised. The electric polarisation, P can be defined in an equivalent way by multiplying both numerator and denominator by d, the thickness of dielectric slab.

$$P = (q^d/Ad) \quad \ldots (5.31a)$$

q^1 d gives the induced electric dipolemoment where as Ad gives the volume of the slab.
So electric polarization can also be defined as the induced electric dipolemoment per unit volume in the dielectric. This suggests that P should be a vector quantity since the induced dipolemoment is also a vector. The direction of P is from the negative induced charge to the positive induced charge.

If q and q1 are the free and induced charges on the plates respectively. 'A' is the area of the plates, they are related by the equation.

$$\frac{q}{A} = \varepsilon_0 \frac{q}{(\kappa\varepsilon_0 A)} + \frac{q1}{A} \quad \ldots (5.32)$$

We can rewrite this equation as

$$\frac{q}{A} = \varepsilon_0 \vec{E} + P \quad \ldots (5.33)$$

The quantity on the right hand side of Eqn. (5.33) occurs so often in electrostatic problems, we give it the special name 'electric dispalcement', D.

$$\text{So } \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad \ldots (5.34)$$

$$\text{Where } D = q/a \quad \ldots (5.35)$$

Since E and P are vectors, D must also be a vector.

In more complicated problems, however, D, E and P may vary both in magnitude and direction from point to point. From the definitions. We observe the following aspects.

(1) The electrical *displacement* D, is connected with free charges only, the vector field of D may be represented by lines of D, just as

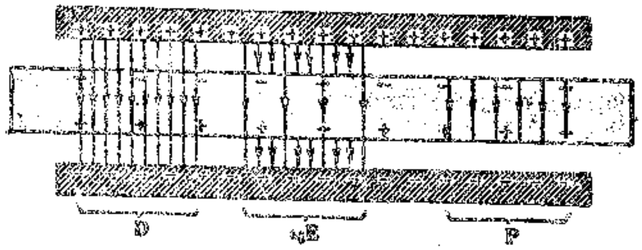


Fig 5.6 D,E,P, in parallel plate condenser with compound dielectric

E is represented by lines of force. (Fig. 5.6) The lines of D begin and end on free charges.

(2) The electric polarization, P is connected with the polarization charges only. It can also be represented by line, which begin and end on the polarization charges.

(3) the electric field, E is connected with all charges present, whether they are free or polarization charges. The lines of E reflect the presence of both kinds of charges.

The units of D and P are same (Coul/m^2) where as that of E is N/coul.

The electric field E, is determined by the force that acts on suitably placed test charge and is of fundamental interest. D and P are auxiliary vectors useful only in the solution of more complex problems.

$$d = \frac{q}{A} = \kappa\varepsilon_0\left(\frac{q}{\kappa\varepsilon_0 A}\right) = \kappa\varepsilon_0 E \qquad \ldots (5.36)$$

$$P = \frac{q^1}{A} = \frac{q}{A}\left(1 - \frac{1}{k}\right) = D\left[1 - \frac{1}{k}\right] \qquad \ldots (5.37)$$

$$P = \kappa\varepsilon_0 E\left(1 - \frac{1}{K}\right) = \varepsilon_0\,(\kappa - 1)\,E \qquad \ldots (5.38)$$

This shows clearly that in vacuum. K = 1, the polarization vector 'P' is zero. * Thus it is clear that polarization exists only in dielectrics.

Certain waxes, when polarized in their molten state, retain a permanent polarization after solidifying even though the external polarization field is removed. They are called "Electrets". These are electrical analogous of permanent magnets in that they possess a gross permanent electric dipolemoment are analogues names of magnets. Materials from which electrets can be constructed are called ferroelectrics. Electrets do not obey Eqn. (5.38) since P # 0 even for E = 0

Eqns (5.36) and (5.38) suggest that for isotropic materials, to which a single dielectric constant κ can be assigned. D and P both point in the direction of E at any given point.

The Gauss' law for, electrical displacement can be given as

$$\oint D.ds = q \qquad \ldots (5.39)$$

In general if there are numbers of dielectrics with dielectric constants κ_1 , κ_2 and or thickness t_1, t_2 ....respectively and are placed in between the plates of parallel plate condenser, we have p.d.

$$= \frac{\sigma}{\varepsilon_0}\left\{d - (t1 + t2 ++ \ldots +) + \frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \ldots\ldots\right\} \quad \text{and the} \qquad \ldots (5.47)$$

$$\text{Capacitance } C = \frac{A\varepsilon_0}{\left(d - \frac{t_1(\kappa_1 - 1)}{\kappa_1} - \frac{t_2(\kappa_2 - 1)}{\kappa_2} \ldots\ldots\right)} \qquad \ldots (5.47)$$

$$\text{or } C = \frac{A\varepsilon_0}{\left(\frac{t1}{\kappa 1} + \frac{t2}{\kappa 2} + \frac{t3}{\kappa 3} + \ldots\right)} \qquad \ldots (5.48)$$

Eqn. 5.47 applies when $d < [t_1 + t_2 + t_3 + \ldots]$ and

Eqn. (5.48) applies if there are various dielectrics in place of air

But there will arise another case when dielectric is partly inside the plates A and B and partly outside. [Fig. 5.7 (b)]

Then the capacity can be worked out on the following lines.

Let A1 and A2 be the areas occupied by air above and the dielectric between A and B respectively. If we assume that the lines of force in air are straight except at the edges of the plates and the dielectric.

$$D = \frac{A2\varepsilon_0}{\left(\frac{d - (\kappa - 1)E}{\kappa}\right)} + \frac{A_1\varepsilon_0}{d} \qquad \ldots (5.49)$$

Thus if A_1 is decreased, C (i.e., capacitance) decreases.

It t = d [Fig. 5.7(6)], $C = \frac{\kappa A_2\varepsilon_0}{d} + \frac{A_1\varepsilon_0}{d}$ ...(5.50)

Also if '1' is the length of the plate and 'x' is the length of the dielectric inside, and A is the area of the whole plate, then

$$\frac{A_1}{A_2} = \frac{(l - x)}{l} \quad \text{and } A_1 + A_2 = A$$

$$\text{And } A1 = \frac{1 - x}{l} A; \; A2 = \frac{x}{l} A$$

substituting the values of A_1 and A_2 in Eqn. 5.50 and substituting t = d

$$C = \varepsilon_0 \frac{X}{l} A + \frac{(-x+l)}{l} A \varepsilon_0 \qquad \ldots (5.51)$$

$$C = \frac{\kappa\varepsilon_0 (xA)}{ld} + \frac{(l-x)\varepsilon_0 A}{ld} \qquad \ldots (5.52)$$

The potential difference $V = \frac{q}{C} = \frac{\sigma A}{C} = \frac{\sigma ld}{\varepsilon_0 [(\kappa - l) x + l]}$ ...(5.53)

The charge q = CV

The electric field $E_o = \frac{q}{\varepsilon_0 A}$

The electric displacement $D = \kappa\varepsilon_0 E = (\kappa q / A)$

Example – 1:

The plater of parallel plate condenser is having an area of 2 sq. They are 5×10^{-3} m Apart, and a potential difference of 10^4 volts is applied across the plates.

i) find (a) the capacity (b) charge on each plate
(c) surface density of cahrge on each plate
(d) electric intensity and (e) electric displacement in the space between them.

ii) When the battery is disconnected and dielectric constant of κ = 5 and thickness 0.5×10^{-2} m is inserted find C, E, p.d. and electric displacement.

Solution:

1) $C = \frac{\varepsilon o \kappa A}{d} = \frac{8.9 \times 10^{-12} \times 1 \times 2}{5 \times 10\text{-}3} = 3.56 \times 10^{-3}\ \mu F.$

$q = CV = 3.56 \times 10^{-9} \times 10^4 = 3.56 \times 10^{-5}$ coul.

$\frac{q}{A} = \sigma = \frac{3.56 \times 10^{-5}}{2} = 1.78 \times 10^{-5}$ Coul/m^2

$E = \frac{\sigma}{\varepsilon o} = \frac{1.78 \times 10^{-5}}{8.9 \times 10^{-12}} = 2.0 \times 10^6$ Volt/m

iii) $C^1 = C\kappa = 3.56 \times 10^{-9} \times 5 = 17.8 \times 10^{-3}\ \mu F.$

$q^1 = c^1 V = 17.8 \times 10^{-9} \times 10^4 = 17.8 \times 10^{-5}$ Coul

$\sigma^1 = \frac{q^1}{A} = \frac{17.8 \times 10^{-9} \times 10^4}{2} = 8.9 \times 10^{-5}$ Coul/m^2

$$E = \frac{\sigma l}{\kappa \varepsilon_0} = \frac{8.9 \times 10^{-5}}{5 \times 8.9 \times 10^{-12}} = 2.0 \times 10^{6} \text{ Volt/m}$$

$$D = \varepsilon o \kappa\ E = \frac{5 \times 8.9 \times 10^{-5} \times 8.9 \times 10^{-12}}{5 \times 8.9 \times 10^{-12}} \text{ Volts/m}$$

$$\text{p.d } E^{1} d = 2 \times 10^{6} \times 0.5 \times 10^{-2} = 1 \times 10^{4} \text{ Volt}$$

Example – 2:

A dielectric slab of thickness o.5 cm and dielectric constant 7.o is palced between the plates or parallel plate condenser of plate of area 100 cm^2 and separation 1.0 cm (A) p.d. of 100 V is applied without the dielectric. Calculate te capacitance C_0 before the slab is inserted.

Solution :

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.9 \times 10^{-12} \text{ Coul}^2/\text{Nm}^2 (10^{-12} \text{ m}^2)}{1 \times 10^{-2} \text{ m}}$$

$$C = 8.9\ \mu\mu F$$

The free charge $q = CV = 8.9 \times 10^{-12} \times 100$

$$= 8.9 \times 10^{-10} \text{ Coul.}$$

Because of the technique used to charge the capacitor the free charge remains uncharged as the slab is introduced. If the charging battery is disconnected, this would not be the case.

An application of Gauss' law indicates.

$\varepsilon o \int \kappa\ \vec{E}.\vec{ds} = \varepsilon_0 \kappa\ EA = q$ (Since $\kappa = 1$, because the surface over which we evaluate the flux integral does not pass through the dielectric)

$$\text{So, } E = \frac{q}{\varepsilon_0 A} = \frac{8.9 \times 10^{-10}}{8.9 \times 10^{-12} \times 10^{-2}} = 1 \times 10^{4} \text{ Volt/m}$$

Note that E remains unchanged when the slab is introduced. But electric field strenght in the dielectric medium change and is given by

$$\varepsilon_0 \kappa\ E^{1} A = q$$

Here κ appears because the surface cuts throughout the dielectric and that, only the free charge q appears on the right. Thus we have

$$E1 = \frac{q}{\kappa \varepsilon_0 A} = \frac{E}{\kappa} = \frac{1 \times 10^{4}}{7} = 0.14 \times 10^{4} \text{ Volt/m}$$

The potential difference between the plates

$$V = \int \vec{E}.\vec{dl} = E\ (d - t) + E^{1} t$$

$$= 1 \times 10^{4} (5 \times 10^{-3}) + (0.14 \times 10^{4} \times 5 \times 10^{-3})$$

V = 57 Volt.

This contrasts with the original applied potential difference of 100 VCapacitance with the slab in between the plates is given by

$$C = \frac{q}{V} = \frac{8.9 \times 10^{-10}}{57} = 15.61\ \mu\mu F$$

When the dielectric slab is introduced. The potential difference drops from 100 V to 57V While the capacitance raises from 8.9 to 15.61 μμ F. a factor of 1.71 times. If the dielectric slab had filled the capacitor, the capacitance would have raised by a factor of κ (= 7)

In the dielectric :

The electric filed is calculated to be

$$E = \frac{q}{\kappa \varepsilon_0 A} = \frac{8.9 \times 10^{-10}}{7 \times 8.9 \times 10^{-12} \times 10^{-2}} = \frac{1}{7} \times 10^4$$

$= 1.43 \times 10^3$ Volt/m

D = the electric displacement = $\kappa\varepsilon_0 E$

$= 7 \times 8.9 \times 10^{-12} \times 1.43 \times 10^3 = 8.9 \times 10^{-8}$ Colu/m^2

P = the electric polarization = $\varepsilon_0 (\kappa - 1)$ E.

$= 8.9 \times 10^{-12} \times 6 \times 1.43 \times 10^3 = 76.26 \times 10^{-9}$ Coul/m^2

With the air gap : $E = \frac{q}{\varepsilon_0 A} = 10^4$ Volt/m

$D = \varepsilon_0 E = 8.9 \times 10^{-12} \times 10^4 = 8.9 \times 10^{-8}$ Coul/m^2

With the dielectric :

$D = \varepsilon_0 E + P = (8.9 \times 10^{-12} \times 1.43 \times 10^3) + 7.64 \times 10^{-8}$

$= 8.9 \times 10^{-8}$ Colu/m^2

Outside the dielectric (within the air gap):

$\varepsilon_0 E + p = (8.9 \times 10^{-12} \times 10^3) +) = 8.9 \times 10^{-8}$ Coul /m^2

So D has the same value both inside the dielectric and in the air gap.

Example –3

A parallel plate condenser is charged to difference of potential V and the plates are then insulated. A slab of glass of thickness 4 mm., is then inserted between the plates and it is found necessary to increase the distance between the plates 3.2 mm in order to restore the p.d to the intial value V. find the dielectric constant of glass.

Solution :

On introducing the slab, the potential falls, So the increase in distance between the plats is given by

$$X = \frac{\kappa - 1}{\kappa} t \text{ or } \frac{1}{\kappa} \ (1 - x/t)$$

$$\kappa = \frac{1}{1-(x/t)} = \frac{1}{1-(3.2/4)} = \frac{1}{0.2} = 5$$

So the dielectric constant of glass is 5.

10. would you except the dielectric constant for polar molecules to vary with temperature? Why?
11. Discuss the following statement: the permitivity is a measure of how easily a dielectric will permit the establishment of electric field lines with the dielectric

III Solve the following problems

1. When a slab of insulating material of thickness 8×10^{-3} m is introduced between the plates of a parallel plate condenser, it is found that the distance between the plateshas to be increased by 7×10^{-3} m. to restore the condenser capacity to its original value. Calculate the dielectric constant of the material

   (Ans : K = 8)

2. The distance between the plants of parallel plate condenser, is 2.4×10^{-2} m. A rectangular slab of thickness 1.2×10^{-2} m and dielectric constant 5 is place din between them and the distance

   (Ans : 3.36×10^{-2}m)

3. Two rain drops, a long way apart, have radius of 2 and 2mm respectively. Their potentials are 40 to 60 esu. Respectively. What will be the change in energy if they coalesce? What will be their potential?

   (Ans : 408 ergs,856.esu)

4. A 100 μμ F capacitor is charged to 100 V After charging, the battery is disconneccted. The capacitor is connected is parallel to another capacitor. The final voltage is 30 V. What is the capacitance of all second capacitor.

   (Ans: 267 μμ F)

5. For a given parallel condenser A = 0.01 sq.m, d = 0.05m, p.q = 100 V, when air is used. If air is replaced by glass of K = 6, calculate the new capacity and new p.d.

   (Ans 640 μμ F; 16.67V)

6. Find the mechanical stress per sq.cm on the glass plate of a condenser charged to a potential of 30,000 V.K and t of glass are 4 and 4 X 103 m. respectively. Find the electrostatic forece per unit area of an insulated spehre of 5 X 10-2 m radius, charged to 177×10^{-10} Coul.

   (Ans : 99.51 N/m^2 ; 44.78 N/m^2)

5.13 RECOMMENEDED BOOKS

1. Halliday, D And Resnck, R	Physics – part II	Wiley Eastern Pvt Ltd. New Delhi
2.Vaudeva, D.N.	Fundamentals of Electricity and Magnetism	S.Chand and Company New Delhi.
3.Duckwork, E.	Electricity and Magnetism	Holt Reinhart and Winston Publications, New York.

Example – 4:

Derive the expression for cahnge in energy ofparallel plate condenser when a dielectric constant K and thick ness t is introduced between the plates:

(a) When the charge remains the same.
(b) When the potential remains the same.

Solution:

Charge remains the same

(a) When the capacity = $\dfrac{A\varepsilon_0}{d-t\,(1-1/k)}$

So energy $\dfrac{q^2}{2C} = \dfrac{q^2}{2A\varepsilon_0}\left(d - t\,(1 - \dfrac{1}{k})\right)$

Thus the energy is reduced on the insertion of the slab by an amount equal to

$$\frac{q^2}{2A\varepsilon_0}\, t\,(1 - 1/\kappa)$$

(a) If d = t i.e when the slab thickness is equal to the air gap of the parallel plate condenser the decrease in energy is $\dfrac{q^2 t}{2\kappa A\varepsilon_0}$

(b) When the potential remains the same i.e., when the battery is kept connected to the plates.

The new capacity = $\dfrac{A\varepsilon_0\kappa}{\left[d - t\,(1-1/k)\right]}$

and hence the energy = ½ CV^2 = ½ $\dfrac{A\varepsilon_0\kappa V^2}{[\,d - t\,(1 - 1/\kappa)]}$

Hence the energy is greater on introducing the slab than without the slab.

Thus the above two cases indicate that (a) the expression is to be used when the charge remains the same and (b) ½ CV^2 is to be used when the potential remain the same.

5.11 SUMMARY

The capacity of a parallel plate condenser is directly proportional to the are a of Cross section A and is inversely proportional to the distance d between the plates.

$$C = \frac{A}{4\pi d}$$

Positive and negative charges will be separated by the introduction of a dielectric material in the electric field. This separation of charges is called polarization. Molecules
Can be divided into two categoreis; polar and non-polar molecular, Polar molecules have permanent dipolemoment where as non polar molecules do not have permanent dipolemoment. The dielectric constant is defined as the ratio of the capacitance with the dilectric to that without the dilectric. The dielectric constnat depends upon pressure, temperature crystalline state and the frequency of the applied electric field. Dielectric constant varies with *temperature*.

Check your progress: Answers.

1. Capacity $C=kA/4\pi d$

   $C=A/4\pi d$.
2. External electric field E_0
3. $E=1/2q^2c$

5.12 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail

1. Calculate the capacity of a parallel plate condenser.
2. Derive an expression for the capacity of a parallel plate condenser and deduce the same using Gauss' theorem.
3. Derive the condtion for energy stored in a charged parallel plate condenser. Discuss the effect of including the dielectric slab in between the parallel plates.
4. Find the expression for the force per unit area of the surface of a conductor due to its charge.
5. Show that the total capacitance of a multiple-plate capacitor containing N plates separated by air is given by.

   $C_o = (N-1)\,\varepsilon o A$; Where A = area of each palate; D = separation of plates.
6. Derive an expression for the capacity of a parallel plate condenser with compound dielectric.
7. A capacitor remains connected to a battery and a dielectric slab is slipped between the plates. How do capacity, charge, potential difference and electric field strength vary ?
8. On introducing a slab of dielectric constant k into a parallel plate condenser, the energy is reduced while the charge remains constant. Energy increases when the potential is maintained the same. Explain.

II Answer the following questions briefly.

1. What do you understand by the potential and capacity of a conductor?
2. why is the energy loss, when the capacitor is connected to another capacitor of lower potential?
3. On introducing a slab of dielectric constant K into a parallel plate condenser, the energy is reduced if charge remained the same.Explain.
4. Can you introduce a dielectric slab into the space in between the plates, of a charged parallel plate condenser slowly? If not why? Explain.
5. If a capacitor is connected across a battery, Why does each plate receive a charge of exactly same magnitue?
6. Does the charge depend on the size and shape of the plates?
7. Can there be apotential difference between two adjacent conductors have same positive charge ?
8. An isolated conducting sphere is given a positive charge. Does its mass increase, decrease or remain the same ?
9. A spherical capacitor consists of two concentric spherical shells of radius a and b with b >a what are its (a) potential and (b) capacity.

BLOCK – 2: CURRENT DENSITY, STEADY CURRENTS AND CIRCUITS

UNIT 6: ELECTRICAL CONDUCTIVITY

Contents

6.1 OBJECTIVES

This unit introduces the concept of current density and resistance, resistivity and conductivity to make you understand the concept, the unit examines.

1. The motion of electrons in a conductor.

2. The flow of current in a conductor.

3. The relation between the conductivity of a conductor and the mobility of the charge carriers; (2) the variation of resistivity with temperature.

After going through the unit you will be able to

1. Explain what the current and the current density in a conductor are;

2. Distinguish between the velocity of the variation of the electrical field and the drift velocity.

3. Understand why the resistivity of a conductor does not depend on the size or shape of the conductor; and

4. Identify non-ohmic type of conductors.

6.2 INTRODUCTION

A 'conductor' is one in which plenty of free charges are available. When it is subjected to electric field, the charges will move due to the force exerted by the applied field. These free charges in a metallic conductor are electrons. In the case of an electron these free charges are ions both positive and negative. A gas under proper condition (such as neon gas in neon bulb) is also a conductor and its free charges are positive ions negative ions and electrons. The motion of these free charges constitutes a "current"

If we wish to maintain a continuous current in a conductor, we must continuously maintain the field or the potential gradient constant. Even though the magnitude of the field fluctuates if we can maintain the field direction same the resulting current is called "direct". If the field reverses the direction periodically the charge flow also reverses. Hence the resulting current is alternating'. Direct and alternating currents are abbreviated as DC and AC respectively.

If the ends of a metal wire-are connected to dry cell, or a storage battery, a potential gradient or an electric field will be maintained within the wire. Under the influence of this field the free electrons in the direction of the force. The free electrons collide with the stationery positive in cores and their motion is slowed down, after which they again accelerate and so on. There motion is thus a succession of accelerations and deceleration, but they will acquire a certain average velocity in the opposite direction to the field and they will all move steadily with the average velocity. In addition to this motion of the electrons these are subjected to the thermal energy and this will result in a random motion of the electrons. Their thermal motion is a random one and for our present purpose it may be ignored.

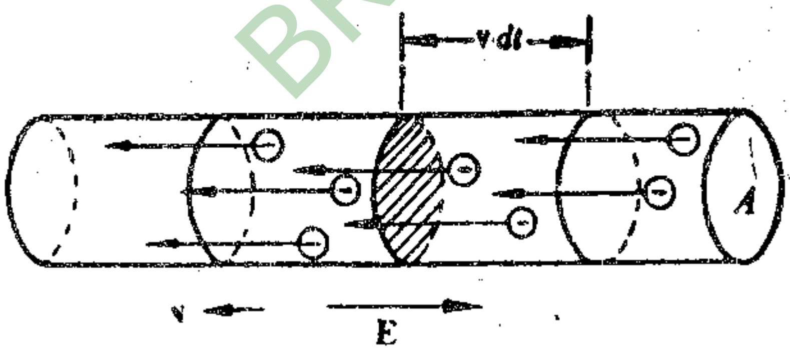


Fig. 6.1 Motion of free electrons in a wire

Fig 6.1 illustrates a portion of a wire in which there is a field acting towards the right., It is assumed that each electron moves with an average constant velocity. V_d. In time interval 'dt' which advances a distance vdt. The number of electrons crossing any plane, such as the one shaded, in time dt, is the umber contained in a section of the wire, of length vdt or volume AV_d dt. Where A is the area across section of the wire. If there are N number of electrons per unit volume the number crossing the plane in time dt is A NV_d dt. If 'e' is the charge on each electron the total charge crossing the area in time dt is

$$dq = NA\ e\ V_d\ dt \quad \text{...(6.1)}$$

The rate at which the charge is transported across a section of the wire is dq and it is called the current in the wire. Current is represented by i.

$$i = \frac{dq}{dt} \quad \text{...(6.2)}$$

From Eqns (6.1) and (6.2) the i is given by

$$i = NA\ eV_d \quad \text{...(6.3)}$$

The MKS unit of current is one "Coulomb per second" and is called one "ampere". Generally small currents are expressed in milli amperes (ma = 10^{-3} amperes) or in micro amperes ($\mu a = 10^{-6}$ amperes)

In general, if any number of different kinds of charged particles are present (as in the case of neon tube) in different concentrations and moving with different velocities, the net charge crossing a surface in time dt is

$$dq = Adt\ (N_{1q1}\ V_1 N_{2q2}\ V_2 + N_{3q3}\ V_3 + \ldots\ldots) \quad \text{...(6.4)}$$

and the current is

$$i = \frac{dq}{dt} = A\ \Sigma N_q\ V_d \quad \text{...(6.5)}$$

The free electrons in a metallic wire carrying current are distributed uniformly throughout the wire and the current in the wire of constant across section is distributed uniformly across any section.

The "current density' in the wire, usually represented by J is a vector quantity and it depends on the ratio of the current to the cross sectional area A.

$$J = i\ A = N_e\ V_d \quad \text{...(6.6)}$$

The above equation defines the average current density over an area A. But if the current is not uniformly distributed, one has to consider an infinitesimal area dA across which the current is 'di' and has to define the current density as

$$J = \frac{di}{dA} \quad \text{...(6.7)}$$

Current i is a characteristic of a particular conductor. Like mass of an object, length of an object, density of an object current is also a macroscopic quantity. The corresponding related microscopic property is the current density j. In a conductor j is characteristic of a point rather than the conductor as a whole. The relationship between J and i is that, for a particular surface 'ds' in a conductor is the flux of vector J over that surface or ds' in a conductor is the flux of vector j over that surface or

$$i = \oint \vec{J} . \vec{ds} \qquad \ldots(6.8)$$

Where ds is the elemental surface area over which the integral is taken.

Example 1:

Consider a copper conductor of 1 cm diameter in which 100 ampers current is flowing, what is the current density and the drift velocity of electrons?

Solution:

The current density is

$$J = \frac{i}{A} = \frac{100}{\frac{1}{4\pi}(0.01)^2} = 1.27 \times 10^6 \text{ amp/m}^2$$

If there are 8.5×10^{28} electrons/m^3 in copper metal, the drift velocity of electrons is

$$V_d = \frac{J}{Ne} = \frac{1.27 \times 10^6}{8.5 \times 10^{28} \times 1.6 \times 10^{-19}} = 0.93 \times 10^{-5} \text{ m/s}$$

The velocity is therefore quite small.

The average velocity of the free electrons should not be confused with the velocity of propagation of light or in general velocity of propagation of electromagnetic waves in free space which is equal to 3×10^8 m/s

6.3 DRIFT VELOCITY

For many purposes, we can think of the charge carries i.e., electrons in metallic conductors, as small particles of charge. These charges undergo random thermal motion. As a result, in a metal the electrons are moving even in the absence of the external electric field. However, if an electric field exists in the metal, the motion of the electrons will be slightly modulated (or biased) by the force exerted on them by the field. The electrons while undergoing rapid random motion (typically 10 km/sec) due to thermal energy will slowly drift in a particular direction under the action of the external electric field (with a typical speed o approximately 0.1 cm/sec.

This situation is shown in *Fig 6.2* (Here the drift effect is exaggerated to explain the concept.)

Fig 6.2 an impressed electric field causes the electrons to drift with a velocity V_d towards the right. What is the direction of E?

The drift velocity Vd can be computed from the current density J. fig 6.3 Shows the conduction electrons in a

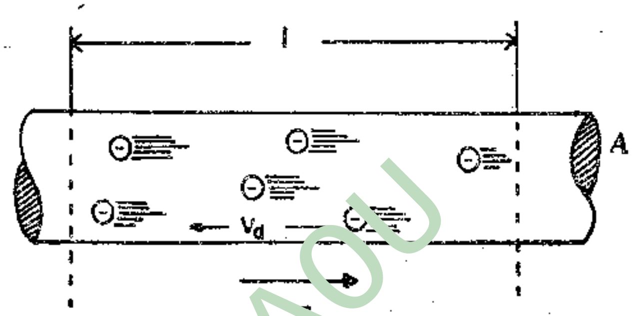


Fig 6.3 Electrons drifting in a direction opposite to the electric field in a conductor

Metallic wire moving the left side of the wire with constant drift velocity V_d and where as the field E is acting toward right. If N is the number of conduction electrons per unit volume the number of electrons in a volume 'A1' is NA1. A change of magnitude

$$q = (NAl)E \qquad \text{...(6.9)}$$

passes through the wire, through its right end in a time 't' and is given by,

$$t = \frac{l}{V_d} \qquad \text{...(6.10)}$$

The current i is given by

$$i = \frac{q}{t} = \frac{NAle}{l/V_d} = N\,A\,eV_d \qquad \text{...(6.11)}$$

Solving for V and by putting J = i /A , Eqn. (6.6) yields

$$V_d = \frac{i}{NAe} = \frac{JA}{NeA} = \frac{J}{Ne} \qquad ...(6.12)$$

Hence the drift velocity is directly proportional to the current density j and inversely proportional the number of electrons (charge carries)

Example 2:

A copper wire of diameter is 0.25 cm is welded to a silver wire with a diameter of 0.16 cm. The composite wire carries a steady current of 5 Amps. What is the current density in each wire?

Solution:

The current distributes uniformly over the cross section of each conductor except at the junction. Hence the current density may be taken as constant for all points in each wire. The cross sectional area of the copper wire is 0.049 cm^2. Thus, from Eqn. (6.6)

$$J_{cu} = \frac{i}{A} = \frac{5Amp}{(0.049cm)^2} = 102\ Amp/cm^2$$

The cross sectional area of the silver wire is 0.2 cm^2

Thus

$$J_{Ag} = \frac{i}{A} = \frac{5Amps}{(0.02cm)^2} = 250\ Amp/cm^2$$

The fact that drift velocity for copper wire of different materials does not enter into consideration here.

Example 3:

What is the drift velocity for copper wire in the problem 2. The current density for copper wire is 102 Amp/cm^2.

Solution:

If we assume each atom is contributing one free electron in the copper metal. The number of atoms per unit volume is given by

$$N = \frac{n_o d}{M}$$

Where n_0 = Avagadro number, d is the density, and M is the weight (o+copper), Hence the number of free electrons per unit volume is

$$N = \frac{n_o d}{M} = \frac{(6 \times 10^{22} atoms/mole)(1 electron/atom)(8.93 gm/cm^3)}{63.45}$$

$$\simeq 8.4 \times 10^{22}\ electrons/cm^2$$

The drift velocity is $V_d = \frac{J}{Ne}$

$(8.4 \times 10^{22} electrons/cm^3)(1.6 \times 10^{-19} Coulelectron)$

$V_d = 0.008\ cm\ s$

It means, the electrons in this copper wire will take 125 sec to drift 1 cm. So the drift speed of electrons should not be confused with the speed at which changes in electric field configuration travel along wires, a speed which approaches the speed of light.

6.4 RESISTANCE

Even in conductors, charges are not perfectly free to move. As shown in Fig 6.2 the charges follow a zigzag path. This path is the result of collisions of charges with the stationary portions of atoms consisting the conductor. During these collisions, as we have discussed earlier, the moving charges lose much of their energy flow acquired as a result of the electric field in the conductor. This lost energy always appears as heat in the conductor. In short, this conversion of electrical energy to heat can be viewed as being due to the frictional force of the moving charges.

If the potential difference of same magnitude V is applied between the ends of a copper rod and of iron rod different currents result. The characteristic of the conductor that enter here is its resistance. We can define the resistance as the ratio of the potential difference v applied between the ends of the conductor to that of the current I flowing through it. It is represented by the sign.

$R = V/i$

To understand the concept of resistance it is customary to compare the flow of charge through a conductor with the flow of water through a pipe, which occurs due to the difference in pressure between the ends of the pipe, established by a pump. This pressure difference can be compared with the potential difference established by a battery between the ends of a resister. The flow of water (let us say cm^3/sec) is compared with the current (coul/s or Amp). The rated of flow of water for a given pressure difference is determined by the nature of the pipe. Is it narrow or wide? Is it short or long? Is it empty or filled with sand or gravel? Etc. These characteristics are analogous to the resistance of a conductor.

6.5 RESISTIVITY (SPECIFIC RESISTANCE) AND ELECTRICAL CONDUCTIVITY

We have seen earlier when current is passing through a conductor it offers resistance to the flow of current. It is analogous to viscous type friction force acting on the moving charges even though the actual force is not such simple. Since the viscous retarding forces are proportional to the speed of the object, one would expect, approximately, the drift velocity Vd of the charge 'q' to be proportional to the electric force tending to make it move, namely Eq. Then we shall write

$$\simeq V_d Eq \qquad ...(6.13)$$

If the charge can move very freely in the conductor, the force Eq. Will give relatively large value of Vd. Then we say in this case the mobility of charge is high. The mobility is defined as the proportionality constants between Vd and E in a conductor and we have then

$$V_d = \mu E \quad \text{...(6.14)}$$

The Eqn. (6.12) for v may however be used here to find the current density in the conductor. We have upon substitution that

$$J = N_q V_d = N_{q\mu} E \quad \text{...(6.15)}$$

The quantity μ Nq is often referred to as conductivity 'σ' of the material. Its reciprocal, is often called the resistivity and is denoted by 'ρ'. Both ρ and σ are the properties of the material alone and do not depend on the size or shape of the conductor. The relation for current density j in terms of resistivity or conductivity becomes.

$$J = \sigma E = \frac{E}{\rho} = \frac{i}{A} V_d \quad \text{...(6.16)}$$

From a practical viewpoint we are more interested in the relation between the current flowing through a conductor and voltage difference between the ends of it. For example, if the electric field I in the conductor (say a write) segment of length 'l' shown in the Fig 6.4 in uniform, the potential difference between the two ends of the wire is

Fig 6.4 if E is uniform $V_k - V_l = E_l$

$$V = V_k - V_l = \int_l^R \vec{E}\cdot d\vec{s} = El \quad \text{..(6.17)}$$

Dropping the vector notation and substituting for E in the Eqn. (6.4) we find,

$$V = \frac{\rho l}{A} i \quad \text{...(6.18)}$$

This relation is more frequently written as

$$V = iR \quad \text{...(6.19)}$$

Where in the case of straight uniform wire, R = $^{\rho}/A$. The quantity R is called the resistance of the wire. The microscopic quantities E, and J are of primary importance when we are concerned with the fundamental behavior of matter.

Example 1:

A cylindrical carbon rod has a diameter 2 cm and a length of 50cm. What is the resistance measured between the two ends? The resitivity of carbon is 3.5 X 10^{-5} Ohms at 20^0C.

Solution: $$R = \frac{(3.5 \times 10^{-5}\ \text{Ohm-m})(0.50\text{m})}{\Pi (0.01\text{m})^2}$$

= 0.055 Ohm.

Eqn. (6.7) can be used to define the resistance of any circuit element. If a voltage difference 'V' exists between its two ends, and if a current 'i' flows through it, the resistance of the element is defined to be

R = V/i

The units of resistance are volts/Coulomb per second or volts per ampere. This unit is called ohm (Ω). The units for resistivity are Ohmmeter. The units for conductivity are Ohms (Ω)/meter

6.6 OHM'S LAW

Eqn. (6.7), V = iR is always true provided a steady current is maintained through a resistance element by a fixed voltage V. The ratio V/i is defined to be the resistance of the element. This equation was first found experimentally by George Simon Ohm (1789-1845). He implied that R is independent of V and i over reasonable ranges of V and i. The relation R = V/i is known as Ohm's law. In general R is not a constant. We have to discard Ohm's idea that R does not vary. Moreover heating of material changes its resistance.

As a result Ohm's law fails. Eqn.(6.7), one major portion of Ohm's law, is always true if V and I can be reproduce. As long as the temperature is kept constant almost all the metallic conductors obey ohm's law. (Fig 6.2) Many conductors don't obey Ohm's law. For

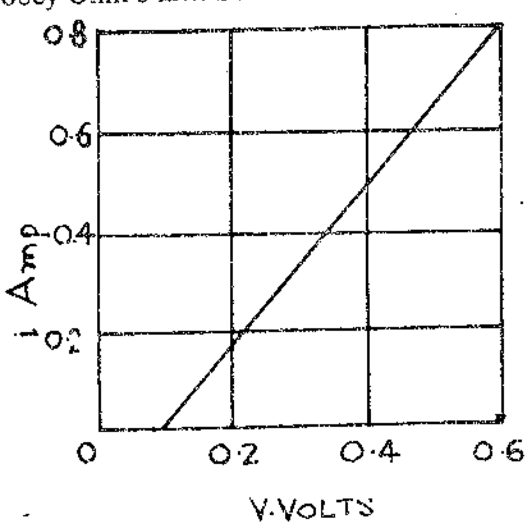


Fig. 6.5. The current variation as a function of potential difference in a copper wire. This obeys Ohm's law

Example Fig (6.6) shows a V-I plot for a vacuum tube. The plot is not straight and the resistance depends on the voltage used to measure it.

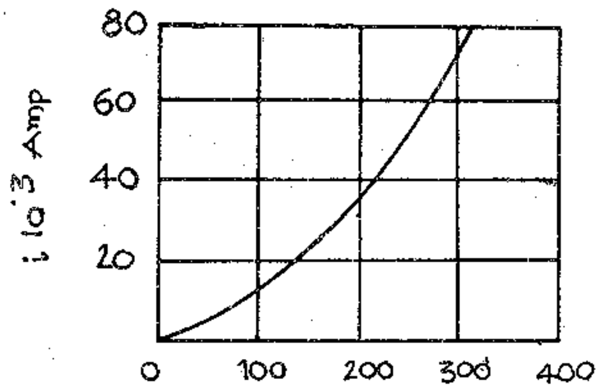


Fig 6.6. The current in a vacuum tube as function of potential difference. This Conductor does not obeys Ohm's law

Fig 6.7 shows a typical V-i plot for another non-Ohmic device, a thermistor, this is a semiconductor, (a class of materials called semiconductors is intermediate between conductors and insulators in their ability to conduct electricity e.g.: Germanium and Silicon) device with a large and negative temperature coefficient of resistivity that varies greatly with temperature. Many conductors such as vacuum tubes and transistors, do not obey ohm's law.

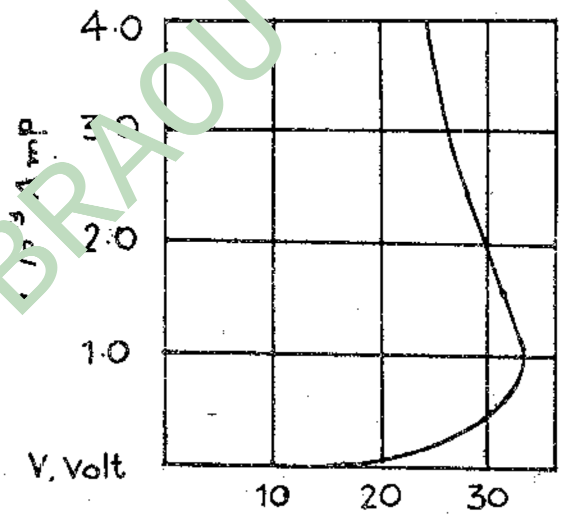


Fig 6.7.A current voltage plot in a typical thermistor. The curve indicates the voltage across the thermistor as the current varies through it. The shape of the curve can be attributes to the large negative temperature coefficient of resistivity of the material used to prepare the thermistor.

6.7 TEMPERATURE COEFFICENT OF RESISTANCE

It is found experimentally that the resistivity of material varies with temperature. For metals the resistance usually increase with increasing temperature. In the case of semiconductors and insulators, however, the resistivity frequently decreases with increasing temperature. Over restricted temperature ranges the following relation is often found to be applicable.

$$\rho = \rho_{ref}\,[1 + \alpha\,(t - t_{ref})] \qquad \text{...(6.20)}$$

$$\text{or } \rho = \rho_{ref}\,[1 + \alpha\,(t - \Delta t)]$$

$$\therefore \Rightarrow \frac{1}{\rho}\frac{\Delta\rho}{\Delta t} = \frac{1}{\rho_{ref}} \bullet \frac{\rho - \rho ref}{t - t_{ref}} \qquad \text{..(6.21)}$$

In this relation p is the resistivity at temperature t, and ρ_{ref} is the resistivity at some reference temperature, t_{ref} α is an experimental constant called the temperature coefficient of resistivity and is given by the relation (6.8a)

The resistivity of copper is 1.7×10^{-8} Ohm-m and that of quartz is 10^{16} Ohm-m. Few physical properties are measurable over such a range of values; Table. 10.1, lists some values of ρ and α for common substances.

Table 6.1 Resistivities and their Temperature coefficients.

Material	*Resistivity (P) at 200C*	*α at 20^oC (per ^oC)*
Silver	1.6×10^{-3}	3.8×10^{-3}
Copper	1.7	3.9
Aluminium	2.8	3.9
Tungsten	5.6	4.5
Nickel	6.8	6.0
Iron	10.0	5.0
Manganin	44.0	1000.0
Graphite (carbon)	3500.0	-0.5
Glass	10^{11}	-
Amber	5×10^{14}	-
Quartz (Fused)	75×10^{16}	-

Check Your Progress

1. The current density in a wire represented by J given interms of..............
2. Drift velocity of an electron is given by
3. Equation for temperature co-efficient of resistance is given by
4. Conductors which do not obey Ohm's law are called

Note:

a. Space is given below for your answers.
b. Compare your answers with those given at the end of the unit.

....

....

....

6.5 RESITIVITY FROM ATOMIC VIEW POINT AND MEAN FREE PATH

In a metal the valance electrons are loosely bound to the atomic nuclei and are free to move within the lattice (array of atoms arranged in straight lines). Theses are called conduction electrons. In silver there is one such electron per atom, the other 46 remaining electrons are bound to the silver nuclei to form ionic cores. In the absence of the electric field the electrons move in random fashion like those of molecules in a gas confined in a container. The electrons 'collide' constantly with the positive ion cores of the conductor, that is, they interact with the lattice, often undergoing sudden changes in velocity and direction. These are very much similar to molecular collisions in a gas confined to a container. So we, can describe the lattice-electron collisions by the concept of mean fee path 'λ' where 'λ' is the average distance that an electron travels between collisions.

In an ideal crystal of metal at O'K, electron lattice collisions should not occur since the lattice is almost frozen at that temperature. According to quantum physics $\lambda \rightarrow 0$ as $T \rightarrow 0$ for ideal crystals. But in real crystals the collision take place because

(i) The positive ion cores at any temperature T are vibrating about their mean equilibrium position in a random way.

(ii) The foreign atoms, that is impurity atoms, may be present

(iii) the crystal may contain lattice imperfections such as missing of rows of atoms, Or displaced atoms etc.

(a) raising its temperature

(b) adding small impurities and

(c) straining it severely, such as by drawing it through a die to increase the number of imperfections.

When such a conductor is conductor is subjected to and electric field, the electrons modify their random motion in such a way that they drift slowly in the opposite direction to the field, with an average drift velocity V_d. The drift speed is much less than the effective average speed V mentioned earlier (see worked example-3 Unit-9). Fig 6.5 shows the relation between the two speeds. The solid lines indicated the possible random path followed by an electron in the absence of the electric field. The electron proceeds from K to L, making six collisin on the way. The dotted show the path of the same electron under the influence of the applied E. One can see from the Fig 6.5., the electron drifts streadily to the right, ending at L rather than at L. Here the drift exhibited by electron is greatly exagerated to understand the concept.

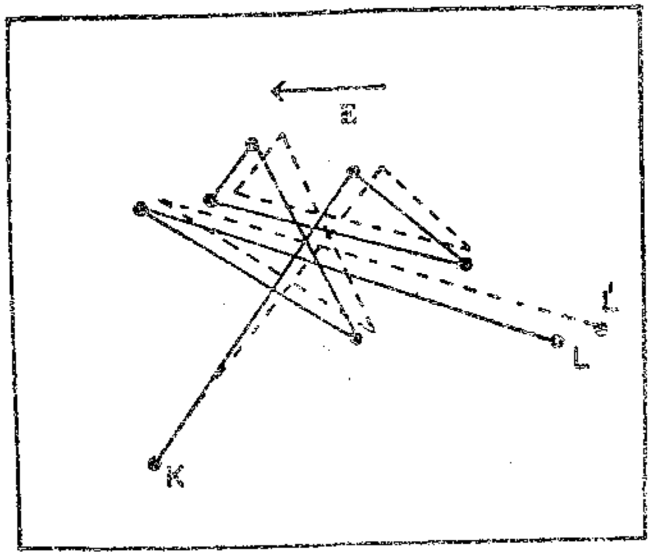


Fig 6.8 the solid lines indicate an electron moving from K to L making six collisions. The dashed line show what could be the electron path in the presence of an external field E. Note the steady drift in a direction opposite to E.

The drift velocity can be obtained in term of the applied electric field E and v and λ. When a field in applied to the electron in the metal it will experience a force eE which will impart to it an acceleration a given by 2nd law of Newton.

$$a = \frac{\vec{F}}{m} = \frac{e\vec{E}}{m} \qquad \ldots(6.22)$$

Let us consider that an electron has collided with one positive ion core.

Naturally at its next collision the electron's velocity will have changed the average by a (1/v), where (1/v) is mean time t taken between collisions. The drift speed Vd, is

$$V_d = a\left[\frac{\lambda}{v}\right] = \frac{E_e}{m}\,\frac{\lambda}{v}, \quad T = \frac{\lambda}{v} \qquad \ldots(6.23)$$

We may write Vi in terms of the current density J [(Eqn. (6.22)] and combining Eqn. (6.10) to get.

$$\vec{V}_d = \frac{\vec{J}}{Ne} = \frac{Ee}{mv}\lambda$$

Combining this with Eqn. (6.16) (E/J = ρ) leads finally to

$$\rho = \frac{mv}{Ne^2\lambda} \qquad \ldots(6.24)$$

This equation can be taken as a statement that metals obey Ohm's law if we can show that v and A do not depend on E. In this case ρ will not depend on the applied electric field E, which is the criterion for a material to obey Ohm's law. The quantities λ and v depend mainly on the velocity distribution of conduction electrons. v is of the order of 10^{-8} cm/see and V_d is of the order 10^{-2} to 10^{-3} cm/see. The ratio is approximate 10^{10}. Hence for all practical purposes the right hand side of Eqn. (6.2) is independent of E and the material obeys Ohm's Law.

Worked Example 2:

What are (a) mean time r between the collisions and (b) the mean free electrons in silver?

Solution:

From Eqn. (6.24) we have

$$a)\ \tau = \frac{\lambda}{v} = \frac{m}{Ne^2\rho}$$

$$= \frac{(9.1 \times 10^{-31} kg)}{(6 \times 10^{28}/m^3)(1.6 \times 10^{-19} Coul)^2 (1.6 \times 10^{-8} Ohm\text{-}m)}$$

$$= 3.7 \times 10^{-14}\ sec.$$

b) The mean free path is $\lambda\ \tau v$

$$\lambda = \tau v = (3.7 \times 10^{-14} S)(1.6 \times 10^{8} cm/s) = 5.9 \times 10^{-6} cm$$

Hydrostatic pressure applied to a metal modifies its resistivity very slightly. The resistivity of bismuth increases if it is placed in a magnetic field and this change in resistivity, can he made use of, for the measurement of magnetic fields. The resistivity of the material selenium decreases when the metal is illuminated with light. The light liberates photo-electrons within that rendering it a better conductor.

6.9 JOULE'S LAW

For simplicity or convenience we have been talking of the current in a conductor as if all the free electrons are moving with the same constant velocity. But strictly speaking the electronic motion may be regarded as a series of acceleration each of which is terminated as collision with one of the fixed ion cores of the conductor. These free electronics gain kinetic energy in the free paths between the collisions and in turn give up the same amount of energy to the positive ion cores which they have gained through each collision. Hence the positive ion cores acquire the energy and increase their amplitude of vibration about their mean position. In other words this would appear as thermal energy or heat.

In order to arrive at an expression for the rate of development of heat we have to workout a general expression for the power input to any portion of a circuit. Let us imagine, as shown in Fig 6.9, a rectangle represent a portion of a circuit element in which there is a convention current 'i' from left to the right. $V\alpha$ and $V\beta$ are the potentials at terminals α and β respectively. The element between the terminals may be a conductor, or generator, battery or a motor

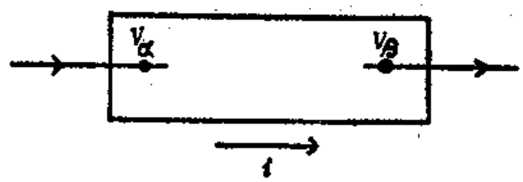


Fig 6.9 Transfer of charge in a portion of the circuit

In a time interval 'dt' a quantity of charge dp= idt enters the portion of the circuit under consideration at terminal α and in the same time an equal quantity of charge leaves the terminal β. Thus there is transfer of charge up from a potential V_α to a potential V_β. The energy W given p by the charge is

$$dW = dq\,(V_\alpha - V_\beta) = idtV_{\alpha\beta} \qquad \ldots(6.25)$$

The rate at which energy is given up is given up, or power input

$$P = \frac{dw}{dt} = iV_{\alpha\beta} \qquad \ldots(6.26)$$

So the power input only depends on the magnitude and relative directions of currents and terminal potential difference. The power input is equal to the product of the current and the potential difference. If it is in amps or Coul/s, and the potential difference is in volts or Joules/Coul, the power is in Joules/s or Watts, since

$$\frac{\text{Coul}}{\text{S}} \times \frac{\text{Joules}}{\text{Coul}} = \text{Joules /s} = \text{Watts}$$

Eqn.(6.25)is a general relation and holds good for many circuit element between α and β.

In a special case in which, the circuit element between α and β is a pure resistance, R all of the energy supplied is converted into heat and in this case the potential difference. $V_{\alpha\beta}$ is given by

$$V_{\alpha\beta} = iR \qquad \ldots(6.27)$$

Hence $P = iV_{\alpha\beta} = i \times iR = i^2R$

$$P = i^2R \qquad \ldots(6.28)$$

$$\text{or } P = \frac{V^2\alpha\beta}{R} \text{ (Since} V\alpha\beta = iR) \qquad \ldots(6.29)$$

Here in this case we may set

$P = \frac{dH}{dt}$, where dH is the heat developed in time dt,

$$\frac{dH}{dt} = i^2R \qquad ..(6.30)$$

If the conductor is linear and the resistance R is independent of the current 'i' then Eqn. (6.29) states that 'the rate of development of heat is directly proportional to the square of the current'. This fact was discovered experimentally by Joule and hence it is known as' Joule's law'. A material which obeys Ohms law in variably obeys Joule's law also; the resistance in Eqn. (6.30) is expressed in Joules/s or Watts. This can be converted into cal/s using the relation, 1 cal = 4.186 Joules.

One should notice that the rate of development of heat in a conductor is not the same thing as the rate of increase of temperature of the conductor. The latter dependents on the heat capacity of the conductor and the rate at which heat can escape by conduction; convection and radiation. Generally the rate of loss of heat increases as the temperature of the conductor increases. The temperature of the conductor raises till the rate of loss of heat equals to the rate of development of heat. Then the temperature remains constant. Thus when the circuit is closed with an incandescent lamp, the temperature of the filament raises rapidly until the rate of heat loss due to radiation equals to the rate of development of heat. i.e. Ri^2. On the contrary, a fuse wire is constructed in such a way that when the current in it exceeds a certain prescribed value, the fuse melts before its final equilibrium temperature is reached.

Example 1:

You are given a 1-meter length of heating wire made of an alloy Nichrome; it has a resistance of 48 Ohms. Can your get more heat by cutting the wire into two pieces and winding two separate coils? In each case the coils are to be connected individually across a 220 volt/line.

Solution:

The power P for the single coil of resistance 48 Ω is given by Eqn. $P = V^2 R$

$$P = \frac{V^2}{R} = \frac{(220)^2}{48} = 1008 \text{ Watts}$$

The power for a coil of half the length is given by

$$P^1 = \frac{(220)^2}{24} = 2006$$

This would seem to suggest that we could buy a 1000 Watt heating coil cut into half, and rewind it to obtain 4000 Watts. Why this is not a practical idea?

Example 2:

A current of 0.25 Amp flow!; through a 2002 resistor. How much power is lost in the resistor?

Solution:

Applying Eqn. i.e, $P = iV = i^2R$

$P = (0.25)^2 \times 200 = 12.5$ Watts.

So, in this case 12.5 J of energy is lost each second and hence 12.514 .185 or about 3 cal of heat is generated each second.

Example 3:

A bulb rated 220V/100 W is operated from a 220 V power source. Find the current flowing through it and its resistance.

Solution:

$P = Vi$

$$i = \frac{p}{V} = \frac{100W}{220V} = 0.45 \text{ Amps}$$

Since the potential drop across the bulb is 220V and the current is 0.45 amps. Ohm's law tells us

$$R = \frac{V}{i} = \frac{220}{0.45} = 484\,\Omega$$

6.9 SUMMARY

In a good conductor there are number of free charges. These charges move under the influence in an external electric field. Current per unit area is called the charge density. Charge density is proportional to the magnitude of the charge (q), number of charges carries (N), and the average drift velocity.

Conductivity of a conductor depends on the mobility of the charge carriers & the resistivity and conductivity are the intrinsic properties of the material and depends on the temperature but not on the size or shape of the conductor. At a given temperature the Potential difference between the ends of conductor is directly proportional to the current flowing through it. It is known as Ohms Law. Vacuum tubes gas tubes, semiconductors and thermistors and thyristors do not obey Ohm's Law. These are called no-Ohmic type of conductors.

Electrons are the carriers of current. Moving charges constitute the current. While moving electrons colliode with positive ion cores. Which results in the transfer of their

Kinetic energy to the ion cores. This increase in energy appears as heat. The rate of amount of heat developed in a linear conductor is directly proportional to the square of the current passing through it. The unit of electrical power is a watt.

6.10. Check your progress: Answers

1. $J = i \mid A$

2. $V_A = J \mid Ne$

3. $$\alpha = \frac{e_2 - e_1}{(e_1 t_2 - e_2 t_1)}$$

4. Conductors which do not obey Ohms Law are called non-ohmic type of resistors.

6.11 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Deduce the expression for the resistivity of a conductor. What are the factors that govern the resistivity of a conductor?
2. Derive an expression for the drift velocity V_d electrons in a conductor.
3. State the laws of the development of heat when an electrical current flows through a wire of uniform material.
4. Show that in a linear conductor the rate of development of heat is directly proportional to the square of the current passing through that conductor.

5. State and explain Ohm's law. What are non-Ohmic resistors and give some examples.
6. Derive an expression connecting the mean free path of an electron and resistivity of a c1 conductor. What is meant by mean time.
7. Discuss the similarities between electrical conductivity and thermal conductivity. Try to obtain an expression relating to the thermal conductivity 'K' and electrical conductivity σ

   Show that K = 3 big (K)T, where K is Boltzmann constant, c the electric charge and T and the absolute temperature. What is the electric charge and T and absolute temperature. What is this law?
8. Derive the expressions for drift velocity and mobility in a conductor.
9. Explain what is drift velocity? Distinguish between the terms drift velocity and electromagnetic wave velocity.
10. Derive an expression for the drift velocity v_d of electrons as a conductor.

II. Answer the following questions briefly

1. Derive the expression for electrical power in a conductor.
2. Basing on the conductivity scale can you classify the elements? If so what are they? Give some examples in each case.
3. Can you imagine a phenomenon at very low temperature (near0^0, K) the resistance of a metallic conductor becomes almost zero? What is that phenomenon? Write a brief note on this phenomenon.
4. What is semiconductor? Explain why the resistance decreases with increasing temperature in the case of a semiconductor.
5. What is magneto resistance? Give some examples.
6. Distinguish clearly between the electron flow and the conventional current.
7. What is wrong with the following statement: 'the resistivity f a material is directly proportional to its length'? Rectify the statement and explain.
8. Distinguish between current and current density.
9. What is the difference between electromotive force and potential difference? Are they same? What are the units?
10. Explain the concept of resistance of flow of current in a conductor.

III. Solve the following problems.

1. A silver wire has a radius of 1.0 mm and it carries 2 amps current. Find the current density n the wire

   (Ans:6.4 X 10A/m^2)

2. What voltage difference is required to sent a current of 2 amps through 50 cms of wire in the above example?
   (Hint: Use the expression R = A/1) (Ans: 5.4 X 10^{-3}V)

3. Calculate the drift speed in the problem-1

   (Ans:Vd = 7 X 10^{-5}m/sec)

4. A current of 5 amps exists in a 10 Ohm resistor for 4 min. (i) How many Coulombs and (ii) how many electrons pass through any cross-section of the resister in this case?

   (Ans:1200 Coul; 7.5 X 10^{21} electrons)

5. A square aluminium rod is edge.1.0 meter long and 5.0 mm on edge

   (a) What is its resistance between the ends?

(b) (b) What must be the diameter of a circular copper rod of 1.0 meter long if-its resistance is to he the same?

(Ans: $1.12 \times 10^{-3}\Omega$, 2.2mm)

6. Number 12 gauge copper wire can carry maximum current to about 30 amperes before heating. Its diameter is 0.26cms. Find the resistance of a 1-meter length of the wire. How large voltage drop occurs along it over one meter when it carries a current of 30 amps.

(Ans: $3.2 \times 10^{-3}\Omega$; 0.096V)

7. A piece of wire is 10 meters long ad has a diameter of 0.2 cm. When its two ends are connected to a battery of 1.5 V, a current 0.70 amperes flows through it. Find its resistance and resistivity of the material from which it is made.

(Ans: 2.14Ω; 6.7×10^{-7} Ohm-meter)

8. Heat is developed in a resistor at a rate of 100 watts when the current is 3.0 amps. What is the resistance in Ohms.

(Ans: 11Ω)

9. A nichrome wire heater dissipates 500 Watts when the applied potential difference is 110V and the wire temperature is 800°C. How much power would it dissipate if the wire temperature were held at 20.0^0C by immersing in a bath of cooling oil? The applied potential difference remains the same. α For nichrome is $4 \times {}^{-4}$/C.

10. Am electric toaster is rated 900W/120V. How much current does the toaster draw? What is its resistance when it is in operation?

(Ans: 7.5 amps; 16Ω)

11. A rod certain material is 1.00 meter long and 0.55 cm in diameter. The resistance between the ends (at 20^0C) is 2.81×10^{-3} Ohm. A round disc is forced of this same material, 2 cm in diameter and 1.00 mm thick (i) what is resistance between the two opposing round faces? (ii) What is the material?

(Ans: 2.2 X 10Ω; Nickel)

12. At what temperature would be resistance of aluminium conductor be double its resistance at 0^0C. Does this temperature hold for all aluminum conductors irrespective of their shape or size.

(Ans: 260^0C. Yes)

13. A square of a aluminum rod is 1.0 m long and 5.0 mm on edge (i) what is its ends? (ii) What must be the diameter 1.0 m silver rod if its resistance is to be the same.

(Ans: $1.12 \times 10^{-3}\Omega$; 213mm)

14. The resistance of number 2^1 gauge manganin wire is 1.73 Ohms/meter at 20^0C. Evaluate its resistance at 0^0c

15. Evaluate the mean free path for free electron in Nickel.

[Hint: Use in = $\dfrac{\text{Avagadro Number x density}}{\text{Atomic Weight}} = \dfrac{6.0 \times 10^{23} \times 8.9}{59}$]

6.12 GLOSSARY

1. Incandescent light produced by glowing filament.
2. Statical machine A machine that produces high potential using static electricity principle.
3. Thermoelectricity Thermoelectric effects involve conversion of heat energy in to electrical energy, Example: Thermocouple
4. Thermistor thermally sensitive resistor, in which the resistance decreases with increasing temperature.

UNIT-7: KIRCHOFFS'S LAWS

Contents

7.1 OBJECTIVES

This unit explains the concept of an emf (electro motive force) and Kirchoff's Laws. To make you to understand them the unit explains the laws by applying them to single and multiple loop circuits.

After going through this unit you will be able to

1. Make out what an emf is;

2. That at any junction in a circuit the algebraic sum of current is zero; and that the algebraic sum of the changes in the potential encountered in a complete traversal of the circuit will be zero.

7.2 INTRODUCTION

In this unit we will discuss the Kirchoff's Laws and application of these laws to different circuits.

7.3 ELECTROMOTIVE FORCE

In certain devices like batteries and generators there exists an ability to maintain a potential difference between two points to which they are attached. Such devices are called seats of Electromotive Force. In a short form they are known as 'seats of emf Fig 7.1 (a) shows a seat of emf E represented by a battery connected to a resistor R. The seat of emf maintains its upper terminal positive (+) and its lower terminal negative (-). The function of a source if emf in an electric circuit is very much similar to the function of water pump maintaining the continuous flow of water through a system of pipes as shown in Fig. 7.1 (b). The source of emf must do certain amount of work on each unit of charge, which passes through it in order to raise it to a higher potential. The work must be supplied at the rate at which energy is lost in flowing through the circuit.

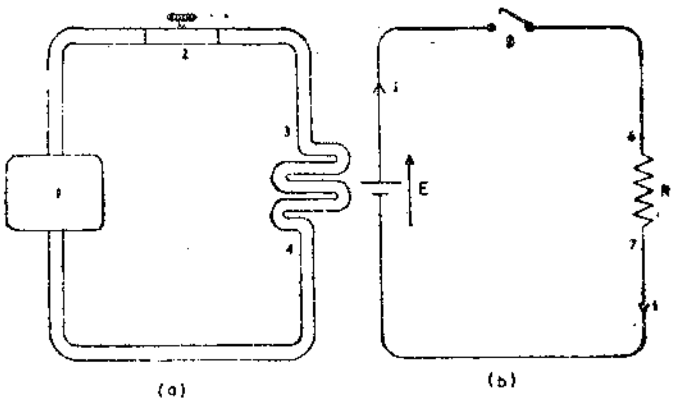


Fig 7.1 (a) Mechanical analogy of a water pump
7.1 (b) Sources of emf in an electrical circuit.

1. Water pump 2. Valve 3. High-pressure 4.
Low pressure 5. Switch 6. High potential 7. Low potential.

By convention, we assume that the current consists of a flow of positive charge even though in most cases it is negative electrons. Hence the charge loses energy in passing through the resistor from a higher potential to a lower potential. In the analogy of water pump, the water flows from high pressure to low pressure. When the shut off valve is closed, pressure exists but no water flows. Similarly when the electrical switch is open there is voltage but no current. Since the emf is the work done on the unit charge. It is expressed in the same unit as potential difference i.e., Joule per second or volt. If dq is the charge crossing a section through the source of emf in time dt and dw is energy transformed in this time then the emf is

$$E = dw/dq. \quad ...(7.1)$$

Therefore the work done by the source in time dt is

$$dW = E\,dq. \quad ...(7.2)$$

And the rate of work done or the power is

$$P = \frac{dw}{dt} = E\frac{dq}{dt} = Ei. \quad ...(7.3)$$

" A source of emf of one volt will perform one Joules of work on each Coulomb of charge which passes through it"

For example, a 6v battery performs 6 Joules of work on each Coulomb of charge, which passes transferred from the low –potential side (-terminal) to the high-potential side (+terminal). An arrow (→) is usually drawn by the side of the symbol E an emf, to indicate the direction in which the source, would cause positive charge to move through the external circuit. The conventional current is directed away from the positive terminal of a battery, and the hypothetical positive charge flows down hill through external resistance to the negative terminal of the battery.

7.4 KIRCHOFF'S LAWS

An electrical network is a complex circuit consisting of number of current loops or meshes. For complex networks containing several loops and a number of sources of emf, the application of Ohm's law becomes very difficult. A easiest way of analyzing such circuits was developed in the 19^{th} Century by Gustav Kirchoff, a German scientist. His method involves two laws namely Kirchoff's laws.

Kirchoff's First Law states that ' the sum of currents entering a junction is equal to the sum of current leaving that junction or at any junction the algebraic sum of currents must be zero.

$$\sum I \text{ entering} = \sum I \text{ leaving.} \qquad \ldots(7.4)$$

Kirchoff's Second Law states that 'the sum of the emfs around any closed loop of current is equal to the sum of all the voltage drops across, the impedance's in that close circuit

$$E = iR \qquad \ldots(7.5)$$

A junction is a point where three or more wires meet together

7.5 ILLUSTRATION OF KIRCHOFF'S LAW TO A SINGLE LOOP CIRCUIT

Fig.7.2 shows a circuit, which stresses that all seats of emf have an intrinsic internal resistance 'r'. This resistance cannot be removed, though we wish to do so, because it is an inherent part of the device. The emf and internal resistance are shown separately in the figure. Applying Kirchoff's 2^{nd} rule, which is other wise known as loop theorem

$$E = ir + iR$$

$$\text{or } E - ir - iR = 0 \quad \& \; E = I(r+R)$$

$$i = \frac{E}{r + R} \qquad ..(7.6)$$

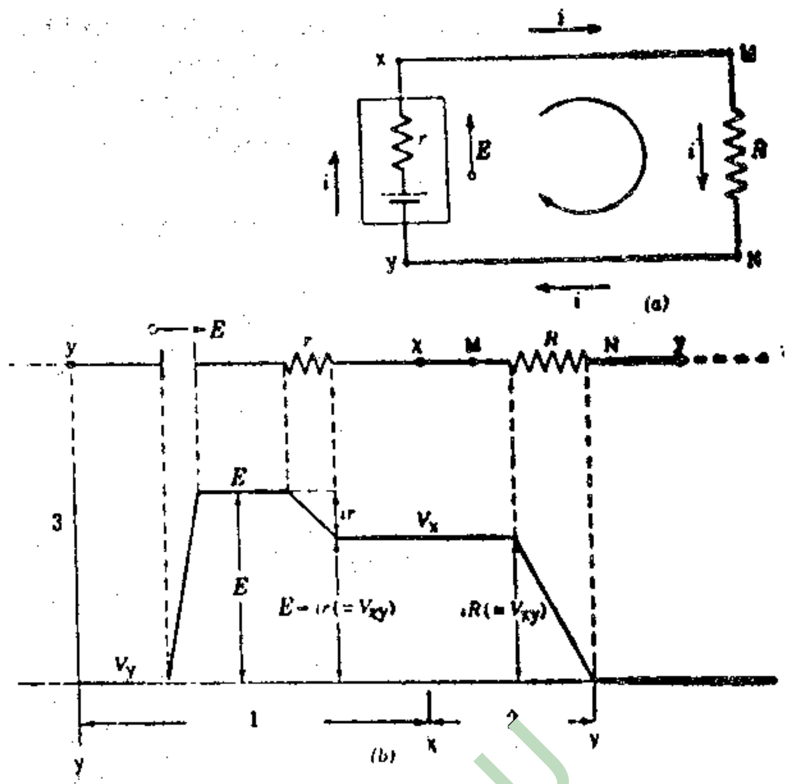


Fig 7.2 (a) A single loop circuit. The rectangular block is a seat of emf E with an internal resistance r.

(a) The same circuit is drawn for convenience as straight line. Directly below are shown the potential changes that one comes across in traversing the circuit clockwise. (1) Seat of emf (2) Eternal resistor (3) Potential, Volts

Example 1:

When four resistance are connected between x and y in series so that there is only one conducting path through all these resistance as shown in fig 7.3 what is the effective resistance R of all these resistances?

Solution:

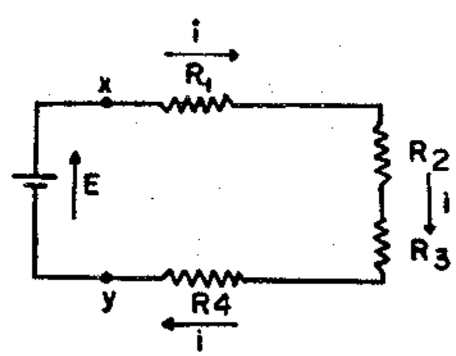


Fig 7.3 Four resistances are connected in series between the terminals x and y

The effective current flowing through the circuit is the same; hence if we apply the loop theorem to the problem (Fig 7.3) it yields

$$-iR_1 - iR_2 - iR_3 - iR_3 - E = 0$$

$$\text{or } E = (R_1 + R_2 + R_3 + R_4)\ ; \quad \text{but } \frac{E}{i} = R_{eff}$$

$$E/i = R_1 + R_2 + R_3 + R_4$$
$$R_{eff} = R_1 + R_2\, R_3 + R_4$$

Potential Difference

Usually we evaluate the Potential difference between two points in a circuit. Now let us find out the relation between the Potential difference $V_{MN} = (V_M - V_N)$ between the points M and N, and the circuit Parameters like, R and r. In order to arrive at this we have to start from the Point X and traverse (see Fig 7.2) the circuit to the point passing through the point M, R (the resistor) and N against the current. If V_M and V_N are the Potentials at points M and N respectively we have.

$$VM = VN + iR$$

We can write this as $V_M - V_N = iR$

$$V_{MN} = V_M - V_N = iR. \qquad \ldots(7.7)$$

From Eqn. (7.7) we can infer that the potential difference V_{MN} between the point M and N has the magnitude iR. Point M is more positive than point N. Combining the Eqn. (7.6) and (7.7) we get

$$V_{MN} = iR = \left(\frac{E}{r+R}\right) R \text{ or } \left(\frac{R}{r+R}\right) E \qquad \ldots\ldots(7.8)$$

So, the potential difference between any two points in a circuit is the algebric sum of potential changes encountered in traversing the circuit from one point to the other.

Example 2:

A circuit is connected as shown in Fig (7.4). Let E_1 and E_2 be 4 volts and 8 volts respectively and the resistance r_1, r_2 and R be 2 ohms 4 Ohms, and 10 Ohms respectively. What is the current? What are the potential differences between (a) points x and y and (b) between points z and x?

Solution:

The emfs E_1 and E_2 opposes each other. Since E2 is larger it controls the direction of current. Thus I will be counter clockwise. Starting from point x going in the counter clockwise direction the loop theorem yields.

$$-E_2 + ir_1 + iR + ir_2 + E_1 = 0$$

Solving for Yields

$$i = \frac{E_2 - E_1}{R + r_1 + r_2}$$

$$i = \frac{(8 - 4)\ \text{volts}}{(10 + 2 + 4)\ \text{Ohms}} = 4/16 = 0.25\ \text{Amp}$$

(a) The potential difference between x and y can be obtained by starting at y and traversing the circuits to x

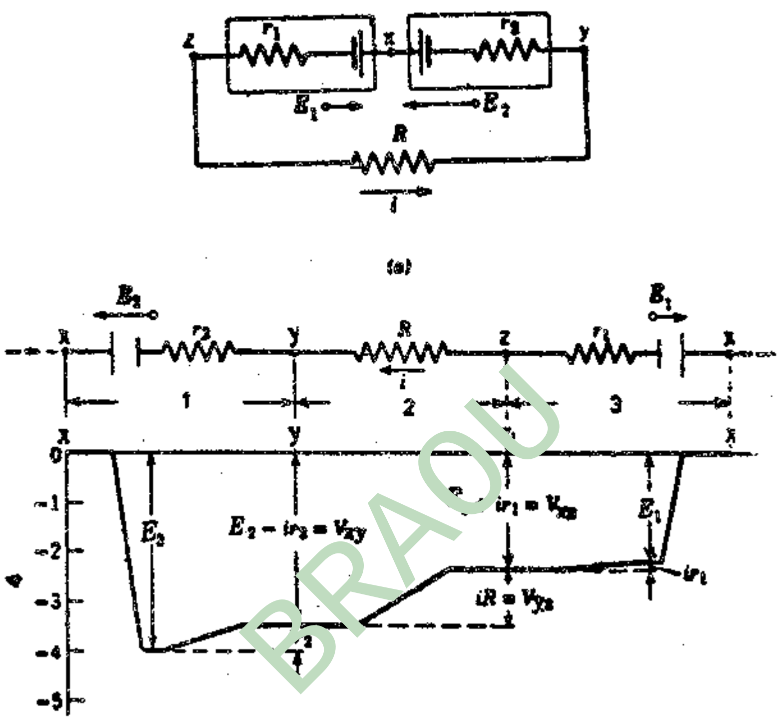


Fig 7.4 a) Single loop unit

a) The same circuit for convenience is schematically shown as a straight line.
The potential differences encountered in traversing the circuit clockwise direction from Point x being represented directly below. In the lower figure the potential at point z was assumed to be zero.

1.Seat of emf 2, 2. External resister, 3. Seat of emf 1,
4. Potential volts. $E_1 = 4V$, $E_2 = 8V$, $r_1 = 2\Omega$, $r_2 = 4\Omega$,

$V_{xy} = (V_x - V_y) = ir_2 - E_2 + E_2 = E_2 = ir_2$

$= 8$ *volts* $-$ *(0.25 Amp) (4 ohm)*

$= 7.0$ *Volt.*

The x point is more positive than y and the potential difference (7.0 volts) is less than the emf (8.0 volts) . See Fig 7.4 (b)

(b)For points z and x we start at z and traverse the circuit to x

$$V_{xy} = (V_x - V_y) = E_1 + ir_1 = 4\ volt + (0.25\ Amp)\ (2\ ohm)$$

$$= 4.5\ Volt.$$

This tells us that x is at a higher potential than z. The terminal potential difference of E1 (4.5 Volts) is larger than the emf (4.0Volt) See Fig 7.4 (b)

Now let us test the first result by proceeding from y to x along a different path, namely through R, ri and E1. We have now

$$V_{xy} = iR + ir_1 + E_1$$

$$= (0.25\ \text{Amp})\ (10\ \text{ohm}) + (0.25\ \text{Amp})\ (2.0\ \text{ohm}) + 4.0\ \text{Volt}$$

$$= 2.5\ \text{Volt} + 0.50 + \text{Volt} + 4.0\ \text{Volt}$$

$$= 7.0\ \text{Volt}.$$

Which is the same as the earlier result.

7.6 ILLUSTRATION OF KIRCHOFF'S LAWS TO MULTIPLE LOOP CIRCUIT

Let us consider a loop as shown in Fig. 7.5 There are two junction points in the circuit. They are C and D and there are three branches namely CAD the left branch, the central branch CD and the right branch CDB.

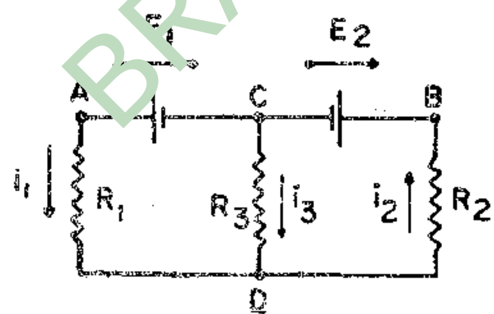


Fig 7.5 A multiple loop circuit

The currents are designated as i_1, i_2 and i_3 respectively. These are flowing through the resistance R_1, R_2 and R_3 respectively. The currents i_1, i_2, and i_3 carry current to the junctions D are away from it. Suppose if we arbitrarily choose the current coming towards the junction as positive and the one leaving the junction as positive and the one leaving the junction as negative, then

$$i_1 + i_3 - i_2 = 0 \qquad ...(7.9)$$

Since the algebric sum of currents at any junction must be zero according to Kirchoff's first law.

Applying the loop theorem to various loops. We can solve this, If we traverse the left loop of Fig 7.5 in a counter clock wise direction, the loop theorem gives

$$E_1 + i_3R_3 - i_1R_1 = 0 \qquad \ldots(7.10)$$

The right loop gives

$$-E_2 - i_2R_2 - i_3R_3 = 0 \qquad \ldots(7.11)$$

using the three equations and solving for i_1, i_2 and i_3 we get

$$i_1 = \frac{(R_1 + R_2)\ E_1 - E_2E_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \ldots(7.12)$$

$$i_2 = \frac{E_1R_3 - E_2\ (R_1 + R_3)}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \ldots(7.13)$$

$$i_3 = \frac{-E_1R_2 - E_2E_1}{R_1R_2 + R_2R_3 + R_3R_1} \qquad \ldots(7.14)$$

Suppose if R_3 is infinite then

$$i_1 = i_2 = \frac{E_1 - E_2}{R_1 + R_2} \quad \text{and } i_3 = 0$$

When the loop theorem is applied to the entire loop ACBDA of Fig 7.5 the loop theorem yields.

$$i_1R_1 - i_2R_2 - E_2 + E_1 = 0 \qquad \ldots(7.15)$$

Which is nothing more than the sum of Eqs. (7.10) and (7.11)

Check your progress:

1. The two laws of Kirchoff's (a) Law of currents (b) Law of emf's are stated as..........
2. Kirchoff's laws are applied in ....

Note: a. Space is given below for your answers.
b. Compare your answers with those given at the end of the unit..........

....
....
....
....

Examples –3:

When four resistances R_1,R_2, R_3 and R_4 are connected against the same seat of emf and parallel what is effective resistance combination.

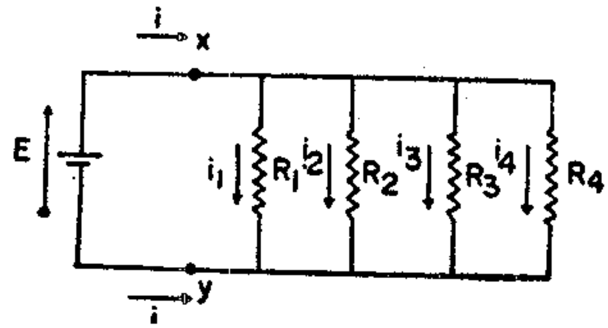


Fig 7.6 Four resistances are connected in parallel between the points z and y

Solution:

The currents in the four branches are

$i_1 = \frac{E}{R_1}$; $i_2 = \frac{E}{R_2}$; $i_3 = \frac{E}{R_3}$ = and $i_4 = \frac{E}{R_4}$

Where E is the potential difference between the points x and y

Applying the junction theorem at x we get

$$i = i_1 + i_2 + i_3 + i_4 = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

But I= E/R, Combining these two we get

$$R_{eff} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Example-4: Solve for the unknown current in Fig 7.7 using Kirchoff's laws.

Solution:

Current directions are assumed as indicated in the figure for i_1, i_2, i_3. If we apply Kirchoff's law to the function m, we obtain

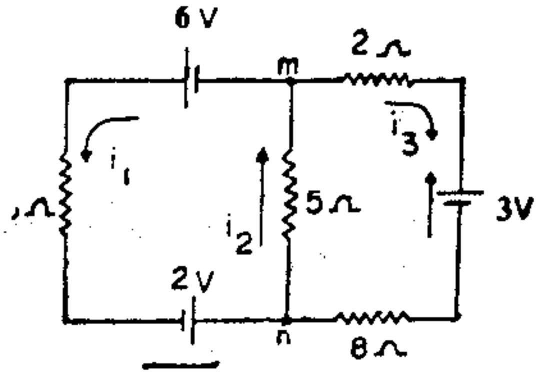


Fig 7.7

$\Sigma i(\text{entering}) = \Sigma\ i\ (\text{leaving})$

$i_2 = i_1 + i_3$

Starting from 'm' and tracing counter –clock wise around the left loop, we write law voltage equation.

$E = iR$

$6V + 2V = i_1\ (3\Omega) + i_2\ (5\Omega).$ ...(I)

$8V = (3\Omega)i_1 + (5\Omega)\ i_2$

Dividing throughout by 1 Ω and transposing we get

$3i_1 + 5i_2 = 8$ Amps. $(1V/\Omega = 1A)$ .. (II)

Another voltage equation can be written by starting from 'm' and tracing clockwise around the right loop

$-3V = i_3\ (\Omega) + i_2\ (5\Omega) + i_3\ (8\Omega).$

The negative sign arises from the fact that the output of the source opposes the racing direction Simplifying we have

$2i_3 + 8i_3 + 5i_2 = -3A$

$10i_3 + 5i_2 \quad = -3A$ ...(III)

The three simultaneous equations which must be solved for i_1, i_2, and i_3 are

$i_1 + i_2 + i_3 = 0$ ...(I)

$3i_1 + 5i_2 = 8A$ ...(II)

$10i_3 + 5i_2 = -3A.$ ...(III)

From the Eqn. (I) we have

$i_1 = i_2 - i_3$

Which when substituted into Eqn. (II), Yields

$$3(i_2 - i_3) + 5i_2 = 8A$$

$$3i_2 - 3i_3 + 5i_2 = 8A$$

$$8i_2 - 3i_3 = 8A \qquad ...(IV)$$

$$5i_2 + 10i_3 = -3A \qquad ...(III)$$

Solving for i_2 and i_3 wet get

$i_2 = 0.75$ and $i_3 = -0.67A$

Subtracting in Eqn. (II) we get $i_1 = 1.42$ A

$i_1 = 1.42A$. $i_2 = 0.75A$, $i_3 = 0.67A$

The negative value obtained for i_3 indicates that our assumed current direction was incorrect. Actually the current flows opposite to the assumed direction. As a check on the above results, we can write one more voltage equation by applying Kirchoff's second law to outside loop. Starting from 'm' and tracing in counter-clock wise direction we obtain.

$$6V + 2V\ 3V = i_1 (3\Omega) - i_3 (8\Omega) - i_3 (2\Omega)$$

$$\frac{11V}{\Omega} = 3i_1 - 10i_3$$

$$11A = 3i_1 - 10i_3$$

Substituting for i_1 and i_3 we obtain

$$4.3A - (-6.7A\ 11A = 11A$$

$$11\ A = 11\ A$$

Note that again the negative value for i_3 was used in the mathematics even though it indicates an incorrect assumption

7.7. SUMMARY

Certain devices like generator, motor, and battery have the capacity to maintain a potent difference between two points to which they are connected. These are called sources of emf.

In an electrical circuit at any junction the algebraic sum of currents is zero and the algebraic sum of the charges in the potential encountered in a complete traversal is zero. These two are known as Kirchoff's Laws.

Check your progress: Answers

1. (a) First law states that the sum of currents entering a junction is equal to the sum of currents leaving that junction or at any junction the algebraic currents must be zero.

   Σ^{1} I entering = Σ^{1} leaving

   (b) Second law states that the sum of emf's around any loop of current is equal to the sum of all the voltage drop across the impedances in that closed circuit.

2. Kirchoff's laws are applied in multi loop circuits.

7.8 SAMPLE EXAMINATION QUESTIONS

I Answer the following quotations in detail

1. Define Kirchoff's laws. Apply them to a single loop circuit and derive the expressions for current and potential difference.
2. Apply the Kirchoff's laws to a multiple loop circuit and obtain the expressions for the potential difference and current.

II Answer the following questions briefly.

1. What is electromotive force? What is internal resistance of a source of emf.
2. Discuss the meaning of Kirchoff's laws in terms of the conversion laws.
3. Why would one connect two batteries in series? In Parallel? Why should unlike batteries never be connected in parallel?
4. Distinguish between terminal potential difference and emf.
5. In an electric circuit, it is desired to decrease the effective resistance by adding resistors. Should these resistors be connected in parallel or series? Why?
6. Defend the following statement; the effective resistance of a group of resistors connected in parallel will be less than any of the individual resistances.

III.Solve the following problems.

1. Solve for the unknown currents shown in the following figure 1 using Kirchoff's laws.

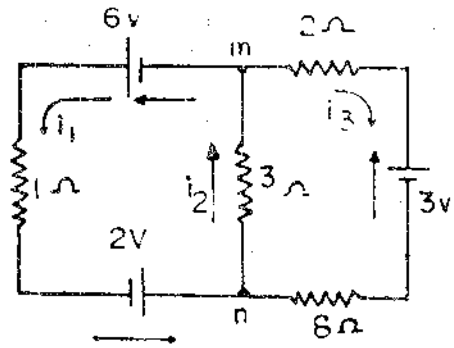


Fig 1

2. An 18Ω resistor R_1 and a 9Ω resistor R_2 are connected in parallel and then series with a 24 Volt battery. What is the effective resistance for each connection? Which combination draws more battery current?

(Ans; 6Ω, 27Ω, Parallel)

3. Given three resistors R_1 100Ω, R_2= 150Ω, and R_3 = 80n, find their equivalent resistance when connected in (i) series (ii) in parallel

(Ans; (i) 330Ω; (ii) 34Ω)

4. Determine the equivalent resistance in the Fig; 2 (a) and (b).

[(a) (Ans: 8Ω (b) (Ans; 2.3Ω)

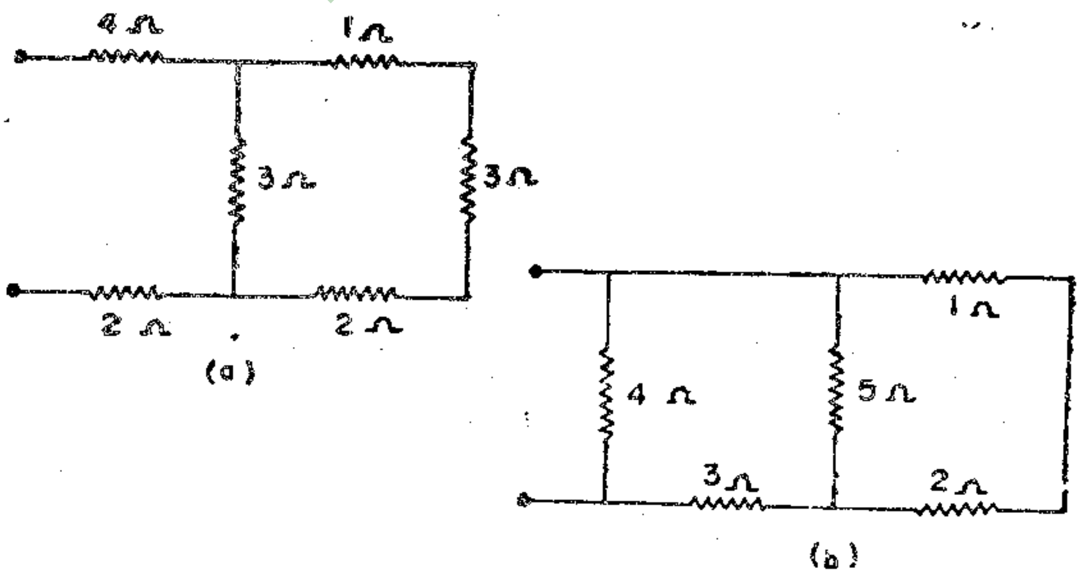


Fig. 2 (a)(b)

P.D. across $Y_1 = E_1 - E_a$

$Y_2 = E_a - E_b$

$Y_3 = E_b - E_c - E_2$

$Y_4 = E_a$

$Y_5 = E_b - E_c$

$Y_6 = E_c$

Let i_1, i_2, i_3, i_4, i_5 and i6 be the currents in different admittances respectively, applying kirchoffis Ist law for Junction 'a'

$$i_1 - i_2 - i_4 = 0$$

$$y_1 (E_1 - E_a) - Y_2 (E_c - E_b) - Y_4 Ea = 0$$

$$E_a (-y_1 - y_2 - y_4) + Ey_2 = - y_1 E_1 \qquad \ldots(8.1)$$

For kirchoffs 1st law for jn 'b'

$$i_1 - i_3 - i_5 = 0$$

$$Y_2 (E_a - E_b) - Y_3 (E_b - E_c - E_2) - Y_5 (E_b - E_c) = 0$$

$$E_a (Y_2) + E_b (-Y_2 - Y_3 - Y_5) + E_c (Y_3 + Y_5) = - Y_3 E_2 \qquad \ldots(8.2)$$

From kirchoffs 1st law for jn 'c'

$$i_3 + i_5 - i_6 = 0$$

$$Y_3 (E_b - E_c - E_2) + Y_5 (E_b - E_c) - Y_6 (E_c) = 0$$

$$E_b (Y_3 + Y_5) + E_c (-y_3 - y_5 - y_6) = Y_3 E_3 \qquad \ldots(8.3)$$

From the above three equations E_a E_b and E_c can be calculated

8.4 SUPERPOSITION THEOREM

In a circuit containing resistances and Networks, the current flowing through a point is equal to, the sum total of the individual currents flowing in each network. But while deciding flow of current through a network we have to imagine their internal resistance in place of other networks.

Further more, this theorem shows that in a circuit. If there are voltage networks and current networks working at a time, each network will work independently. That is why we can calculate the effect due to individual network.

The current flowing through a resistance is equal to the sum total of current through individual networks. Similarly the potential difference developed across a resistance is equal to the sum total of the potential difference developed due to networks. To make use of this theorem, we have to follow the method given below.

8.4.1 Proof of Theorem:

To verify the theorem let us consider the simple Network with 2 generators of emf E_1 and E_2 as shown in fig 8.2 with internal impedance Z_1 and Z_2 respectively.

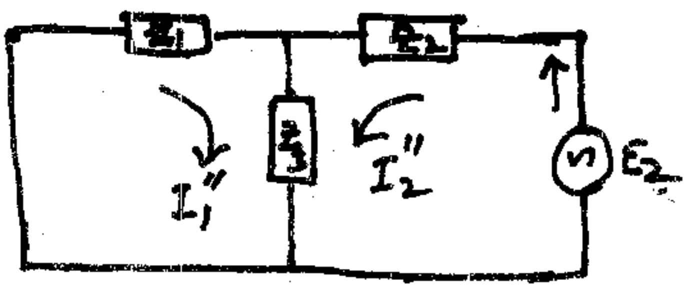


Fig 8.2

Let the currents due to E_1 and E_2 acting together be I_1 and I_2. Fig 8.3(a) and let the current due to the emf E1 acting alone be I_1 and I_2. Fig 8.3 (b) and those due to E2 acting alone be I_1'' & I_2'' Fig 8.3

Applying kichoff's second law to the mesh of fig 8.3 (a), we have

$$E_1 = I_1(Z_1 + Z_3) + I_2 Z_3 \quad \ldots(8.4)$$

$$E_2 = I_2(Z_2 + Z_3) + I_1 Z_3 \quad \ldots(8.5)$$

When E_1 is considered to act alone, mesh 8.3 (b)

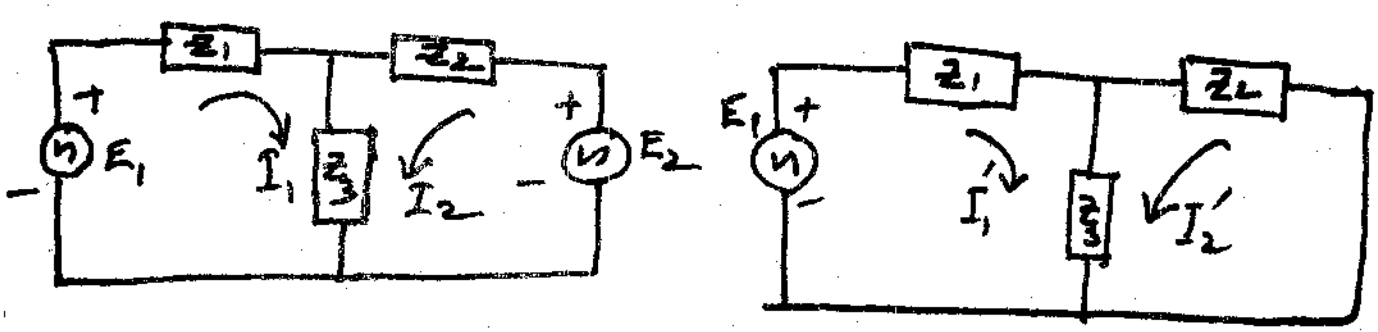


Fig 8.3 (a) & (b)

$$E_1 = I_1 (Z_1 + Z_3) + I_2{}'Z_3 \quad \ldots(8.6)$$

$$\text{And } O = I_2{}' (Z_2 + Z_3) + I_1{}' Z_3 \quad \ldots(8.7)$$

5. Use Kirchoff's law to solve the currents through the circuit illustrated in 3.

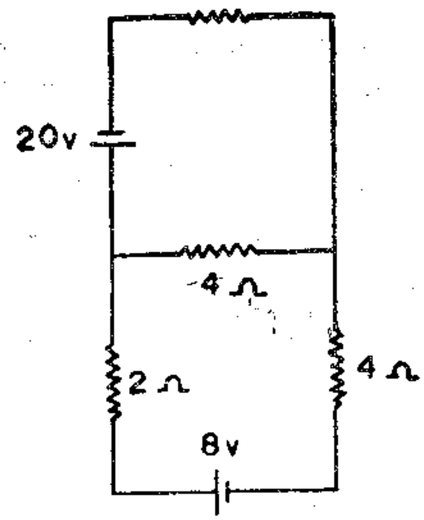


Fig 3

[Ans: i_1 = 23.8ma, i_2 = 190ma, i_3 = 214ma]

6. Apply the Kirchoff's laws to the following circuit and obtain the currents through each branch.

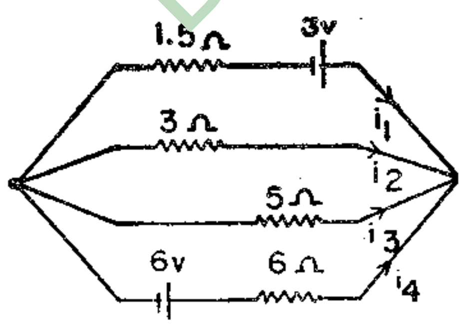


Fig 4

[Ans: i_1 = 546ma, i_2 = 732 ma, i_3 = 439 ma, i_4= 634 ma,]

7.9 RECOMMENDED BOOKS

1. Tippens, P.E (1978	Applied Physics	Mc Graw-Hill Book company New York
2. Sears, F.W. (1979)	Electricity and Magnetism	Addison-Wesley Publishing Company Massachusetts
3. Halliday, D. and Resnick (1978)	Physics Part –II	Wiley-Eastern Limited New Delhi.
4. Bueche, F.J. (1980)	Introducing to Physics for Scientists	Mc Graw-Hill International Book Company, New Delhi

UNIT – 8 NETWORKS

Contents

8.1 OBJECTIVES

This unit introduces the concept of Networks. To help you understand them, the unit explains:

(i) Mesh and node analysis

(ii) Super position theorem

(iii) Thevenins & Norton's theorem.

(i) Understand the circuit analysis

(ii) Explain the application of network theorems:

8.2 INTRODUCTION

An electrical circuit containing impedances (or elements like Resistance, inductance, Capacitance etc.) and generators of emf source is known as electrical network. It means an electrical network is nothing but a combination of circuit elements.

Linear and Non linear Network : A network is said to be linear when the current in all branches is directly proportional to the driving voltage. If in case voltage and current in any branch of Network is non linear then the network is said to be Non-linear.

Network theorem; network problems; can very often be solved by the application of ohms law and kirchoff's laws. But in complicated circuits, calculations based on these laws are very tedious and cannot be worked out on these laws, Therefore a more convenient method has to be chosen. Some theorems provide us a very easy solution to such problems and complicated network can be solved. These theorems are called network theorems of which we are going to deal with superposition theorem, Reciprocity theorem. The Thevenin's theorem and Nortons's theorem.

8.3 MESH & NODE ANALYSIS

In the process of circuit analysis ammeters are used to measure current in different components. But ammeter works effectively for supplies of low frequency. For high frequency supplies voltmeter work more effectively than ammeter. Hence voltage information at different node points will be taken.

We know $I = \frac{V}{Z} = Vy$

When y is 1/z known as admittance.

The method of analysis using voltage information is known as Nodal analysis.

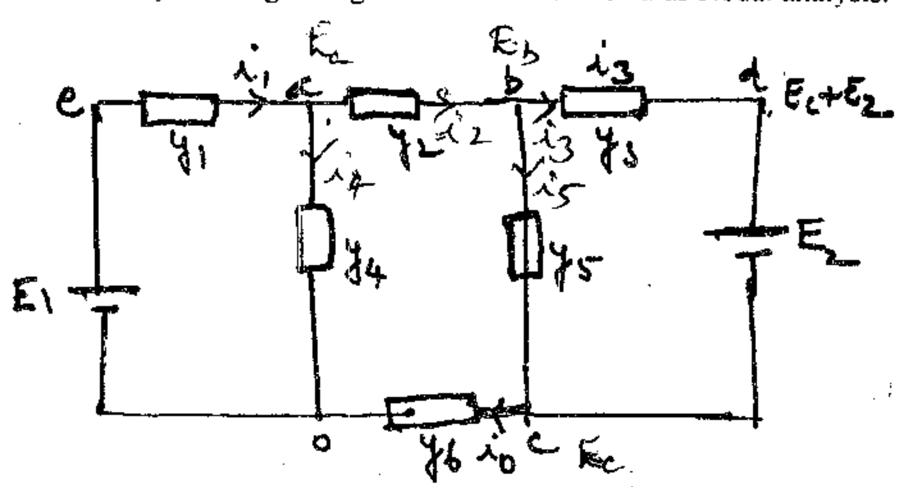


Fig.8.1

Let us consider six resistors of admittance's $Y_1, Y_2, Y_3, Y_4, Y_5, Y_6$ connected to emfs E_1 and E_2 shown in fig 8.1 Let the potential at 'O' be considered as zero. Then pot at e is E_1 let the pot at a, b, c be respectively E_a, E_b & E_c.

When E2 is considered alone, mesh of Fig 8.2 gives.

$$O = I_1^{11}(Z_1 + Z_3) + I_2^{11} Z_3 \qquad .....(8.8)$$

$$\text{And } E_2 = I_2^{11} (Z_2 + Z_3) + I_1^{11} Z_3 \qquad .....(8.9)$$

Adding Equations 8.6 & 8.8 we get

$$E_1 = (I_1^{1} + I_2^{1})(Z_2 + Z_3) + (I_2^{1} + I_2^{11})Z_3 \qquad .....(8.10)$$

Adding Equations 8.7 & 8.9 gives

$$E_2 = (I_2^{1} + I_2^{1})(Z_2 + Z^3) + (I_1^{1} + I_1^{11})Z_3 \qquad ...(8.11)$$

Equations 8.10 and 8.11 are identical with the equation 8.4 & 8.5 respectively, if

$$I_1 = I_1^{1} + I_1^{11}$$

$$\text{And } I_2 = I_2^{1} + I_2^{11}$$

This proves the truth of superposition theorem. The superposition theorem simplifies network calculations when several generators are present.

8.4.2 Procedure for application

The procedure to apply superposition theorem is as given below.

(i) Only one source is considered at a time and all other sources are replaced if there is a current source, it is replaced by an open circuit because its internal resistance is infinite. We must keep only one source E_1

(ii) Current in various resistors and their voltage drops is then calculated due to this single source .

(iii) This procedure is repeated for other sources one by one i.e E_2 also.

(iv) Algebraic sum of current voltage drops over a resistor due to different sources is then calculated to obtain the net current and voltage drop in any branch / resistor.

Note: In case of AC networks, according to the superposition theorem.

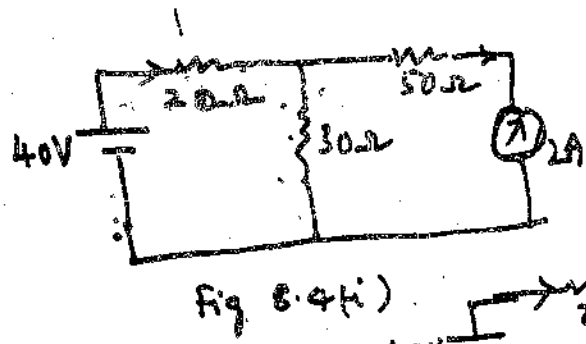
In any network containing more than one voltage or current source, the current through any branch is the phasor sum of the currents due to each source acting independently.

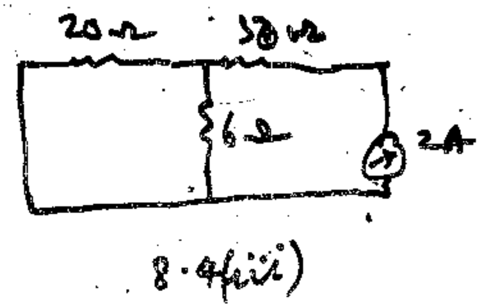
In the above-mentioned way problems can be solved.

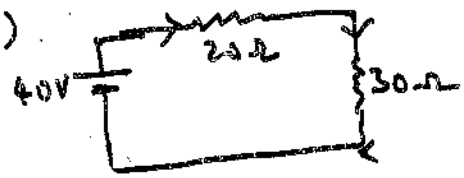
Examples: Find the current I in the circuit given below using super position theorem.

Solution: Considering first the voltage source alone, the circuit is reduced to fig, 8.3 when current source is open circulated, then

$$I1 = \frac{40}{50} = 0.8 \text{ Amps.}$$







Next considering the 2 Amps current source alone (short circuiting the voltage source) Fig iii is obtained.

8.4 (ii)

Fig 8.4
(i), (ii), (iii)

The current flow through 20 oms resistor.

$$I_2 = 2 \times \frac{20}{(20+30)} = \frac{40}{50} = 0.8 \text{ Amp}$$

Applying principles of superposition. The total current through 20 oms branch

$I = I_1 + I_2 = 0.8 + 0.8 = 1.6$ Amp

8.5 RECIPROCITY THEOREM

Statement : If an emf E applied in one mesh of a network of linear impedance, produces a certain current I in the second mesh, then the same emf acting in the second mesh will give an identical current in the first mesh. That means E & I can be mutually be exchanged that is why E/i is known as Transfer resistance.

In the circuit shown in fig 8.5 the first mesh due to emf E, there is a current of i_1 in the second branch. Now if sources of current and emf are mutually transferred with each other then there will be i_1 reading observed in ammeter. This value is equal to the reading of ammeter when it was in first position that means E & I can mutually be exchanged.

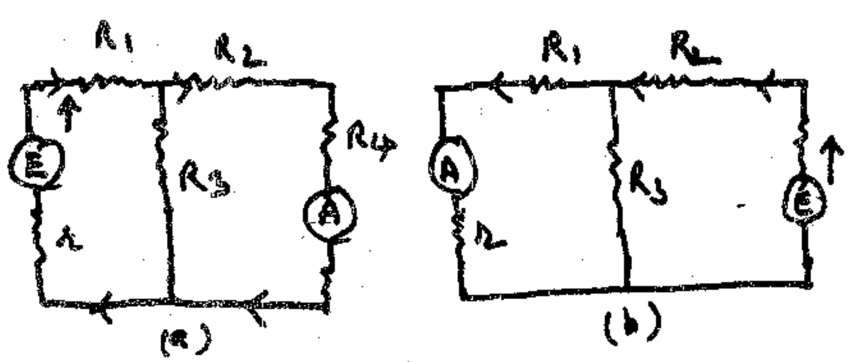


Fig 8.5 (a) & (b)

Proof : To prove the theorem, consider the arrangement shown in fig 8.6 in which E is a emf source is in the first mesh. Let the current in first & 2nd meshes be I_1 and I_2 respectively taking 8.6 (a) alone.

Applying kirchoff II law to the two meshes, we have

$$I_1 (Z_1 + Z_2) - I_2 Z_2 = E \qquad \ldots (8.12)$$

$$\text{And } I_2 = (Z_2 + Z_3) - I_1 Z_2 = E \qquad \ldots (8.13)$$

Substituting the value I_1 from 8.12 in to 8.13 we get,

$$I_2 \left[\frac{(Z_1 + Z_2)(Z_2 + Z_3)}{Z_2} - Z_2 \right] = E$$

$$\text{Or} \qquad I_2 = \frac{EZ_2}{[(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2]} \qquad \ldots (8.14)$$

Considering Fig No 8.6 (b) in which the source of emf is in second mesh, let the current in the two meshes be I_1^1 & I_2^1

Applying kirchoff's II law to the two meshes we get

$$I_1^1 (Z_1 + Z_2) - (I_2^1 Z_2 = E \qquad \text{---8.14 (a)}$$

$$\text{And } I_2^1 (Z_2 + Z_3) - I_1^1 Z_2 = O \qquad \text{--8.14 (b)}$$

Substituting the value of I_2^1 from eq 8.14 (a) into Equations,8.14 (b) we get

$$I_1 \left[\frac{(Z_1 + Z_2)(Z_2 + Z_3)}{Z_2} - Z_2 \right] = E$$

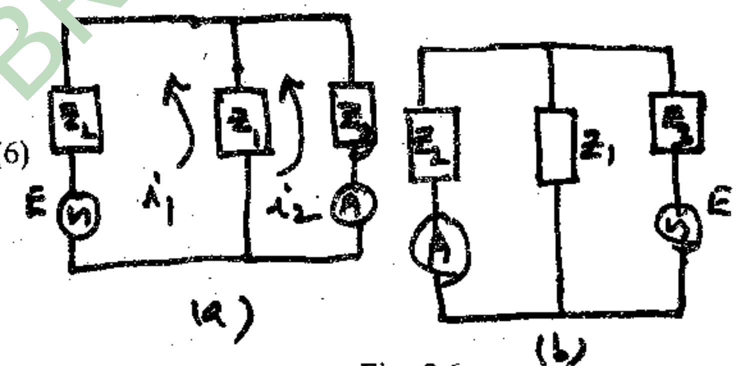
Or

$$I_1^1 = E Z_2 \;/\; [Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2] \text{ ---(6)}$$

From equations (8.3) & (8.6) We get

$I_2 = I_1^1$

Which proves the reciprocity theorem.



Figs 8.6
(a) &(b)

The ratio of emf in one branch to the current in another branch is called the Transfer impedance.

8.6 THEVENIN'S THEOREM

This theorem is useful in reducing a complicated network containing several voltage generators & resistances into a simple equivalent voltage i.e. generator equivalent voltage is E_o & resultant resistance R_o, It can be treated as a circuit containing these two

STATEMENT: The current in a load resistance connected between two terminals of a network of generators and liner resistances is same as due to a single voltage generator of emf equal to open circuit (when there is no load) voltage between the same terminals of the network & internal resistance equal to the resistance of the network between the same terminals of the network when all the generators in the network have been replaced by their internal resistances.

The Thevenin's theorem can be elaborately be understood by the circuit as given below. To apply the theorem the following steps have to be adopted.

8.6.1 Procedure for application of theorem

(i) First of all the load resistance RL is disconnected from terminals A & B.

(ii) With the load terminals A and B open, the open circuit voltage E_o between them is calculated by any convenient method.

(iii) Now the generators are removed from the network leaving behind their internal resistances and equivalent reisstance's R_{th} between terminals A and B is calculated by usual methods

(iv) Finally the load resistance R_L is connected between A & B terminals, so that we get a simple series circuit as shown in fig 8.7

This way Thevenin emf (E_o) and the Thevenin internal resistance R_{th} can be obtained

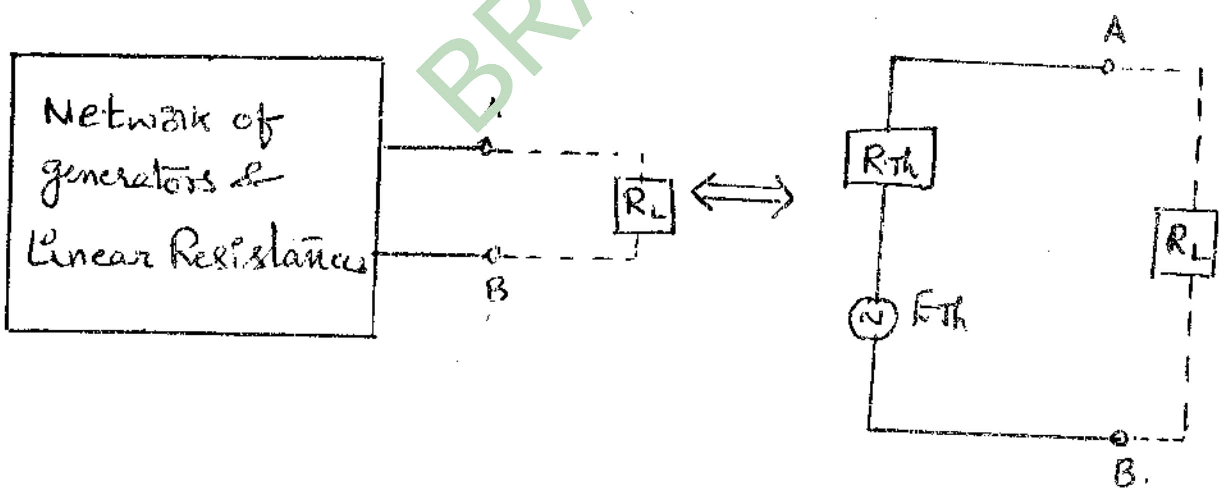


Fig 8.7

8.6.2 Proof of Theorem

To prove the theorem let us consider a network containing resistive resistances R_1, R_2 and R_3 one generator of emf E & internal resistance zero (i.e ideal generator)
Applying kichoffs II law to meshes I & II we get

$I_1 R_1 + (I_1 - I_2) R_3 = E$

i.e $I_1 (R_1 + R_3) - I_2 I_3 = E$ ...(8.15)

and $I_2 R_2 + I_2 [R_2 (I_1 - I_2] R_3 = 0$

i.e. $I_2 (R_2 + R_2 + R_3) - I_1 R_3 = 0$ ...(8.16)

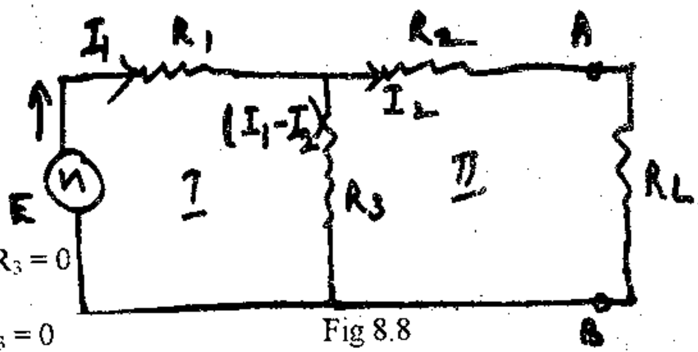


Fig 8.8

From (2), $I_1 = I_2 \dfrac{(R_2 + R_3 + R_2)}{R_3}$

Substituting this value of I_1, in equation 8.15 we get

$I_2 [(R_2 + R_3 + R_1)(R_1 + R_3) - R_3^2] = ER_3$

This gives $I_2 = \dfrac{ER_3}{R_2 [R_1 + R_3] + R_1 \ R_3 (R_1 + R_3)}$ ... (8.17)

This is the current flowing through load impedance R_2 Equation 8.17 may be put in the following form

$$I_2 = \frac{E R_3 / R_1 + R_3}{R_2 + \left(\frac{R_1 R_3}{R_1 + R_3}\right) + R_L}$$

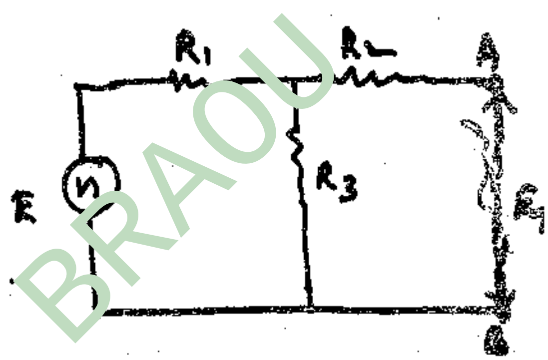


Fig 8.9 ...(8.18)

This current is same as it would flow in R_L if R_L were connected to a generator of emf

$\dfrac{ER_3}{R1 + R3}$ and internal resistance $\left[R2 + \dfrac{R_1 R_3}{R1 + R3}\right]$

If R_L is removed (i.e terminal A and B are open circuited) the current in R_1 & R_3 will be

$$I_1 = \frac{E}{(R_1 + R_3)} \quad ...(8.19)$$

i.e voltage across R3 will be

$$V = I_1 \ R_3 = \frac{E R_3}{(R_1 + R_3)} \quad ....(8.19(a))$$

As terminal A and B are open circuited, there is no current in R_2 and the potential difference E_{Th} across

$$A \ \& \ b \text{ is } E_{Th} = V = \frac{E R_3}{R_1 + R_3} \qquad \ldots(8.20)$$

Further if E is replaced by its internal resistance Fig 8.10(a) the resistance of network between A & B will be

$$R_{Th} = R_2 + \frac{1}{1R/_1 + 1/R_3} = R_2 + \frac{R_1 \ R_3}{R_1 + R_3} \qquad \ldots(8.21)$$

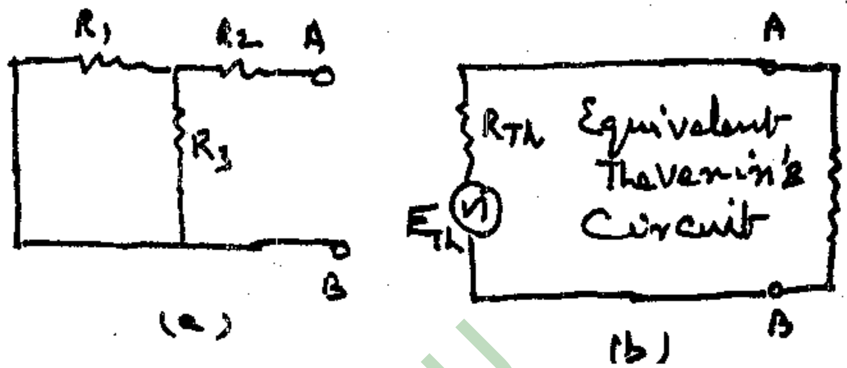


Fig 8.10 (a) & (b)

Thus equation 8.18 is equivalently written as

$$I_2 = \frac{E_{Th}}{R_{Th} + RL} \qquad \ldots(8.22)$$

This proves Thevenin's theorem. The equivalent circuit is shown in Fig 8.10

Check your progress:

1. The method of analysis using voltage information is known as……….
2. Thevenivis Theorem is useful…………..

Note: a) Space is given below for your answers.
b) Compare your answers with those given at the end of the unit.

…………………………………………………………………………………………

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8.7 NORTON'S THEOREM

This theorem is very useful in converting complicated network into a simple parallel circuit, consisting of an ideal current Source and a parallel resistance.

The theorem states as follows:

Statement : The current in a load resistance between two terminals of a network of generators & resistances In is the same as if the load resistance were connected to a constant current source, whose generated current is equal to the short circuit current between the same terminals of network & which is placed is parallel with a resistance Rn equal to the resistance of the network looking backs into terminals when all the generators in the network have been replaced by the resistance equal to their internal resistances.

The schematic figure of Norton's theorem is shown in fig.8.11

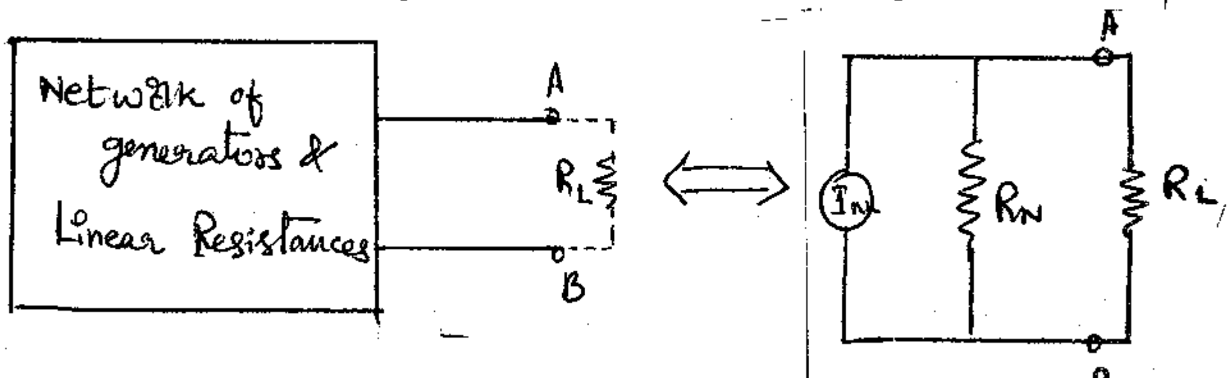


Fig 8.11

To apply the theorem, the following steps are adopted.

8.7.1 Procedure: For application of the Norton's theorem:

Suppose we want to find the current through resistance R_L connected to terminals A & B of the network of generators & linear resistances.

(i) R_L is disconnected from terminals A and B and terminals A & B are short circuited.

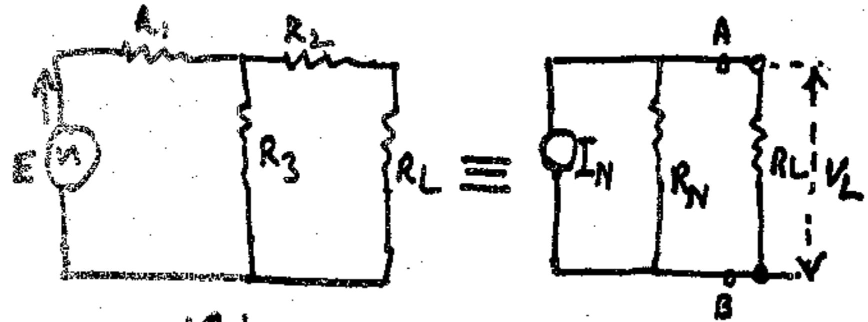
(ii) The current in the short circuit is found by usual methods. This current is usually called Norton's current I_N

(iii) The short circuited terminals A & B is removed so that they are again open & the generators are removed from the network leaving behind their internal resistances & equivalent resistance of the network as looked from open terminals A and B is found. This resistance is Norton's resistance Rn

(iv) Norton's equivalent circuit is sketched, keeping current source. In & resistance Rn is parallel & again load resistance RL is connected between A & B finally current in load resistance R_L is calculated.

8.7.2 Application of Norton's theorem

To practically show the application of this theorem to problems we shall consider a network as shown in Fig 8.12 (a) . Norton's equivalent is shown in fig 8.12 (b).



(a) *Two figs 8.12(a) & (b)*, (b)

The equivalent resistance of Rn & RL is

$$\frac{1}{R} = \frac{1}{R_N} + \frac{1}{R_L} \quad \text{i.e. } R = \frac{R_N R_L}{R_N + R_L}$$

The P.D. across load is given by.

$$V_L = I_N R = \frac{I_N R_N R_L}{(R_N + R_L)} \qquad \ldots(8.23)$$

Current in Load $I_L = V_L / R_L = I_N R_N / (R_N + R_L)$ ...(8.23)(a)

The magnitude of current I_N is found by short circuiting the out put terminals A & B fig 8.12 Applying Kichoffs laws to the meshes

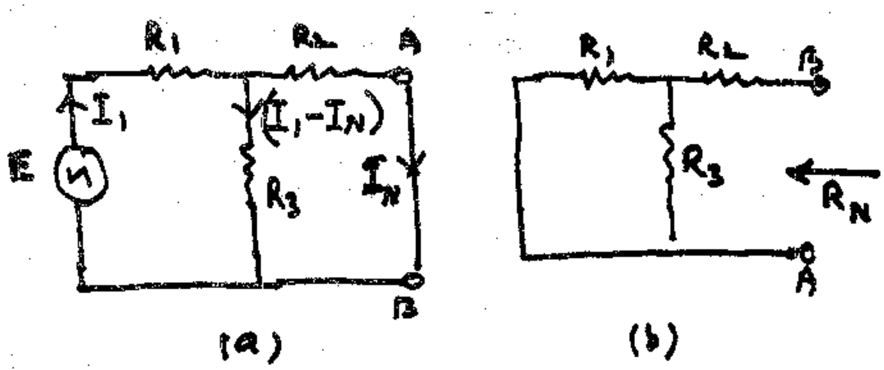
$$I_1 R_1 + (I_1 - I_N) R_3 = E$$

$$\text{i.e } I_1 (R_1 + R_3) - I_N R_3 = E \qquad \ldots(8.24)$$

$$\text{and } I_N R_2 - (I_1 - I_N) R_3 = 0$$

$$\text{i.e } I_N (R_2 + R_3) - I_1 R_3 = 0 \qquad \ldots(8.25)$$

$$I_1 = \frac{(R_2 + R_3)}{R_3} I_N$$



Figs 8.13 (a) & (b)

Substituting this value of I_1 in equation 8.24 we get

$$(R_2 + R_3)\left[\frac{(R + R_3)}{R_3}\right] I_N - I_N R_3 = E$$

Solving for I_N, We get $I_N = \dfrac{ER_3}{R_1 \; R_2 + R_3 + R_1 + R_3 + R_2 + R_3 - R_3^2}$

$$= \frac{ER_3}{R_1 R_2 + R_3 R_1 + R_2 R_3} \qquad \ldots(8.26)$$

The parallel impedance RN is found by looking back from the out put terminals. When All sources are removed leaving behind their internal resistances.

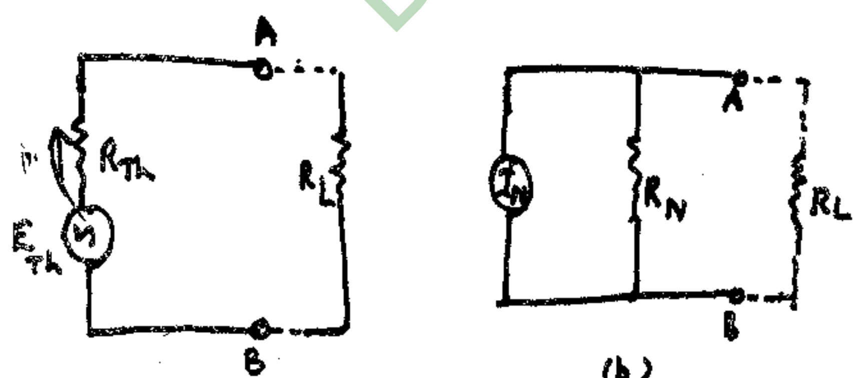
Therefore equivalent circuit is shown in fig 8.13(b) the resistance as viewed from out terminals in Norton's resistance given by

$$R_N = R_2 + \frac{R_1 R_3}{R_1 + R_3} \qquad \ldots(8.27)$$

Thus knowing the values of I_N and R_N the current in load may be found by using Equation 8.23 (a).

8.8 DUALITY OF THEVENIN'S & NORTON'S EQUIVALENT CIRCUITS

Consider the thevenin's & Norton's equivalent circuits of same network shown in fig. 8.14 (a)and 8.14 (b)



(a) *Fig 8.14(a) & 8.14(b*

The current I_2 for Thevemins equivalent circuit in load R_L is given below.

$$I_R = \frac{E_{Th}}{R_{Th} + R_L} \qquad \ldots(8.28)$$

The current I_L in load for Norton's equivalent circuit in load R_L is given by

$$I_L = \frac{I_N R_N}{R_N + R_L} \quad \ldots(8.29)$$

But $R_N = R_{TH}$ (from definition of internal resistance in both theorems)

$R_N = R_{Th} = R$ (say)

In view of this equations 8.28 & 8.29 may be expressed as

$$I_2 = \frac{E_{Th}}{R_I + R_L} \text{ and } I_L = \frac{I_N R_N}{R_I + R_L} \quad \ldots(8.30)$$

I_N, the short circuited current of Thevenin's equivalent circuit is

$$I_N = \frac{E_{Th}}{R_{Th}} = \frac{E_{Th}}{R_I} \text{ i.e., } I_N R_I = E_{Th} \quad \ldots(8.31)$$

$$I_2 = I_L = \frac{E_{Th}}{R_I + R_L} \quad \ldots(8.32)$$

Thus thevenin's & Norton's equivalent give the same values of load circuit. Hence either theorem may be applied to any given network as per convenience.
Norton's theorem is preferred for networks containing generators of high internal resistance.
Ex : Equivalent circuit for a Pentode tube amplifier.

8.9 SUMMARY

Many complex networks in electrical engineering and electronics can be analysed making Use of the Thevenin's and Norton's theorems with the help of these two single networks it becomes easy to understand the working of each network in a complex circuitry.

Each of the networks discussed in this chapter have their individual importance in Appropriate context.

These theorems are applicable for both AC & DC networks and also used in complex networks consisting of vacuum tubes and also of transistors.

Check your progress: Answers :

1. Nodal Analysis.
2. Thevenivis Theorem is useful in reducing a complicated network containing several voltage generators and resistances into a single equivalent voltage.

8.10 SOME TYPICAL WORKED OUT EXAMPLES.

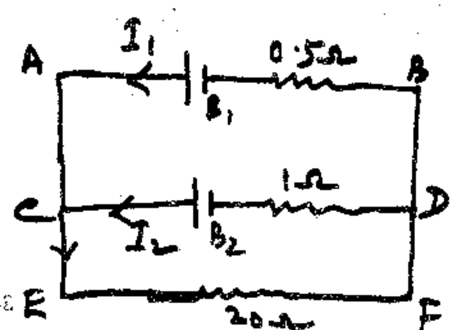
(1) A battery of emf 10 Volts and internal resistance 0.5 Ω is joined parallel with another battery of emf 15 volts and internal resistance 1 Ω. This combination sends a current through an external resistance 20 ohms. Calculate the current through each battery.

Solution : Let the current through batteries B_1 and B_2 be I_1 & I_2 respectively.

Applying kirchoffs II law, to the mesh ABFEA at point C

$I_1 + 0.5 + (I_1 + I_2)\,20 = 10$

Or $20.5\ I_1 + 20\ I_2 = 10$



Applying kirchoffs II law, to CDEFC

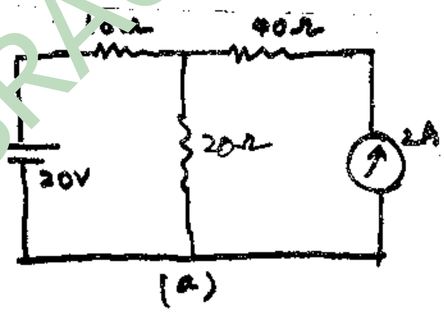
$I_2 + 1 + (I_2 + I_2) \times 20 = 15$

i.e. $21\ I_2 + 20\ I_1 = 15$

Solving equations (1) & (2) we got
$I_1 = 2.94$ Amp & $I_2 = 3.525$ Amp.

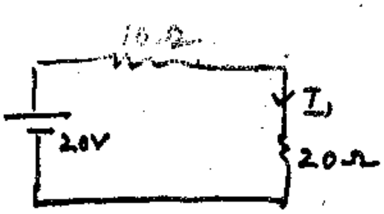
Fig I

(2) Find the current I in the circuit given below using superposition theorem.



Solution : Considering first the voltage source alone the circuit is reduced to fig. Below when current source is open circuited Then I1 = 20/30 = 0.66 Amps.

Now considering the 2 Amps. Current source alone (short circuiting the voltage source) fig. Below is obtained.



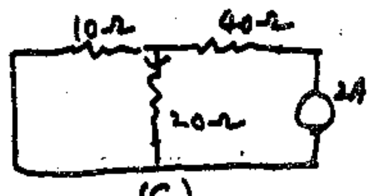


Fig II a,b,c

Fig II

The current flowing through 20 Ω resistance.

$$I2 = 2 \times \frac{10}{10+20} = \frac{20}{30} = 0.66 \text{ Amp}$$

Applying principle of superposition the total current through 20 Ω resistance in

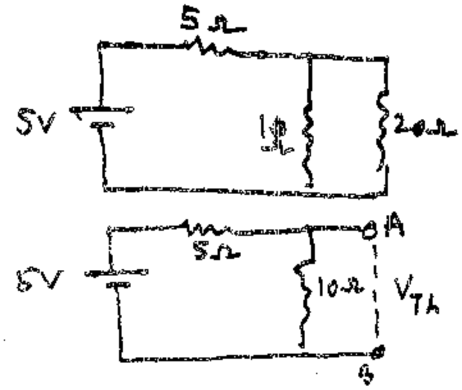
$I = I_1 + I_2 = 0.66 + 0.66 = 1.32$ Amp.

(3) Find the current in 20 Ω resistors in the circuit shown III by thevenins theorem.

Solution : The open circuit voltage or thevenins voltage V_{Th} disconnecting the load (Fig beside) is given by

$$V_{Th} = I \times 5\,\Omega = \frac{20}{5+10} \times 5 = 6.6\ V$$

The thevenins resistance is found by removing load & short circuiting voltage source fig III(b)



Thus $R_{Th} = 5/50 = 0.5\ \Omega$

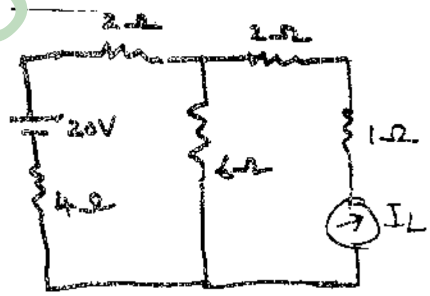
Fig III (a & b)

According to thevenin's theorem the given circuit can be replaced by the equivalent Circuit shown in fig III (d). Therefore current through load 20 Ω is

$$IL = \frac{6.6}{0.5 + 20} = 0.32\ \text{Amp}$$

(4) Prove reciprocity theorem for 2 Ω branch in the fig IV

Solution : From Fig total current in the circuit



$$= \frac{20}{2+2+4}$$

$$= \frac{20}{8} = 2.5\ \text{Amp.}$$

Fig IV

Current I_L can be found by proportional current formula (i.e., current 1/R)

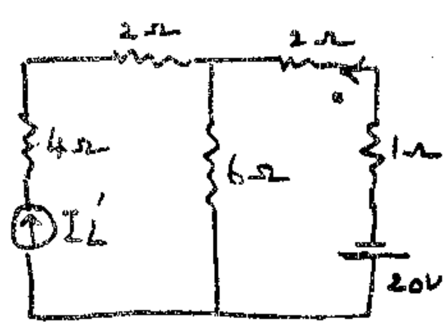
$$\text{Hence } I_L = 2.5\ \frac{6}{6+2+1} = 2.5 \times \frac{V^2}{R^3} = 3.66\ \text{Amp.}$$

According to reciprocity theorem, replacing voltage source is 1 Ω branch by fig. Shown Below.

$I = 20/3 + 2 + 1 = 20/6 = 1.33$ Amps

Evidently $I_L = I_L = 1.33$ Amp.

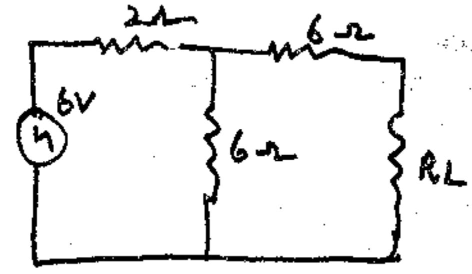
Hence reciprocity theorem is proved.



5) Draw Thevenin's equivalent circuit for the given Network fig V and then obtain Norton's circuit.

SOLUTION : The open circuit voltage i.e. when load R_L is disconnected, is given by

$$E^1 = E \frac{R_3}{R_1 + R_3} = \frac{6 \times 6}{2+6} = 3 \text{ Volts}$$



(a)

Fig V

The impedance across load terminals after disconnecting the B load R_L is

$$Z = R_2 + \frac{R_1 + R_3}{R_1 + R_3} = 6 + \frac{2 \times 6}{2+6} = 6 + 1.5 = 7.5\ \Omega$$

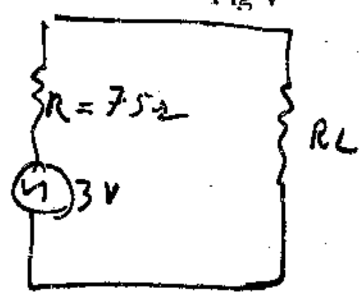
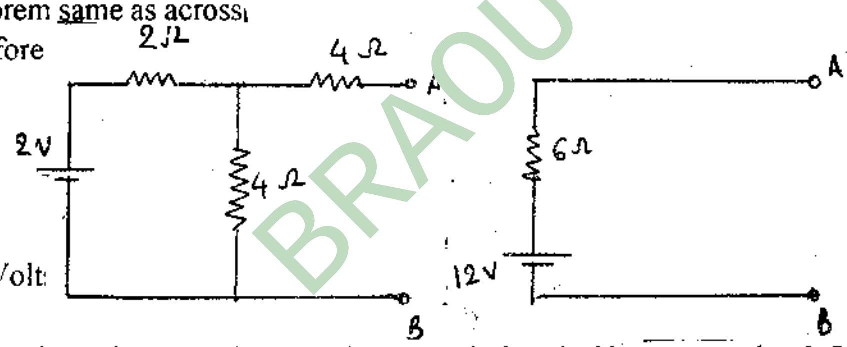


Fig VI (a)

Accordingly Thevenins equivalent circuit is shown as the one beside is Fig VI(b)

6) Find the Thevenin's equivalent for the circuit given below Fig VII(b)

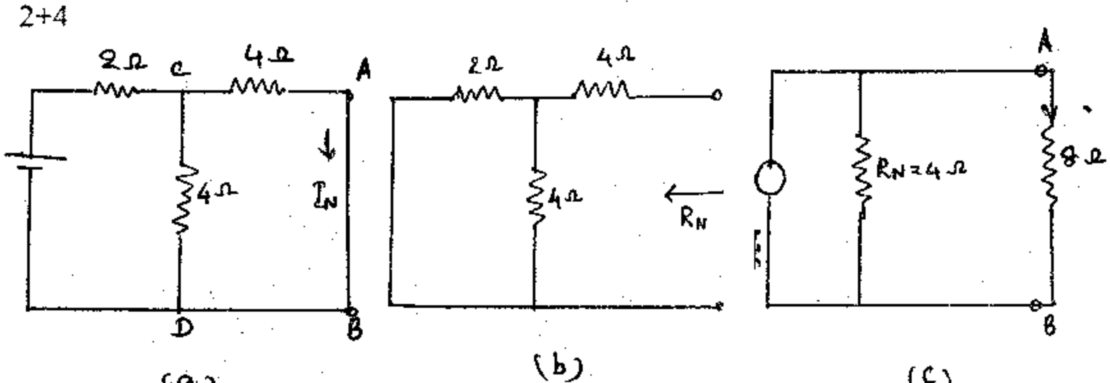
Solution: With terminals A.B open the P.D across A.B is theorem same as across 4Ω Resistance. Therefore



$$V_{AB} = \frac{12 \times 4}{2+4} = 8 \text{ Volt}$$

In order to find the impedance between A.B We short circuit the 12 V battery, so that 2 Ω & 4Ω resistance become parallel, and their effective resistance is

$$R = \frac{2 \times 4}{2+4} = 4/3\ \Omega$$



The resistance between AB is now

$$Z_{A\text{-}B} = 4/3 + 4 = 16/3 = 5.3\Omega$$

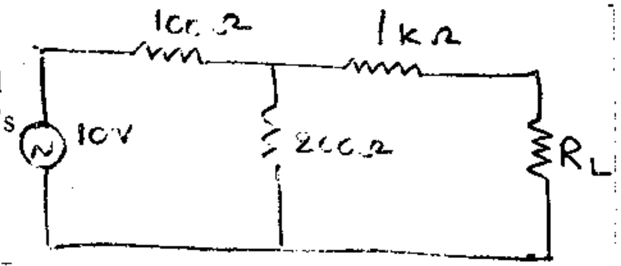
Fig VII (a) & (b)

And the Thervenin's equivalent of the above network is shown in fig VII b

7) Using Norton's theorem, calculate the current flowing through 8 Ω resistor shown in fig below

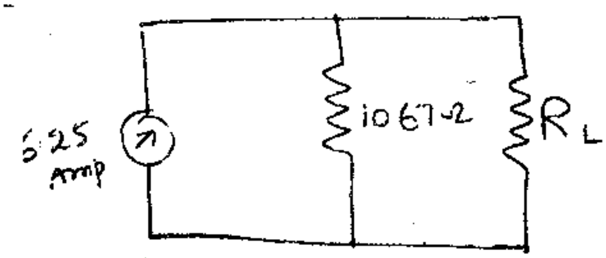
Solution: To solve this problem we proceed as follows

(i) The 8Ω resistor is removed from terminals A & B and they are short circuited & then The current is short circuit is Norton's current I_N



The equivalent resistance of circuit is

Fig VIII (a) and (b)



$$= 2 + \frac{4 \times 4}{4+4} = 2 + 2 = 4\Omega$$

Current I = 12/4 = 3 Amp.

The current at 'C' is divided in to two unequal parts Such that
$(I - I_N) 4 = 4 I_N$ i.e., $8 I_N = 4I$ X $I_N = 0.5I$

$I_N = 0.5I = 0.5 \times 3 = 1.5$ Amp

8) Draw Thevenins & Norton's equivalent circuits for the following Network of resistances. Calculate the current in load in each case

Solution: Thevenin's equivalent circuit.

(i) Thevenin's emf E_{TH} we first remove R_L and leave internal resistance A & B in open circuit. Let I be the current through 1 Ω resistor.
The current in 0.8Ω resister will also be same I since A & B are in open circuit.

Applying kirchoffi II law to mesh of circuit

(a) we get $1\Omega \times I + 0.8 \times I + 0.2 = 12$ or $I = 12/2 = 6$

Current through 1 Ω resistance is zero

$E_{TH} = 6 \times 0.8 = 4.8\Omega$

$$R_{TH} \; R_2 + \frac{R_1R_3}{R_1+R_3} \;\&\; R_{Th} = \frac{1.0\,(1+0.2)\times 0.8}{0.8+1.2}\, 1 + 0.48 = 1.48\,\Omega$$

$$I_L = \frac{E}{R_{Th} + R_L} = 1.256 \text{ Amp.}$$

Norton's equivalent circuit will be as shown in fig given below

$I_N = E_{TH}/R_{TH} = 4.8 / 1.48 = 3.24$ Amp.

Current in Load (I_L) = $R_N = R_N = / (R_N + R_L) \times L$

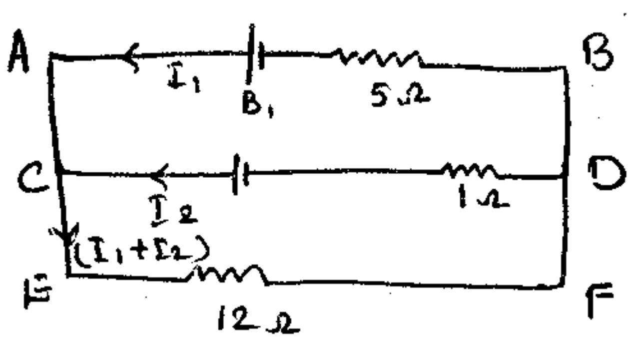
$R_N = R_{Th} = 1.48\,\Omega$

$$= \frac{1.48}{(1.48 + 3.2)} \times 3.24 = 1.025 \text{ Amp.}$$

Problems to be Solved:

1. A battery of 1.5 volts is connected is series to the resistance of 20 & 30 Ω. Find out equivalent voltage and resistance across the points of 30 Ω resistance. Ans:0.9v, 12.5Ω.
2. A battery of emf 6 volts & internal resistance 5 Ω is joined is parallel with another of emf 10 volts & internal resistance 1 ohm the combination is used to send a current through an external resistance 12 Ω. Calculate the current through each battery.
   Hint: Apply Kirchoff's II law to two meshes I.e., ABFEA & CDEFC fig . for above problem given beside

Ans: I_1 = 6/11 Amp.
I_2 = 14/11 Amp



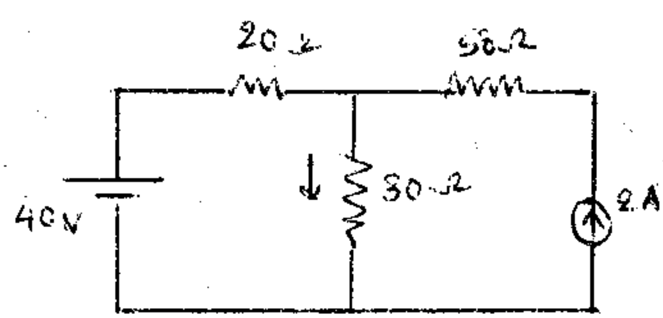


Fig (i)

3. Find the current I in the circuit given below using superposition theorem.
Hint: Open circuit when current source is open the find I_1. Then find current flowing through 20Ω i.e. I_2 Apply superposition theorem i.e. $I = I_1 + I_2$

Ans: 1.6 Amp.

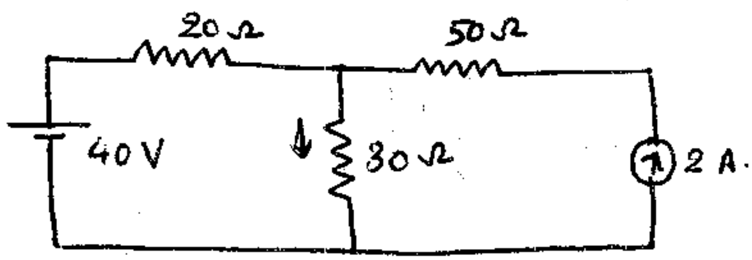
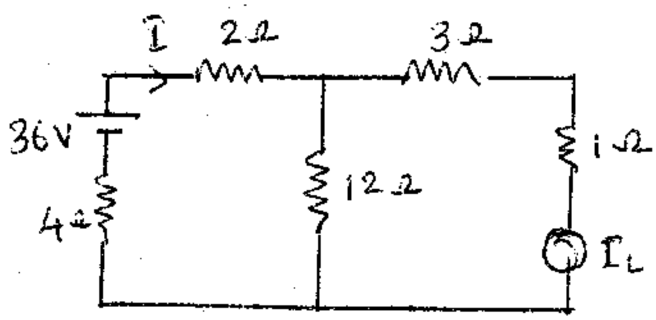


Fig (ii)

4. Prove reciprocity theorem for 1Ω branch is the fig below

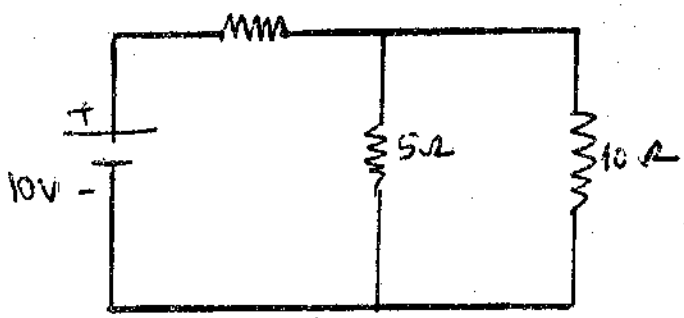
Hint (i) Find total current in circuit Accordingly to reciprocity theorem replace voltage source is 1Ω branch then obtain. I. Find I1 x I1 L prove them equal. Hence theorem is proved

Ans : 3 Amp each.



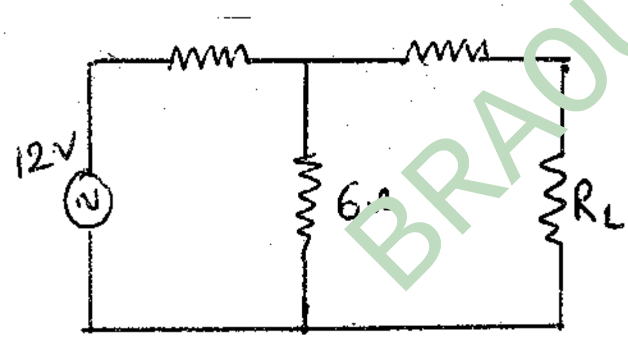
5 Find the current in 10 ohms resistance in the circuit shown by thevemins theorem. Hint : (1) Draw circuit for V_{TH} disconnecting load, the find R_{TH} (2) Draw equivalent circuit and Find current through 10Ω load.

Ans: 0.4 Amps.



6 Draw Thervenin's equivalent circuit for the given network fig. Below & then obtain Norton's circuit.

Hint: 1) Find open circuit voltage when R_L is disconnected i.e E^1 = 8 volts. 2) Then find R_L & impudence
Value = 9Ω then draw Norton's circuit.

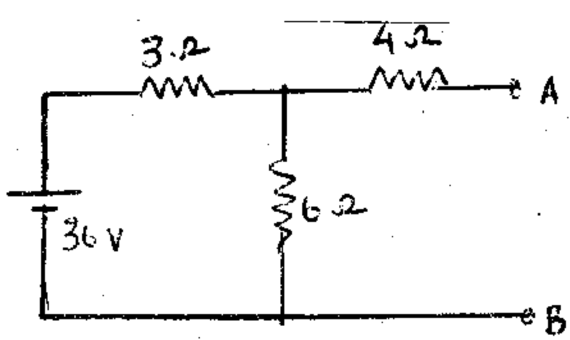


7. Find the thevenins equivalent for the circuit given below

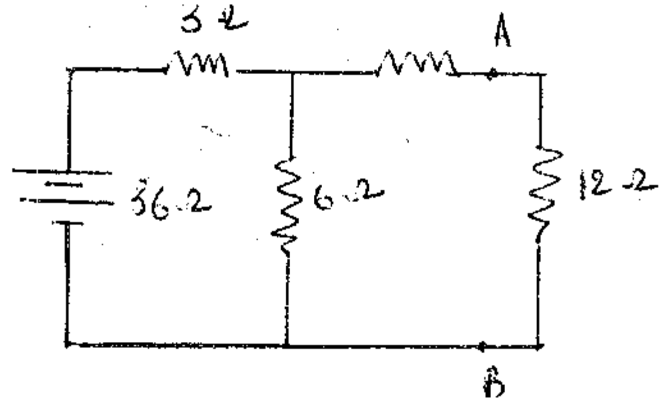
Hint : With A and B open Find P.D across A X B Then resistance between A & B

(2) Then Draw thevenins equivalent of above network.

Ans V_{AB} = 24 V
Z = 6 Ω



8) Using Norton's theorem, calculate the current following through a 1.2 ohms resistance in the circuit shown below.



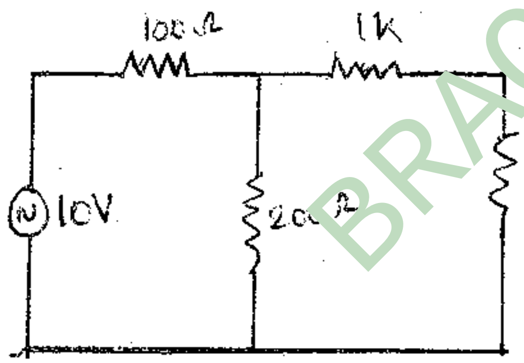
Hint(i) The 12Ω resistance is removed from the terminals A& B and they are short circuited then draw Fig.

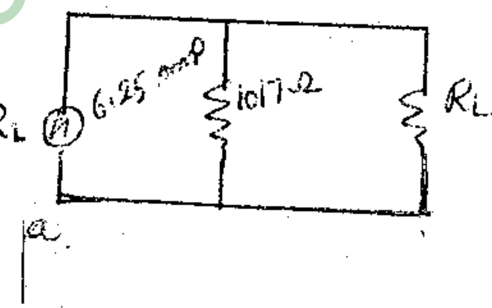
(2) The current is short circuit is Norton's current I_N Find that

Ans : $I_N = 4A$, $R_N = 6\Omega$

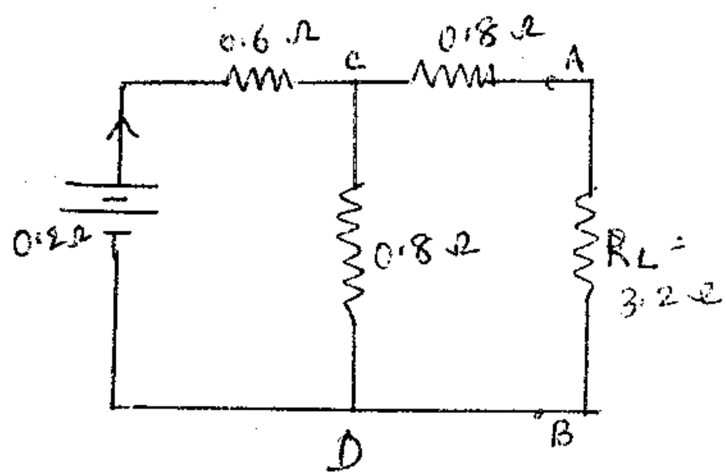
Hence Norton's equivalent of given circuit to be shown with 4 amp is circuit.

9) What is Norton's equivalent to the network shown below.





Hint : (1) Remove load R_L, then short circuit. Find Current them 1 KΩ i.e., 37.5 M.A. (Resistance is circuit = 266.7 Ω)
2) find I_N = 6.25 Amp, R_N also = 1067 Ω
Accordingly Norton's equivalent of Fig. Given as drawn as



10) Draw the Thervenin's & Norton's equivalent circuits for the following & Norton's of resistance. Calculate the current in the load in each case.

2) Then Thevemins emf E_{TH} Apply kirchoff's law II to circuit.
Find E_{TH} & R_{TH} by formula.

Then I_L value to be found

3) find $I_N = E_{TH} / R_{TH}$ & I_L by $\dfrac{R_N}{R_N + R_L} \cdot I$

Ans: $E_{TH} = 12$ V
$I_L = 30/11$ Amp.
$I_N = 10$ Amp.
$R_{TH} = 1.2\ \Omega$

8.11 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail

1. Superposition principle & Reciprocity theorem – state and prove them
2. In general network what is the use of superposition theorem
3. State and prove Thevenins theorem
4. State and prove Norton's theorem
5. Define & Compare Thevenins Norton's Theorems.
6. How is Thevenins equivalent & Circuit related with the Norton's equivalent circuit.

II Answer the following questions in brief

1. Define the following terms:
   a) Network b) Node c) branch
2. Give statements of a) Thevenins & b) Norton's theorems.
3. State and Explain superposition theorem.
4. State and Explain Reciprocity theorem.
5. Distinguish between the two theorems Norton's & Thevenins.

BLOCK – 3: MAGNETOSTATICS

UNIT 9: AMPERE'S LAW

Contents

9.1 OBJECTIVES

This unit discusses of magnetic field and the effects associated with them. To help your understand them the Unit explains.

1) Oersted's experiment

2) Amperes Law

After going through this unit you will be able

1) to calculate the magnetic field caused by current carrying straight wire; and

2) explain the concept of magnetic lines of induction

9.2 INTRODUCTION

Our knowledge of magnetism and magnetic phenomena is as old as science itself. During the 16^{th} century, the English physician name Gilbert studied the properties of magnets and also realized that a magnetic field existed around the earth. The nature of this field was similar to the magnetic field around a magnetic sphere. In 1820 Oersted discovered that a magnetic field exists around a wire carrying electric current. This basic observation proved the way for producing high magnetic fields.

9.3 MAGNETIC FIELD

It is an experimental fact that a bar magnet or a current carrying conductor produce magnetic field in its surroundings. The intensity of a magnetic field at a given point has both magnitude as well as direction. Thus the magnetic field can be represented as vector denoted by $\vec{B}$. The magnetic field vector $\vec{B}$ is known as magnetic induction. The magnetic induction may also be represented in terms of induction. The vector B is related to lines of induction in the following manner.

I. The tangent to lines of induction at any point gives the direction of B at that point.

II. Lines of induction represent qualitatively the strength of the field at that point. Lines of induction crossing per unit area represent the strength of the magnetic induction. Thus the intensity of the magnetic induction at a point may be evaluated by considering the number of lines of induction that pass through unit area.

$$\phi B = \int \vec{B} \cdot \vec{ds} \qquad ...(9.1)$$

ϕB = is known as magnetic flux, the unit Weber is used to measure B. Hence the field B is expressed as Weber /m^2

$$B = \phi B \text{ / Area} \qquad ...(9.2)$$

(The C.G.S Unit for B is gauss)

1Weber /m^2 = 104 gauss = 1 tesla

9.4 DEFINITION OF B

When a magnetic needle is brought near the current carrying conductor, it shows a deflection. This shows that a magnetic field is produced near the wire i.e., a moving charge produces magnetic field. Therefore an electric field also is produced due to a charge. Hence magnetism and electricity are two aspects.

The magnetic induction vector $\vec{B}$ is responsible for the magnetic field.

If a point change moves in the magnetic field it experiences a force in the perpendicular direction. The force will be maximum when the direction of 'B' and the velocity of the particle are perpendicular to each other and will be zero when they are in the same direction. Hence we can write.

$F = q\,V\,B \sin\theta$ (θ angle between V & B)

or $F = q\,\bar{V} \times \bar{B}$

Magnetism of B = $\dfrac{F\,max}{qv} = \dfrac{\text{Newt/ Sec}}{\text{Coul.meters}}$

If there exists both electric and magnetic field, the force on the particle is Lorentz force.

$$\bar{F} = q\,\bar{E} + q\,\bar{V} \times \bar{B}$$

9.5 AMPERE'S LAW

When an electric current flows through a conductor three kinds of effects may be produced. They are (1) the magnetic effect, (2) the heating effect and (3) the chemical effect. Any one of these methods can be used to measure the strength of the currents. We shall consider the magnetic effects only.

It was observed by Oersted that a magnetic needle was placed near a current carrying conductor, the needle was found deflected from its north south setting, indicating thereby an electric current produces as magnetic field. Reversing the direction of the current reverse the direction of the magnetic needle.

Laplace and Ampere showed the law for intensity of the field due to a current carrying, linear conductor in a mathematical form. This is referred to as Ampere's Law.

According to this Ampere's law we write the quantitative relationship between current and the magnetic field **B** as

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i \qquad \ldots(9.3)$$

This equation is known as Ampere's law

It is appreciable to know the historical experiment performed by Ampere which led him to formulate above equation. The experiment consists of measuring B at various distances r from a long straight wire of circular cross section and carrying current i.

Let us put a small needle at a distance r from the wire. Such a needle (we may call this as small magnetic dipole) tends to line up with the external magnetic field, with its north pole pointing in the direction of B. The direction of the B is along the tangent to the circle of radius r centered on the wire.

Let us turn the dipole through an angle θ from its equilibrium position. To do this we must exert an external torque τ, which must be able enough to overcome the restoring torque that will act on the dipole.

The torque τ, angle of deflection θ and the magnetic field B are related by an equation, which gives the magnitude of τ.

$$\tau = \mu B \sin\theta \qquad \ldots(9.4)$$

$$\text{or } \vec{\tau} = \vec{\mu} \times \vec{B} \qquad \text{(Vectorial representation)} \qquad \ldots(9.5)$$

μ is called magnetic moment of the dipole. Thus by measuring μ and θ we can obtain a relative measure of B for various distances and various currents I in the wire and $\vec{\mu}$ will be a constant for a given magnetic dipole.

The experienced results have shown the following proportionality.

$$B \propto \frac{i}{r} \quad \text{...(9.6)}$$

We introduce a proportionality constant is called the permeability.

$$\therefore B = \frac{\mu_0}{2\pi}\frac{i}{r} \quad \text{Which can be also be written as} \quad \text{...(9.7)}$$

$$(2\pi rB = \mu_0 i) \quad \text{...(9.8)}$$

The left hand side can also be written as §B. dl or a path consisting of circle or radius r centered on the wire, dl is a small element with which the line integration is to be carried to get $2\pi r$ i.e., circumference of the circle around the wire.

$$\oint \vec{B}\,\vec{dl} = \oint \vec{B}\,\vec{dl}$$

$$= \vec{B} \,.\oint \vec{dl}$$

$$= B2\pi r \quad \text{...(9.9)}$$

$$\therefore \oint \vec{B}.\vec{dl} = \mu i \quad \text{..(9.9a)}$$

Which is Ampere's law. It is a general equation and true for any magnetic field configuration, for any assembly of current and for any path of integration.

The permeability constant in Ampere's law has an assigned value of
$\mu_0 = 4\pi \times 10^{-7}$ Weber /Amp

Check your progress

1. The unit of magnetic flux is .....
2. According to Ampere's law the equation is ..... And the statement is ....

Note: (a) Space is given below for your answer.
(b) Compare your answers with those given at the end of the unit.

....
....
....
....

9.6 MAGNETIC FIELD AT A POINT DUE TO A CURRENT CARRYING STRAIGHT WIRE

In a long straight wire carrying a current I as shown in Fig. 9.1 an element idl of the current will produce a magnetic induction dB at a point-situated r from it.

$$\therefore dB = \frac{\mu_0}{2\pi} \frac{dl \sin\theta}{r^2} \qquad (9.10)$$

Expressing dl, sin θ and r^2 in terms of the angle θ

We find that

$$B = \frac{\mu o i}{4\pi r} \int_{-\pi/2}^{+\pi/2} \cos\theta d\theta$$

$$B = \frac{\mu o i}{2\pi r} \qquad \ldots(9.11)$$

The magnitude of B thus falls off inversely as the distance from an infinity long wire and is in the direction perpendicular to a plane containing the wire. The lines of induction B are circles lying a in a plane perpendicular to the wire and are centered on it.

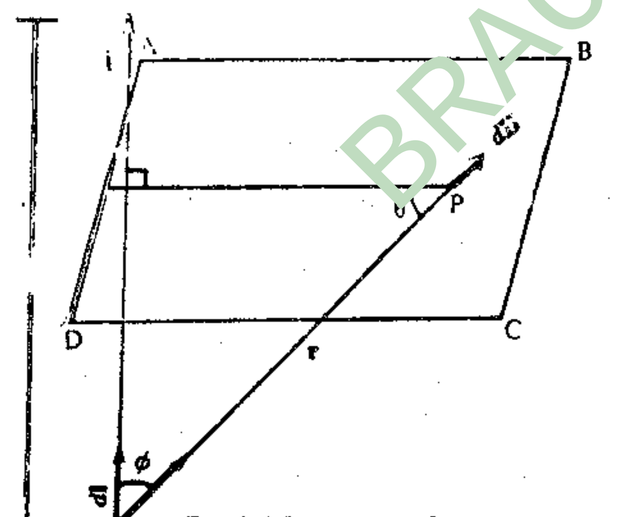
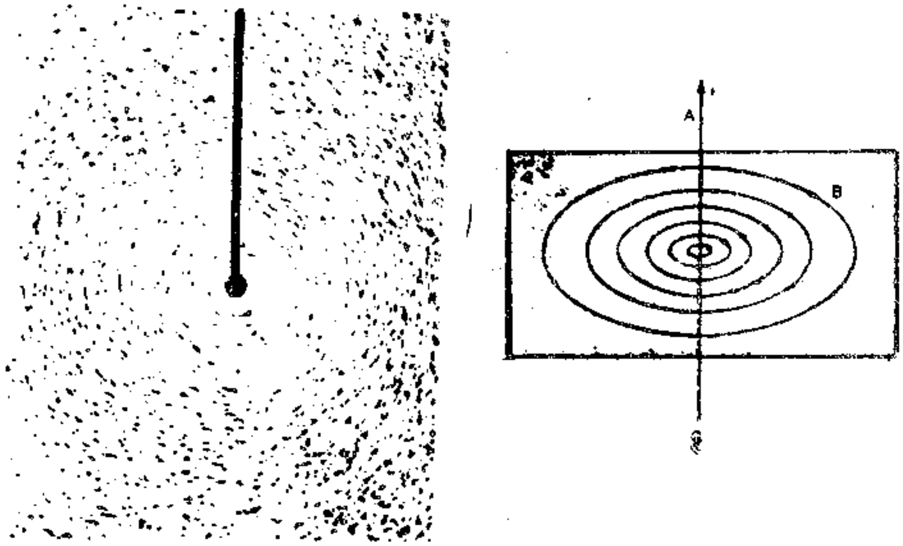


Fig 9.1 Long straight wire carrying current i

9.7 MAGNETIC LINES OF INDUCTION

Suppose AB is a straight wire through which a current is passing upwards. The sense of the magnetic lines of intersection of the card board and the conductor have the common centre. Like wise the lines of induction at any point of the conductor consists of concentric rings with the point at centre.



9.2 (a, b) Magnetic lines of induction of a straight wire as seen by iron filing arranged themselves on a cardboard.

If we carefully look at the lines of magnetic induction representing the field B near a long wire it may be noticed that the increase in the spacing of the lines with increasing distance from the wire. This represents 1/r decrease in B, as predicted in the Eqn. (9.11)

Fig 9.3 Lines of B near a long cylindrical wire cross sectional view.

Thus indicating B is inversely proportional to the distance from the wire. Hence the lines are progressively farther and farther apart as we go out from the centre.

Michael Faraday, who investigated the concept of lines of induction, endowed them with more reality than are give now. He imagined that like stretched rubber bands, the present site of mechanical forces. Today we use lines of induction largely for purpose of visualisation. For quantitative calculations we use the field vectors describing the force on the wire. For example, we will see the relation $\vec{F} = i\vec{l} \times \vec{B}$ in the foregoing sections.

Let us see how the lines of induction due to current carrying wire get modified when the wire is place in an external magnetic field B_e.

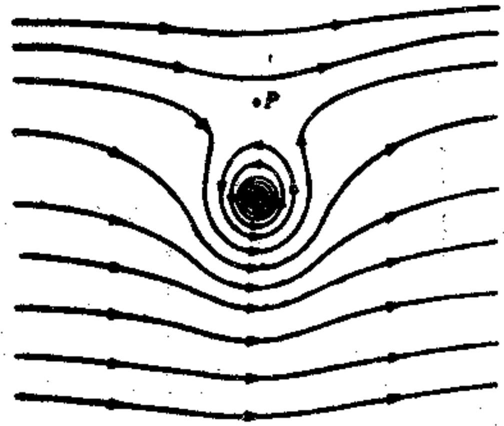


Fig 9.4 Lines of B near a long carrying wire placed in an external field B.

Fig 9.4 shows the resultant lines of magnetic induction associated with a current in a wire, i.e., oriented at right angles to a uniform external field B. at any point the resultant magnetic induction B will be vector sum of B_e and B_i Where Bi is the magnetic induction set up by the current in the wire. The fields B_e and B_i tend to cancel above the wire and reinforce each other below the wire. At one point i.e., at P;B_e and B_i cancel exactly. Very near the wire the field is represented by circular lines and it is essentially due to B_i.

Example 1:

A hollow cylindrical conductor of radii **a** and b carries a current i uniformly spread over its cross section. Show that the magnetic field B for point inside the body of the conductor i.e., a < r < b is given by

$$B = \frac{\mu_0 i}{2\pi(b^2-a^2)} - \frac{r^2-a^2}{r}$$

Check this formula for the limiting case or a = 0

Solution:

Since the current is spread over its cross section uniformly, the current inside the circle of radius r is given by

$$= i \; \frac{\pi(r^2-a^2)}{\pi(b^2-a^2)}$$

$$= i \; \frac{(r^2-a^2)}{(b^2-a^2)}$$

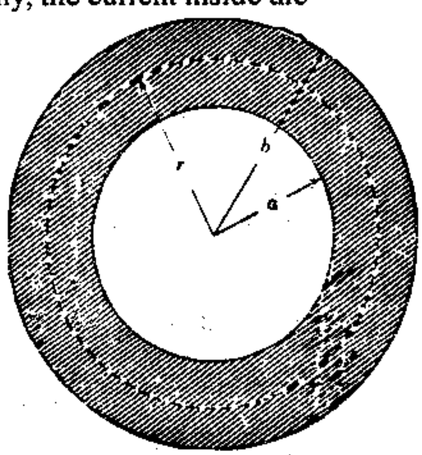


Fig 9.5

(Applying Ampere's law)

The field B for point inside the cross section (a<r<b) is given by

$$B = \frac{\mu o i}{2\pi}\frac{(r^2-a^2)}{(b^2-a^2)}\frac{1}{r}$$

When **a** = 0, $\mathbf{B} = \frac{\mu_0 i r}{2\pi b^2}$

Example 2:

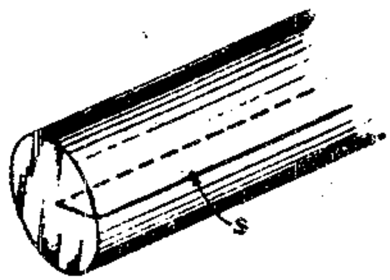
A long cooper wire carries a current of 10 Amp Calculate the magnetic flux ϕ for a plane surface S inside the wire

Solution:

Consider a point at distance the centre.

According to Ampere's law

$$\oint \vec{B}.\vec{dl} = \mu_o i$$



O

Fig 9.

In this problem

$$B = \frac{\mu_o}{2\pi r} \frac{\pi r^2}{\pi a^2} = \frac{\mu_o i r}{2\pi a^2}$$

The average field through the surface shown in figure is

$$B = \frac{1}{a}\int_o^a \vec{B}.\vec{dr} = \frac{1}{a}\int_o^a \frac{\mu_o i}{2\pi a^2} r.dr$$

$$B = \frac{\mu_o i}{2\pi a^3}\int_o^a r.dr = \frac{\mu_o i}{2\pi a3}\left[\frac{r^2}{2}\right]_o^a = \frac{\mu_o i a^2}{4\pi a^3}$$

The magnetic flux per meter of wire for a plane surface S = a x lm^2; inside the wire .

$$B.S = \frac{\mu_o i a^2 a}{4\pi a^3} = \frac{4\pi x 10^{-7}}{4\pi} \times 10 \times 10^{-7} = 10^{-6} \text{ Weber/m}$$

9.8 SUMMARY

Oersted first discovered that a current carrying wire produced magnetic effects.

Magnetic field is a vector and it is denoted by $\vec{B}$. where $\vec{B}$ is a magnetic induction vector. The number of magnetic lines of induction that pass through the Unit area is known as magnetic flux. It is denoted by ϕB

Check your progress: Answers

1. The unit of magnetic flux is Weber.
2. Ampere's law, $\oint B.\ \vec{dl} = \mu_0 i$

According to this Ampere's law, the magnetic induction along a current carrying conductor is the integral of the element of length carrying current i equal to μ_0 times the magnitude of current flowing through it.

9.10 SAMPLE EXAMINATION QUESTIONS

I Answer the following questions in detail

1. How Ampere's law may be use to calculate the magnetic field at a point due to a long current carrying wire? How do you visualize the magnetic lines of induction of a current carrying wirer?

II. Answer the following question briefly.

1. Write the integral form of Ampere's law.

2. What are magnetic lines of induction explain the strength of magnetic field at a
3. point due to a current carrying wire?

UNIT 10:BIOT– SAVART'S LAW

Contents

10.1 OBJECTIVES

This unit discuss the concept of magnetic induction due to a current carrying wire of arbitrary shape and explains its using mathematical equations.

After going through this unit you should be able to make out that the magnetic field arising at any point due to a small element of length of the conductor is

1) Directly proportional to the elemental length of the conductor.

2) Directly proportional to the elemental length of the current flowing through the conductor;

3) Inversely proportional to the square of the distance between the reference point and the elemental length of the conductor; and

4) Directly proportional to the angle made by the line joining the elemental length of the conductor to that point and elemental length.

10.2 INTRODUCTION

Biot-Savart's studied the intensity of the magnetic field around a straight wire carrying current. As a result of their experiments it is showed that the intensity of the magnetic field at a point near the conductor varies directly as the strength of the current through the conductor and inversely to the perpendicular distance of the point from the conductor. This is known as biot-

Savart's Law.In this chapter you will know how to find the force acting between two conductors carrying current.

10.3 BIOT-SAVART'S LAW

Let AB be a linear conductor through which a current 'i' is flowing. According to Biot-Savart's law the magnetic field at any point P due to a small element dl is

(i) directly proportional to the elemental dl of the conductor.

(ii) directly proportional to the strength of the current 'i' flowing the conductor.

(iii) inversely proportional t the square of the distance 'r' of the element from the point, and

(iv) directly proportional to the sine of the angle made by the line joining the element to the point with the element.

Thus $B \propto idl \frac{\sin\theta}{r^2}$ (10.1)

'r' is called a displacement vector from the element dl to P and θ is the angle

between this vector and dl. The direction of B is that of the vector $\vec{dl} \times \vec{r}$

Thus the Biot-Savart's law may be written in a vector form

$$\vec{dB} \propto \frac{i\vec{dl} \times \vec{r}}{r^3} \quad ...(10.2)$$

or

$$\vec{dB} = \frac{\mu_0 i}{4\pi} \frac{\vec{dl} \times \vec{r}}{r^3} \quad ...(10.3)$$

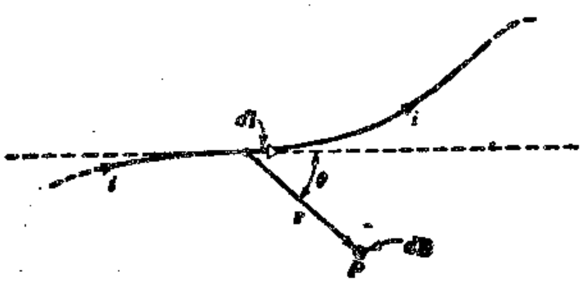


Fig 10.1 a current element dl contributing db at a point P.

The resultant field at P due to the whole length of the wire is found by integrating the Eqn.(10.3)

$$\oint dB = B \qquad (10.4)$$

Ampere's law may be used to calculate magnetic fields only if the symmetry of the current distribution is maximum enough to permit the easy evaluation of the line integral B.dl. This requirement limits the usefulness of the Ampere's law in practical problems. Hence the law becomes difficult to apply in a useful way but it never fails.

Using Biot-Savart's law we shall calculate the magnetic fields due to a straight wire and circular wire.

10.4. A LONG STRAIGHT WIRE

We shall find the magnetic field at a distance Z apart from a long straight wire carrying steady current 'i'.

In the diagram the vector dl x r points out of the page & has the magnitude.

Fig 10.2 A long straight wire carrying current 'i'.

$$dl\sin\theta \ \ \phi = dl\cos\theta$$

$$l = Z\tan\theta$$

$$\therefore\ dl = Z\cos^2\theta\, d\theta \ \text{ and } \ \frac{z}{r} = \cos\theta$$

$$\therefore\ \frac{1}{r^2} = \frac{\cos^2\theta}{Z^2}$$

$$\text{Thus } B = \frac{\mu_0 i}{4\pi}\int_{\theta}^{\theta_2}\left(\frac{\cos^2\theta}{Z^2}\right)\left(\frac{z}{\cos^2\theta}\right)\cos\theta\, d\theta \qquad \ldots(10.5)$$

$$= \frac{\mu_0 i}{4\pi Z}\left(\sin\theta_2 - \sin\theta_1\right) \qquad \ldots(10.6)$$

Eqn.(10.6) gives the filed of any straight segment of wire, in terms of the initial and final angles θ and θ_2. In the case of an infinite wire $\theta_1 = -\pi/2$ *and* $\theta_2 = +\pi/2$

$$\text{So we obtain } B = \frac{\mu_o i}{2\pi Z} \qquad \ldots(10.7)$$

B points out of the page. This is the result we arrived at earlier for this problem. Thus the law of Biot-Savart will always yield results that are consistent with Ampere's law.

10.5 A CIRCULAR WIRE CARRYING CURRENT

We shall find the magnetic field at a distance Z above the center of a circular loop of radius R, which carries a steady current. The field dB attributable to the segment dl points as shown in figure. As we integrate dl around the loop dB describes a cone. The horizontal components cancel, and the vertical components combine to give.

$$B = \frac{\mu_o i}{4\pi} \int \frac{dl}{r^2} \cos\theta \qquad \ldots(10.8)$$

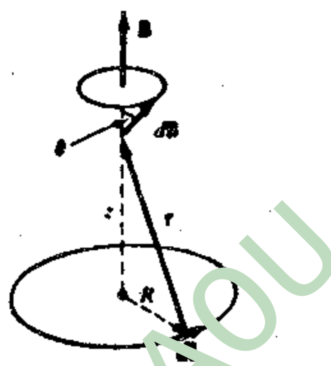


Fig 10.3 A circular wire carrying current

Figure shows that r and θ are not independent of each other. Let us express in terms of a new variable, Z, the distance from the center of the loop to the point P.

The relationships are

$$r = \sqrt{R^2 - Z^2} \qquad \ldots[10.9(a)]$$

$$\cos\theta = \frac{R}{r} = \sqrt{\frac{R^2}{R^2 + Z^2}} \qquad \ldots[10.9(b)]$$

Substituting these values into the expression for B i.e.,

Eqn. (10.8) gives

$$B = \frac{\mu_o i}{4\pi} \oint \frac{R}{(R^2 + Z^2)^{3/2}} \, dl \qquad \ldots(10.10)$$

Integrating this equation, and noting that $\oint . dl$ is the simply circumference, 2π R, So

$$B = \frac{\mu o i}{R} \frac{R^2}{(R^2+Z^2)^{3/2}} \qquad \text{.........(10.11)}$$

If we put Z >> R, Eqn. (10.11) reduces to

$$B = \frac{\mu o i R^2}{2Z^3}$$

This equation may be writen as

$$B = \frac{\mu o}{2\pi} \frac{NiA}{Z^3} \; (\because A = R^2) \qquad \text{...(10.12)}$$

$$= \frac{\mu o}{2\pi} \frac{\mu}{Z^3}$$

When μ = Ni A is the magnetic dipole moment while A is the area with N turns.

In this way current loop can be regarded as a magnetic dipole, which experiences a torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ when placed in an external magnetic field., It generates its own magnetic field for the points to the axis, as given by the equation.

Example 1:

In the Bohr model of hydrogen atom the electron orbits round the nucleus in a path of radius 5.1 x 10^{-11} m at a frequency of 68 x 10^{+15}rev/s. What value of B is set up at the center of the orbit?

Solution:

The current is the rate at which charge passes any point on the orbit and is given by

i = ev – 1.6 x 10^{-19} x 68 x 10^{+15} = 1.1 x 10^{-3} Amp.

B at the center of the orbit is given by =

$$\therefore \quad B = \frac{(4\pi \times 10^{-7} \text{ weber/Amp-m})(1.1 \times 10^{-3} \text{ Amp})}{2 \times 5.1 \times 10^{-11} \text{m}} = 14 \text{ weber/m}^2$$

Example 2:

A circular copper loop of radius of 10 cm carries a currents of 15 Amp. At its center is placed a second loop of radius 1.0 cm, having 50 turns and a current of 1 Amp (a) What magnetic induction B does the large loop set up at its center (b) what torque acts on the small loop? Assume that the planes of the two loops are at right angles and that the induction B provided by the large loop is essentially uniform throughout the volume occupied by the small loop.

(a) B near the center of the loop having 10 cm radius carrying a current of 15 amps is

$$B = \frac{\mu o i}{2R} = \frac{4\pi \times 10^{-7} \times 15}{2 \times 10^{-1}} = 9.4 \text{ weber/m}^2$$

(b) Torque acting on the small loop at right angles to the first to be given by

$$\tau = N\,i\,A\,B$$
$$\tau = 50 \times 1 \times \pi \times (0.01)^2 \times 30\,\pi \times 10^{-6}$$
$$\tau = 1.5 \times 10^{-6} \text{ N.m}$$

10.6 TWO PARALLEL CONDUCTORS CARRYING CURRENTS

Let us now examine the force between two parallel wires a, b carrying currents i_a and i_b separated by a distance d as shown in Fig 10.4

Wire (a) carrying current i_a will produce a field of induction B_a in its surroundings. The magnitude of B_a at the site of second wire is

$$Ba = \frac{\mu o i a}{2\pi d} \qquad \ldots(10.13)$$

The direction of the B_a at the location of wire b is down as shown in Fig 10.4

Fig 10.4 Force of attraction between two parallel wires carrying parallel currents.

Wire B is carrying current I_b finds itself immersed in an external field of magnetic induction B_a. The length of this wire b experience a force (il x B) whose magnitude is

$$F_b = i_b\, l B_a$$

$$= \frac{\mu_0 l i_a i_b}{2\pi d} \qquad \ldots(10.14)$$

The force F_b lies in the place of the wires and points towards the wire a as shown in Fig 10.4

We shall as well start with wire b, compute the field of induction which produces at the site of the wire a, and then the force on wire a. Thus the force on wire 'a' would point towards the wire 'b'. Thus the forces that the two wires exert on each other are equal and opposite. Hence according to Newton's law these must be of action and reaction.

For anti parallel currents, the two wires repel each other.

From the Eqn (10.14) the force per unit length is

$$\frac{F}{l} = \frac{\mu_0 i_a i_b}{2\pi d} \qquad \text{...(10.15)}$$

This equation provides us with definition of the ampere, the unit of current, if the two wires are 1 metre apart and the two current area equal ($i_a = i_b = i$). If this common current is adjusted until (by measurement), the force of attraction per unit length between the wires is 2×10^{-7} N/m the current is defined to be one ampere.

$$\frac{F}{l} = \frac{\mu_0 i^2}{2\pi d} = \frac{4\pi \times 10^{-7}\,(\text{weber/Amp-m})\,(1\ \text{Amp}^2)}{2\pi (\text{m})}$$

$$2 \times 10^{-7}\ \text{N/m} \qquad \text{...(10.16)}$$

This assumes that the diameters of the wires are negligible compared to their separation.

10.7 MAGNETIC INDUCTION DUE TO A SOLENOID CARRYING CURRENT

A long cylindrical coil is called a solenoid. Thus a solenoid is a long wire wound in a close packed helix. The helix is assumed to be long compared to its diameter (Fig 10.5). We are interested to calculate the field set up in and around the solenoid, when a steady current passes through it. If we consider a single turn of a solenoid, the wire is bent as an arc.

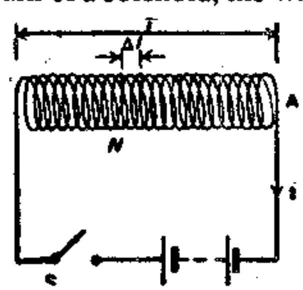


fig 10.5 (a) Solenoid. N-Number of turns, A – Area of cross section

Hence the wire behaves magnetically almost like a long straight wire and the line of B due to this single turn are almost concentric circles. Thus the solenoid field is the vector sum of the fields setup by all the turns that makes up the solenoid.

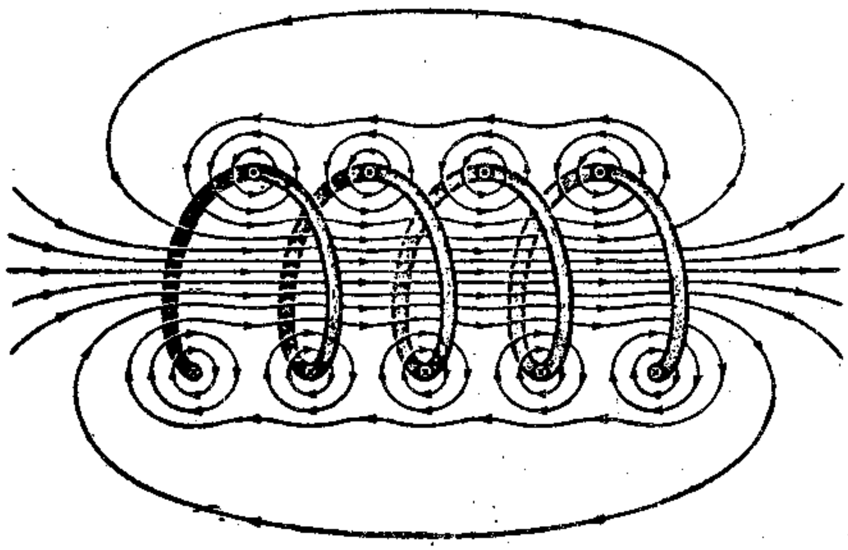


Fig 10.5 (b) Magnetic lines of induction in and around a solenoid.

For the sake of understanding, if we see the induction of a solenoid with widely spaced turns, it suggests that the field partially get cancelled near the wires. It also suggests that B gets reinforced at the center and it is parallel to the solenoid axis for the points which are far from the wires inside the solenoid.

The field setup by the upper part of the solenoid turns points to the left and tends to cancel the field setup by the lower part of the solenoid turns which points to the right. As the length of the solenoid approaches the configuration of an infinitely long cylindrical current sheet, the induction outside the solenoid approaches zero. For practical solenoid, if its length is must greater than its diameter, the field external to the solenoid is weaker or insignificant. For the solenoids whose length is not much greater than the diameter, the external field is much weaker than the internal field.

To calculate the magnetic induction, we take a section of the solenoid as shown in the Fig. 10.5 (c)

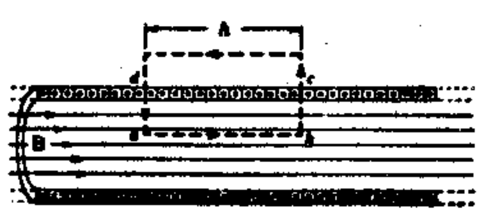


Fig 10.5(c) The field must be zero outside infinitely long solenoid.

Applying Ampere law

$[\oint \vec{B}.\vec{dl} = \mu_0 i]$

to the rectangular path abcd in the solenoid, we may write the integral B.dl as the sum of fou integrals one for each path of the segment.

$$\oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl} \qquad ...(10.17)$$

Thus first integral on the right is 'B' where 'B' is the magnitude of the induction inside the solenoid and h is the arbitrary length of the path from a to b. The path ab is parallel to the solenoic axis. The second and the fourth integrals are zero because for every element of these paths 'B' is at right angles to the path, because we have taken 'B' as zero for all external points for an idea solenoid. The third integral which includes the parts of the rectangle that lies outside the solenoic is zero,

Thus $\oint \vec{B}.\vec{dl}$ for the entire rectangular path has the value Bh.

Let 'n' be the number of turns per unit length and total current passing through all the turns is i_0

Thus the current I due to h, length of the solenoid, is

$$i = i_0\ (nh) \qquad ...(10.18)$$

Ampere's law becomes

$$Bh = \mu_0 i_0 nh$$

$$B = \mu_0 i_0 n \qquad ...(10.19)$$

This equation shows that B does not depend on the diameter or the length of the solenoid. A solenoid is a practical way to setup a known magnetic field for experimental purpose, just as a parallel plate capacitor to setup a known uniform electric field.

Check your progress:

1. Biot Savart's law may be written in vector form as ....
2. The magnetic field expressions for 'B' due to
3. The magnetic induction in a solenoid does not ________________ depend on __________ and ______________ of the solenoid.

Note:

a. Space is given below for your answers.
b. Compare your answers with those given at the end of the unit.

....
....
....

Example 1:

Field of a toroid: Fig 10.6 illustrates a toroid wound with N turns of wire carrying current 'i'. The mean radius of the toroid is 'd'.

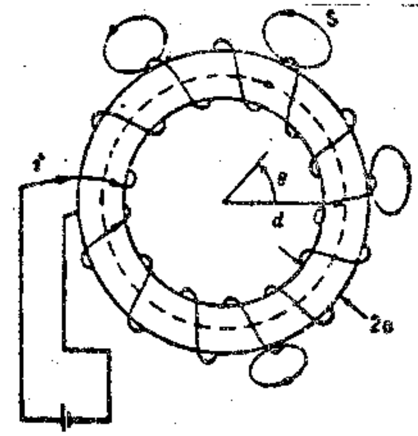


Fig. 10.6 A toroid wound with N turns.

The cross section of the toroid is circular with radius 'a' much smaller than d, i.e., A<<d;

The tangential field b is continuous across the boundary separating the toroid and the air region just outside but inside the helix winding. Therefore the flux density B in the interior is much greater than that outside the toroid. Thus most of the flux lines are concentrated in the interior as shown in the figure 10.7.

Applying Ampere's law to circular path of integration of radius d

$$\oint \vec{B}.\vec{dl} = \mu_0 i$$

$$B(2\pi d) = \mu_0 iN$$

Where 'i' is the current in the toroid windings and N is the total number of turns.

$$B = \frac{\mu_0}{2\pi}\frac{iN}{d}$$

Example 2:

Two long wires at a distanced 'd' apart carry equal and antiparallel currents 'i'. Show that B at a point P which is equidistant from the wires.

$$B = \frac{2\mu_0 id}{\pi(4R^2+d^2)}$$

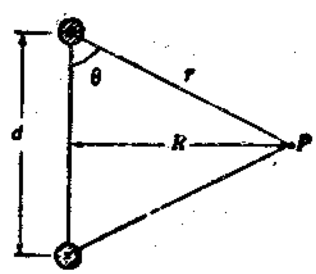


Fig 10.7

Solution:

As shown in the figure, the vertical components cancel each other and the horizonta components are reinforced. The field due to each wire at P at a distance 'r' is given by

$$B = \frac{\mu_0 i \cos\theta}{2\pi r}$$

The field B at point P due to both the wires

$$= \frac{2\mu_0 i \cos\theta}{2\pi r}$$

But from the figure, we have

$$\cos\theta = \frac{d}{2} \Big/ \sqrt{\left(\frac{d}{2}\right)^2 + R^2} = d \Big/ \sqrt{d_2 + 4R^2}$$

$$\text{Since } r = \sqrt{\left(\frac{d}{2}\right)^2 + R^2}$$

$$= \frac{1}{2}\sqrt{d^2 + 4R^2}$$

$$\therefore B = \frac{2\mu_0 i}{2\pi \frac{1}{2}\sqrt{d^2 + 4R^2}} \cdot \frac{d}{\sqrt{d^2 + 4R^2}}$$

$$= \frac{2\mu_0 i d}{\pi(4R^2 + d^2)}$$

Example 3:

Four long 10 SWG copper wires are parallel to each other, their cross section is forming a square of 20 cm on edge A 20 amp current is setup in each wire in the direction shown in Fig. 10.9. What are the magnitude and direction of B at the center of the square?

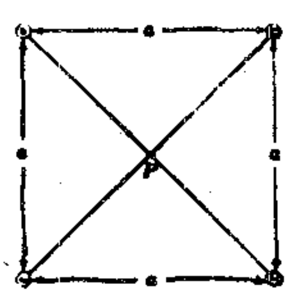


Fig 10.8

Solution:

$i = 20$ Amp $a = 20\text{cm} = 0.2\text{m}$

The field directions are shown in figure, only the vertical components reinforce each other the cross section of wires from a square.

$$B = \text{near the point } p = 4 \times \frac{\mu o i}{2\pi a/\sqrt{2}} \cos\theta$$

$$= \frac{4x4\pi 10^{-7} x\, 20}{2\pi x 0.2/\sqrt{2}} \; \frac{1}{\sqrt{2}} = 8 \times 10^{-5} \text{ weber/m}^2$$

Example 4:

A solenoid 1 meter long and 3.0 cm in mean diameter. It has five layers of windows 850 turns each and carries a current of 0.5 Amp. What is B at the center?

Solution:

$B = \mu_o i_o n$

$= 4\pi \text{ X } 10^{-7} \text{ X } 5 \text{ X } 5 \text{ X } 850$

$= 2.7 \text{ X } 10^{-2}$ Weber / m^2

10.8 SUMMARY

The magnetic field at any point due to a small elemental length of the conductor is directly proportional to he elemental length of the conductor, strength of the current flowing through the conductor, sine of the angle made by the line joining the elemental length of the conductor to that point and elemental length and inversely proportional to the square of the distance between the reference point and the elemental length of the conductor.

There will be attractive force between the parallel wires carrying currents in the same direction while the force is repulsive if the direction of currents are opposite. The resultant force per unit length between two current carrying parallel wires is given by $\frac{F}{I} = \frac{\mu_o i_a i_b}{2\pi d}$

The magnetic induction of a solenoid depends upon the number of turns n, wound on the solenoid and the current 'i' passing through the solenoid.

$B = \mu n i$

Check your progress: Answers

1. $d\vec{B} = \frac{\mu_o i}{4} \frac{dl \times r}{r^3}$

2. (a). $B = \dfrac{\mu_o i}{2\pi r}$

(b). $= \dfrac{\mu_o}{2\pi} \cdot \dfrac{NiA}{r^3}$ (Where r =distance of pt on axis of coil)

3. The magnetic induction of a solenoid depends upon the number of turns n wound on the solenoid and the current 'i' passing through the solenoid.

10.9 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Show that Ampere's Law is a special case of generalized Biot-Savart's Law considering a straight-wire.
2. Show that there exists a force of attraction between two parallel current carrying wires, when the direction of the flow of the current in both the wires is along the same direction.

II. Answer the following question briefly.

1. Applying Biot-Savart's Law, calculate the magnetic induction at a point on the axis passing through the centre of a circular.
2. Describe the working principle of a current balance.
3. Deduce an equation to represent the magnetic induction of a solenoid.

III. Solve the following problems.

1. A conducting washer of thickness 't' has an inner radius r^1 and outer radius r^2. Find the flux density B as a function of 'r' if a current 'i' flows (a) between inner and outer edges (b) between the flat surfaces (c) around the washer (circular loop current)
2. A straight wire segment of length 1 carries a current 'i', show that the field induction b to be associated with this segment at a distance R from the segment along a perpendicular bisector is given in magnitude by

$$B = \frac{\mu_o i}{2\pi R} \frac{l}{(l^2 + 4R^2)^{1/2}}$$

$F_{max} = q\,VB$ ...(11.1)

In the equation 11.1 except q all other quantities are vector quantities. Hence a vector definition of B may be defined as follows:

If a positive test charge q is fired with a velocity V through a region and if a side ways deflecting force F is experienced by the moving charge then a magnetic induction B is present at that region satisfying the relation.

$$\vec{F} = q\vec{V} \times \vec{B} \qquad ...(11.2)$$

i.e $F = QVB \sin\theta$ where θ is angle between $\vec{V}$ and $\vec{B}$ as shown in figure ... (11.3)

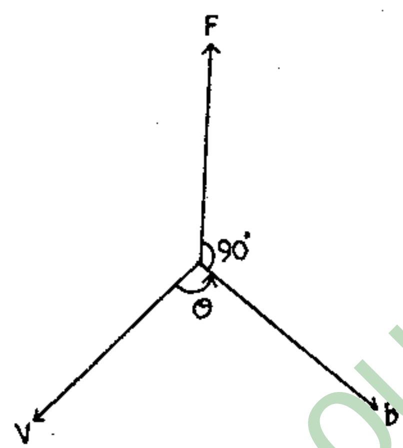


Fig 11.1 Vectorial representation of $\vec{F} = q\vec{V} \times \vec{B}$

If a charged particle moves through a region where an electric field E and a magnetic induction B are present then the resultant force experiencing by the moving charge is

$$\vec{F} = q\vec{E} + q\,\vec{V} \times \vec{B} \qquad ... (11.4)$$

This equation is known as Lorentz force equation.

11.4 MAGNETIC FORCE ON A CURRENT

A current in a wire may be visualized as assembly of moving charges. Hence if a current carrying wire is placed in magnetic field a force of the Lorentz type will act in the wire. We shall calculate this force on the wire.

Let a wire of length i carrying a current I be placed in a magnetic field of strength B. This direction of current density vector J and magnetic field vector B are taken to be Perpendicular to each other.

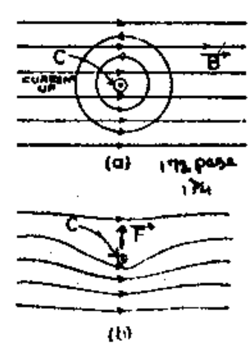


Fig11.2(a) & (b)

Current in a wire is carried by the free electrons. Hence, the magnitude of the force on a single such electron is given by

$$F = qV_d\, B \sin\theta = q_v\ B \text{ for } \theta = 90^\circ \qquad \ldots(11.5)$$

Where V_d is the drift speed of the electron; and

$V_d = \frac{\vec{J}}{Nq}$ and N being the number of electrons per unit volume.

$$\vec{F}^1 = q\ \frac{j}{nq}\ B$$

$$\vec{F}^1 = \frac{JB}{n} \qquad \ldots(11.6)$$

Let the volume of the wire b, 1A. A being the cross sectional area. Then the total number of electrons in the wire is nA1.

The net force on the wire is

$F = n1AF'$

$F = n1A\frac{JB}{n}$

$\therefore F = ilB$ Since $JA = I$ ...(11.7)

Equation 11.7 hold only if the wire is at right angles to B. We can write a more general equation in a vectorial form

$$\vec{dF} = i\vec{dl} \times \vec{B} \qquad \ldots(11.8)$$

and the integration of 11.8 yields

$$\vec{F} = i\vec{l} \times \vec{B} \qquad \ldots(11.9)$$

11.5 TORQUE ON A CURRENT LOOP

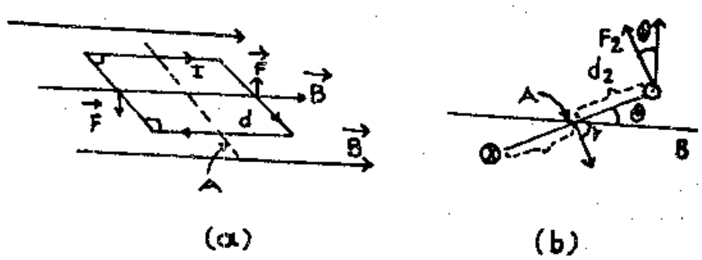


Fig 11.3 (a) (b) Rectangular loop in a field of uniform magnetic field

10.10 RECOMMENDED BOOKS

1.	Kraus, J.D and Carver, K.R	Electromagnetics	Mc Graw-Hill Kognakusha Ltd.,Tokyo
2.	Corsan, D.R. and Lorrian, P	Introduction to Electromagnetic Field Waves	Freeman Toppa, London.
3.	Grifffiths,D.	Introduction to Electromagnetic	Prentice-Hall of India New Delhi.
4.	Plonsey, R. and Collin, R.E.	Principles and Applications of Electromagnetic fields	Mc Graw -Hill Publishing CompanyLtd. New Delhi.
5.	Laud, B.B	Electromagnetics	Wiley Eastern Ltd., New Delhi.
6.	Halliday, D. and	Physics –Part II	Wiley Eastern Ltd. New Delhi.
7.	Grant, I.S. and	Electromagnetism	John Wiley & Sons, Chichester.
8.	Wenham, E.J., Dorling, G.W., Snell, J.A.N. and Taylor,B.	Physics concepts and models	Addision-Wisely Publishers Ltd. London.

UNIT – 11: MAGNETIC FIELD AND MAGNETIC FORCE ON A CIRCUIT TORQUE

Contents

11.1 OBJECTIVES

This unit explains the effect of the magnetic field on moving charge. To make you under stand mathematical equations are used to spell out the effect.

After going through this unit you should be able (1) to describe the effect of external magnetic field on a current loop (2) to calculate the potential energy associated with the loop.

11.2 INTRODUCTION

In this unit we will discuss the effect of magnetic field on a moving charge. We will discuss the force exerted on a current carrying conductor kept in a magnetic field. Also know about the Torque on a current loop.

11.3 ACTION OF THE MAGNETIC FIELD ON A MOVING CHARGE

In this section we consider the detection of magnetic induction B through its effects on a moving charge while moving through it.

Suppose a positive test charge is injected with certain velocity into a region of the field and if a deflecting force is experienced by the charge, we assert that a magnetic field exists at that point.

The direction of $\vec{B}$ may be identified as follows. Let the velocity of the unit charge be $\vec{V}$ and the deflecting force on it is found to be at right angle to $\vec{V}$. Due to the directional change of the velocity the magnitude and direction of F can be inferred from experimental observation It is found that the force $\vec{F}$ will be maximum when the velocity $\vec{V}$ is perpendicular to B i.e.

When a current loop is placed parallel to a magnetic field the force acts on the loop in such a way that it tends to rotate the loop. The product of tangential force and the radial distance at which it acts is called the torque; or mechanical moment on the loop. Torque (or mechanical moment) has the dimensions of force times distance and is expressed in Newton Meters.

Consider the rectangular loop as shown in figure 11.3 (a) with sides of length l and d placed in a magnetic field of uniform flux density B. The loop carries a steady current i.

As per the equation (11.8) the force on any element dl of loop is

$$\vec{dF} = i\vec{dl} \times \vec{B} \qquad \ldots(11.8)$$

If the plane of the loop is at an angle θ with respect to B as indicated in the cross-sectional figure 11.3(b) then the tangential force $F_t = |F| \cos 0$

$$Ft = iB\cos\theta \int_0^l dl \qquad \ldots(11.10)$$

$$= iBl \cos\theta$$

The total torque on the loop is then

$$T = 2F_t \frac{a^l}{2}$$

$$= iBl\, d \cos\theta$$

Since id = A the torque

$$\text{So } \tau = I\, AB \cos\theta \qquad \ldots(11.11)$$

Hence according to equation 11.11, the torque is proportional to the current in the loop, to its area and to the flux density of the field in which the loop is situated.

The product of I and A in equation (11.11) is designated as the magnetic moment or magnetic dipole moment of the loop μ

With $\mu = N\, iA$

Where N is the number of turns in the loop

Then the torque $\tau = U\, B \cos t\theta$

$$\text{Or } \tau = \mu B \sin\alpha \qquad \ldots(11.12)$$

Where α is the angle between the normal to the plane of the loop and the direction of **B** (See figure 11.3 (b))

If the magnetic dipole moment is regarded as a vector μ along the direction $\hat{n}$ normal to the plane of the loop and its positive sense is determined by the right hand rule.

The torque relation of (11.12) can be expressed in a more general form

As $\vec{\tau} = \vec{\mu} \times \vec{B}$ ...(11.3)

The torque τ is considered to be a vector as coinciding with the axis of rotation of the loop as given by $\vec{\mu} \times \vec{B}$ The direction of the torque on the loop is obtained by turning $\vec{\mu}$ into $\vec{B}$

Check your progress:

1. The force exerted on a current carrying conductor when placed in a magnetic field is ...

2.. Torque on a current loop as expressed in vector form is ......

Note: a) Space is given below for your answers
b) Compare your answers with those given at the end of the unit.

....
....
....
....

Potential energy associated with the current loop in a field.

When a magnetic dipole (or a current loop) is placed in an external magnetic field, a torque acts on it. Then it follows that work must be done by an external agent to change the orientation of the dipole. Thus the magnetic dipole has potential energy associated with its orientation in an external magnetic induction B. This energy may be taken to be zero for any arbitrary position of the dipole. We assume that the magnitude of potential energy U is zero when $\vec{\mu}$ and $\vec{B}$ are right angles to each other i.e $\alpha = 90$.

The magnetic potential energy in any position is defined as the work that an external agent must do to turn the dipole from its zero energy position ($\alpha = 90^0$) to the given position.

$$U = \int_o^\alpha \tau d_\alpha \int_o^\alpha NiABsin\alpha d\alpha$$

$$U = \mu B \int_0^\alpha \sin\alpha d\alpha$$

$$= - \mu B\cos\alpha$$

$$\text{or} = - \vec{\mu}_0 \vec{B}$$

$$U = - \mu_o B$$

...(11.14)

Examples:

1. A wire of 60cm length and mass 10 grams is suspended by a flexible leads in a magnetic field of induction 0.40 weber/meter^2 . What are the magnitude and direction of the current required to remove tension in the supporting leads?

   The weight of the wire = mg = 0.001 * 9.8 = 9.8 * 10^{-2}. In order to have no tension in the leads, the force acting on the wire must be equal to the weight of the wire

   $mg = ilB$

   $i = mg/lB \qquad = 9.8 \times 10^{-2} / 0.6 \times 0.4 = 0.41$ Amp

   $i = 0.41$ Amp

2. A rectangular loop of a wire having sides 10cm, 5cm, carries a current of 0.10 Amp and is hinged at one side. What torque acts on the loop if it is mounted with its plane at an angle 30^0 to the direction of a uniform field of magnetic induction 0.50 webers/m^2

   Force on the conductor $= F = NiB \times l$

   $= 20 \times 0.1 \times 0.5 \times 0.1$

   $= 0.1$ N

   $\tau = F \times 0.05 \cos 30^0$

   $\tau = 0.1 \times 0.05 \times \cos 30^0$

   $\tau = 4.3 \times 10^{-3}$ N

11.6 SUMMARY

When the moving charge is placed in magnetic field, it produces a torque on the moving charge.

Check your progress: Answers

1)Force exerted is given by $\vec{F} = i\, \vec{l} \times \vec{B}$ (Where B is the magnetic field intensity & 'i' is the current flowing through a conductor of length l)

2)The torque on a current loop is given by $\vec{\tau} = \vec{\mu}.\vec{B}$ (Where μ is the permeability of the medium.).

11.7 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail

1. Discuss the action of magnetic field on a moving charge.
2. Derive the equation to calculate the force acting on a current carrying wire placed in a magnetic field.

II Answer the following questions briefly.

1. Derive equation for a torque on a current loop.
2. What is the potential energy associated with the loop in the field.

III. Solve the following problem

1. A particle of mass m charge q moving with velocity V enters a region where there is a uniform magnetic field B. The vectors V and B make an angle θ with each other Determine the motion of the particle.

BLOCK – 4: ELECTROMAGNETIC INDUCTION

UNIT 12: MOTION OF CHARGED PARTICLES

Contents.

12.1 OBJECTIVES

This Unit discusses the effects of electric and magnetic fields on electric charges at rest or in motion. To make you understand the effect the basic principles are illustrated.

After going through this Unit you should be able to make out the energy of the particle moving through the electric field would increase and that the motion of charged particles along the lines of force in the uniform magnetic field would not be effected.

12.2 INTRODUCTION

In this unit we will discuss the motion of a charged particle in electric and magnetic fields.

In the period between 1914 and 1916 a controversy has arisen between F. Ehrenhaft and H.A. Millikan about the elementary nature of electron. In 1897 J.J. Thomson showed cathode rays to consist of a stream of negatively charged particles. In the early part of 1897 E. Wieehert and W. Kaufmann in Germany reported results of their measurements on the path of cathode rays in a magnetic field. Kaufmann found that the value of charge to mass ratio(e/m) for cathode ray particles was the same regardless of the nature of gas present in the discharge tube. Wieehert compared this e/m value with those calculated for a hydrogen ion in solution and concluded the particles and a mass of approximately that one-thousandth part of a hydrogen atom. At a later date J.J. Thomson made similar studies. His experiments became classical even though are not of great accuracy.

12.3 CHARGED PARTICLES IN ELECTRIC FIELDS

When a particle of mass 'm' with charge 'e' is allowed to pass through an electric field of strength $\vec{E}$, (The electric field E, is being created by the two parallel plates P_1 & P_2 by the application of voltage) the force $\vec{F}$ on the particle due to electric field E, is given by

$$\vec{F} = q\vec{E} \qquad \ldots(12.1)$$

As per the Newton's 2nd Law of motion

$$\vec{F} = m\vec{a}$$

or $$\vec{a} = \vec{F}/m \qquad \ldots(12.2)$$

A uniform electric field can be achieved by connecting the plates P_1 & P_2 (Figure 12.1) to the terminals of an voltage source (say Battery). If the spacing between the plates is small compared with their dimensions, the field between them will be fairly uniform except near the edges. We should note a point here that in calculating the motion of charged particle in a field set up by external charge the field due to the particle itself i.e. field is ignored. Now we will consider two cases of the acceleration of charge particle in a uniform electric field.

Case(1): The charged particle in an uniform electric field (initially at the rest)

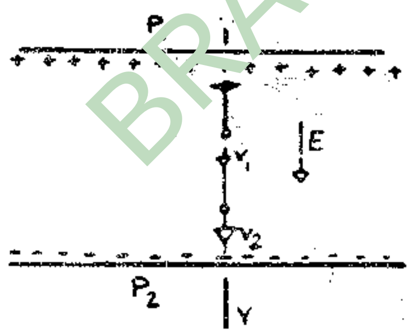


Fig.12.1

A particle of mass 'm' and charge 'q' is released from rest in the uniform electric field The motion of the particle resembles that of a body falling in gravitational field. The acceleration is thus

$$\vec{a} = qE/m$$

having defined the acceleration and electric field.

The equation of kinematics for uniform acceleration can be applied with initial velocity $V_0 = 0$

$$And\ V = \vec{a}t = \frac{qEt}{m} \qquad \ldots(12.3)$$

Since the particle is moving only along 'y' directions i.e. from up to down, the distance traveled along 'y' in time 't' is given by

$$y = \frac{1}{2}at^2 = \frac{qEt^2}{2m} \qquad ...(12.4)$$

$$V^2 = 2ay = \frac{2eEy}{m} \qquad ...(12.5)$$

The kinetic energy attained by the charge particle after moving a distance 'y' is

$$K.E. = \frac{1}{2} mV^2$$

$$= \frac{1}{2} m \frac{(2qEy)}{m}$$

$$= qEy$$

$$= q^2E^2t^2/2m \qquad ...(12.6)$$

Case (2): The charged particle in a uniform electric field (initial velocity being not zero)

Fig 12.2 shows the path of a charged particle of mass 'm' and charge 'q' moving with velocity V_0 at right angles to the uniform electric field E along 'y' direction. We will show that the motion of such charge particle will be parabolic as long as it is in the field. As the electron emerges out of the field, it travels in a straight-line tangent to the parabola at the exit point. The general motion of the particle can be described in terms of its motion along the horizontal direction by

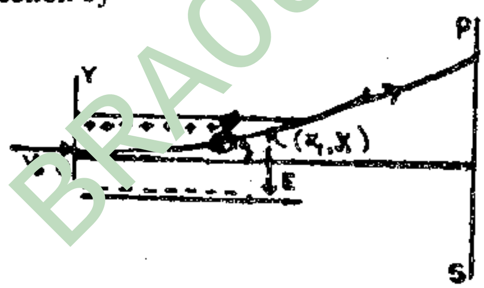


Fig 12.2

$$X = V_0 t \qquad ...(12.7)$$

And the displacement along the vertical direction can be shown to be (using Eqn. 12.4)

$$Y = \frac{1}{2} at^2 = qEt^2/2m \qquad ...(12.8)$$

Combining the equations 12.7 and 12.8 we get

$$Y = (qE/2m V_0^2)X^2$$

Which shows that the path of the particle is a parabola.

Example 1:

An electron moving with a speed 5.0×10^8 cm/sec is shot parallel to an electric field strength 1.0×10^3 N /coul (a) How far will the electron travel in the field before coming to rest? And (b) how much will elapse? (c) What fraction of initial energy will the electron lose in traversing it?

(a) $V_0 = 5 \times 10^8$ cm/s

$= 5 \times 10^6$ m/s

$E = 1 \times 10^3$

And $F = ma = qE$

$a = qE/m$

Thus from the equation of kinematics we have

$$V^2 - V_0^0 = 2aS^0$$

Where V is the final velocity, V0 is the initial velocity and S the distance traveled.

$$-5 \times 5 \times 10^{12} = -2qES/m$$

(- Ve means retardation)

$$25 \times 10^{12} = \frac{2 \times 1.6 \times 10^{-19} \times 9 \times 1 \times 10^3 \times S^0}{9.11 \times 10^{-31}}$$

or $S^0 = 7.11$ m

(b) $V = V_0 + at$

$V_0 = -at$ since $V = 0$

$$t = \frac{V_0}{a} = \frac{V_0 m}{qE} = \frac{5 \times 10 \times 9 \times 11 \times 10^{-31}}{1.6 \times 10^{-19} - \times 10^3 \times 1}$$

$= 26 \times 10^{-8}$S

(c) K.E (initial) = $\frac{1}{2} m V_0^2$

K.E (lost) = qEX

K.E (initial) = $\frac{1}{2} \times 9.3 \times 10^{-31} \times 25 \times 10^{12}$

$= 113.7 \times 10^{-19}$

K.E (lost) = $1.6 \times 10^{-19} \times 1 \times 10^3 \times 8 \times 10^{-13}$

$= 12.8 \times 10^{-19}$

Percentage fraction of loss

$$= \frac{12.8 \times 10^{-19}}{113 \times 10^{-19}} \times 100$$

$= 11\%$

2. The motion of charged particle in the magnetic field

The magnetic field does not effect the stationary charge. But if the charge is In motion, a force will be usually exerted. The motion of a charge is equivalent to current

Case (i) consider a charge in motion with a velocity V at right angles to a magnetic field of strength B. Then time 't taken by it over a distance l is t = l/V. The motion of charged particle constitutes a current 'i'

$$it = q$$

$$i = q/t = \frac{q}{l/V} = \frac{qV}{l}$$

$$\text{or } il = qV$$

But the force on this current element from equation

$$F = Hil = BqV \qquad \ldots(12.9)$$

Fig 12.3 shows a negatively charged particle introduced with a velocity V into the uniform magnetic field of induction B. We assume that V is at right angles to B and thus the motion lies entirely in the plane of the figure. The relation F = qVB shows that the particle will experience a side ways deflecting force of magnitude qVB. This force will lie in the plane of the figure, which means that the particle cannot leave this plane.

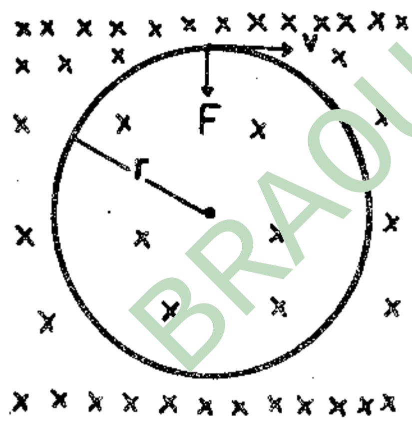


Fig 12.3

Having defined the force, the kinematic relation can be shown as follows:

$$qBV = \frac{mV^2}{R} \qquad \ldots(12.10(a))$$

$$\text{or } R = \frac{mV}{qB} \qquad \ldots(12.10(b))$$

Which gives the radius of the path.

The angular velocity ω is given by V/r from equation 12.9 we have.

The frequency V measured in Hertz is given by

$$\omega = \frac{V}{r} = \frac{qB}{m} \qquad \ldots(12.11)$$

The frequency ν measured in Hertz is given by

$$\nu = \frac{\omega}{2\pi} = qB/2\pi m \qquad \ldots(12.12)$$

here we can notice that V does not depend on the speed of the particle. Fast particles move in large circles and slow ones in small circles but all require the same time (T) to complete one revolution the field.

The frequency is a characteristic frequency for the charged particle in the fields and may be compared to the characteristic frequency, of swinging pendulum, in the earth's gravitational field or to the characteristic frequency of an oscillating mass spring system. It is called the cyclotron frequency of the particles in the field.

Case (ii): when a charged particle enters the magnetic field with a velocity V along a path making an angle θ with the direction of the field, its velocity may be resolved into two components:

$$V = V_{11} + V_{\perp}$$

Where V_{11} is parallel to the field $\vec{B}$ and $V_{\perp}$ is perpendicular to the field $\vec{B}$ still remains a circle: equations 12.10 and 12.11 also apply here provided that V is used for the speed in equation 12.10. The component V_{11} remains constant because there is no acting Force (on the particle moving parallel to B). The path that the particle follows is then a helix as shown in figure. 12.4

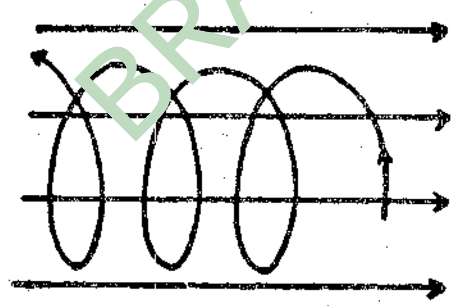


Fig 12.4

Example 2:

A proton moves in a circle of radius r = 0.1 m. S in a magnetic field B of magnitude 1.2 **Weber/m2** (a) what is the cyclotron frequency? (b)What is the K.E. (in MeV) of the **proton**?

(a) from equation 12.10 we have

$$qVB = \frac{mv^2}{r}$$

and $\omega = V/r = qB/m$

$$= \frac{(1.2 Weber/m^2)(1.60 \times 10^{-19} coul)}{1.67 \times 10^{-27} kg}$$

$$= 1.15 \times 10^{8} sec^{-1} \quad \text{(i.e. rad/s)}$$

The corresponding frequency (in Hz) is

$$= \omega / 2\pi = 1.83 \times 10^{7} sec^{-1}$$

(b) $qVB = mV^2/r$

$$V = qBr/m$$

$$K.E = \tfrac{1}{2} mV^2$$

$$= \tfrac{1}{2} m (qBr/m^2)$$

$$= \tfrac{1}{2} q2B^2r^2/m$$

$$= 1/2 (1.60 \times 10^{-19} coul)^2 (1.2 web/m)^2 (0.1m)^2 \times$$

$$= \frac{1 eV/1.60 \times 10^{-19} coul}{1.67 \times 10^{-27} kg}$$

$$= 1.7 \times 10^{-} eV$$

$$= 17 MeV$$

Example 3:

A 5.0 cm length of wire carries a current of 3.0 Amp. There is a uniform B field of magnitude 10^{-13} Weber/m^2, whose direction is shown in the diagram. Calculate the magnetic force exerted on the wire.

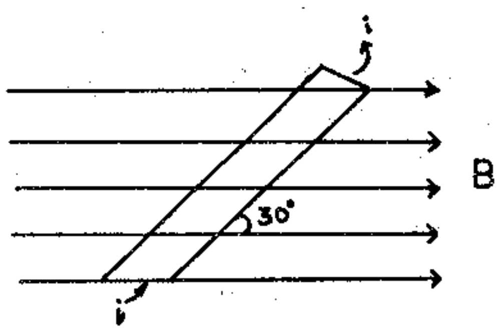


Fig 12.5

F = il x B from equivalent 12.9

For F = ilBsin θ

Where θ = 30, is the angle between i & B

$F = (3.0 \text{ amps}) (5.0 \times 10^{-2} \text{ m}) (10^{-3} \text{ Web/m}) (0.5)$

$= 7.5 \times 10^{-5} N$

The direction of this force is in to the paper.

12.4 CYCLOTRON

From the preceding section we have learnt that the charged particle gains energy while passing through electric field and the motion of charged particle can be found by the use of magnetic field. Thus a positively charged particle traveling in a uniform magnetic field will experience a force $F q\vec{V} \times \vec{B}$. This force is directed perpendicular to $\vec{V}$ and $\vec{B}$ therefore, the particle moves in circular path with radius.

$$r = \frac{mv}{qB} \qquad \text{...(12.10(b)}$$

From this equation it may be inferred that the radius of the particle orbit in a uniform magnetic field is directly proportional to its momentum and inversely proportional to its charge and to the induction of the field. Since the constant negative field cannot change the energy of the particle it keeps the same speed and energy, it had when it entered the field, even though it has been deflected by the magnetic field.

The cyclotron, a device for accelerating charged particle is based upon equation 12.10 (b) the time required for the charged particle in a magnetic field to make a complete revolution, since the distance it moves in that time is the circumference 2π of its circular path is

$$T = \frac{\text{distance}}{Speed} = \frac{2\pi r}{V} = \frac{2\pi m V}{VqB}$$

$$= \frac{2\pi m}{qB} \qquad \text{...(12.11)}$$

Which does not depend upon the speed of the particle or the radius of its orbit. The particle goes around the circle in period time that varies only with mass and charge and with magnetic field.

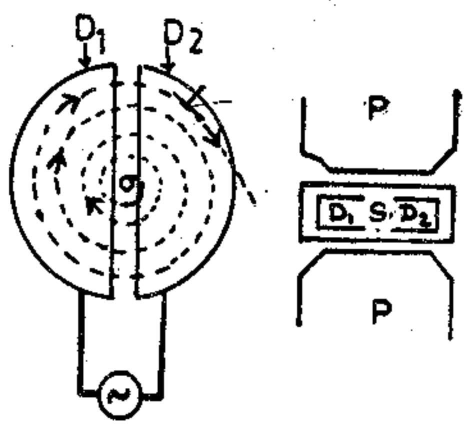


Fig 12.6 cyclotron

P = Magnetic/ pole pieces

S = High frequency oscillator

D_1D_2 = Deflectors

Fig 12.6 is a sketch of a cyclotron. The magnetic field is directed upwards as shown. The Dees' (so called because they are like a letter D) are hollow copper boxes that are connected to a source of alternating potential (called an Oscillator) that their polarity changes regularly. When a charged particle, for instance a proton, is injected into the space between the Dees from a suitable source, it is attracted by the Dee that is negative since its own charge is positive. Within the Dee's the magnetic fields compels the proton to travel in a semicircle, as in Fig 12.6. When it comes out at other side, if the alternating current has the proper frequency, the opposite D will be negative and the proton will be accelerated across the gap between Dees. Then it circulates with in the second Dee and again receives acceleration when it emerges and so on. Ultimately the proton has sufficient energy to leave the cyclotron, through the opening shown. The principle of the cyclotron is to use a relatively small electric field to accelerate charged particles by causing them to be acted upon by the field repeatedly. If the period of the oscillator is exactly equal to the period of the protons in the magnetic field they will always be attracted to the opposite Dee when they reach the gap between the Dees even though their speed (and the radius of their orbit) is greater each time they arrive there.

The accelerated particles are usually protons, deutorons, and particles. The energy acquired will depend upon the size of the Dees as the maximum velocity will correspond to the path of the radius equal to that of Dees

$$V_{max} = \frac{B_g R_d}{m}$$

(where Rd is the radius of the Dee)

$$E_{max} = \frac{1}{2} V^2 max$$

$$= \frac{1}{2} m \left(\frac{B_g R_d}{m}\right)$$

$$= \frac{B^2 R_d^2 q^2}{2m} \quad \ldots(12.12)$$

Example 4:

A typical cyclotron has a magnetic induction of 1.5 Web/m^2 and accelerating protons then calculate the period, frequency? What is the speed of a 10 meV proton?

$$T = \frac{2\pi m}{qB}$$

$$= \frac{2 \times 1.7 \times 10^{-27} kg}{1.6 \times 10^{-19} Coul \times 1.5\ Web/m^2}$$

$$= 4.5 \times 10^{-8} s$$

Since the charge of proton is 1.6×10^{-19} Coul. and its means is 1.7×10^{-27} kg. This means that the Oscillator should have the frequency of

$$f = 1/7 \times 2.2 \times 10^{7} S^{-1}$$

Which is 22 MHz, a perfectly feasible operation of frequency. If the P.D. between the dees is 100,000 volts, which is about as high as it can conveniently be made, each proton receives 10^5 eV of energy each time it passes through the gap. In course of 100 passages, corresponding to 50 completed revolutions, the proton therefore receives a total energy of 10^7eV. (1eV = 1.6×10^{-19} Joules) and so such a proton has kinetic energy.

$$K.E.\ 10^{-7} eV \times 1.6 \times 10^{-19} \times 10^{-19}\ Joules/eV$$

$$= 1.6 \times 10^{-12}\ J$$

Hence

$$K.E. = ½\ mV^2$$

$$V = \frac{2K.E}{m} = \frac{2 \times 1.6 \times 10^{-12} J}{1.7 \times 10^{-27} kg}$$

$$4.4 \times 10^7\ m/s$$

At speeds comparable to that of light, 3×10^8m/sec, the mass of the body increase with increasing speed. Under these circumstances the period of revolutions of a particle in a cyclotron in no longer a constant, independent of its speed, and it will no longer be accelerated since it will not reach the gap between the Dees when the electric field there is favorable. Variable frequency devices an d other new methods of accelerating particles to very high energy have been developed under different names, such as synchrotron, alternating gradient machines

12.5 HALL EFFECT

We have seen that a metallic conductor carrying a current placed in a magnetic field, experiences a force tending to make it move in a direction, perpendicular both to the direction of the current and to that of the field, If the conductor is fixed the moving charged particle, constituting the current should be displaced within the conductor under the action of transverse magnetic field. This then should lead to the potential difference, when a current is passed along a strip of metal foil, and when the magnetic field, perpendicular to the plane of the foil is applied, a measurable transverse emf called the Hall voltage was set up between the points on the opposite edges of the foil. Figure 12.7 shows the arrangement to observe Hall voltage.

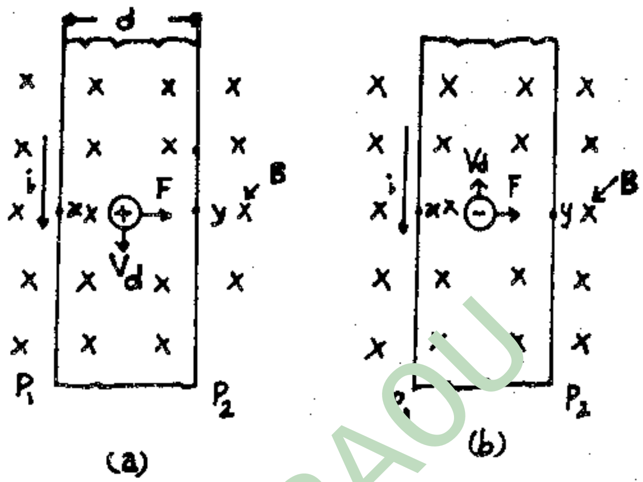


Fig 12.7 Hall effect

If the current is due to the motion of electrons moving upwards (i.e. in the direction opposite to the conventional current, then applying Fleming left hand rule to the conventional current as shown in Fig 12.7 the force on the charge carriers is towards the edge P_2 and the electrons are moved toward P_2, making it negative while the edge becomes positive. Thus an emf set up between P_1 & P_2 Hall was able to show that the current in a metal is the result of negatively charged particles i.e. electrons.

From the above discussion it follows that hall Voltage is due to the transverse force on the carries in the conductor.

If

q is charge o reach carrier and

V the velocity of drift charge along the conductor

N the No. of charged particles per unit volume

B the magnetic field intensity

Then the electric field intensity E set up by hall Voltage between P_1 & P_2 in equilibrium,

Since F = 0 is given by

$$Eq = qVB \quad \ldots(12.13)$$

If d = thickness of the foil

V_H = Hall Voltage, b = breadth of the foil

$$\text{Then } E = \frac{V_H}{bd} = BV \quad \ldots(12.14)$$

But $i = NaVbd$

$$V = i/Nqbd$$

$$V_H/bd = \frac{Bi}{Nebd} = \frac{1}{Nq} \cdot \frac{Bi}{bd}$$

$$= R_H$$

Where $1/Nq = R_H$ is called the Hall coefficient.

Check your progress:

1. One of the most important applications of a charged particle when placed in an intense magnetic field is ..
2. According to Hall effect, the relation between electric field intensity, E and set up Hall Voltage (V_{H}) and magnetic field induction B is...

NOTE:

a. Space is given below for your answers.

b. Compare your answers with those given at the end of the unit.

....

....

....

12.6 DETERMINATION OF CHARGE OF AN ELECTRON BY MILLIKAN'S OIL DROP METHOD

Millikan's method for measuring electronic charge of an electron is simple and elegent. An experiment to measure, it must be carried out with a body having so few charges that the change in one charge should make a noticeable difference. Since the experiment must be done with very little charge, the force of the body experience will be small even though a large electric field is utilized. If the force on the charged body is very small the body itself must be very light. The force of gravity is always there, and if a small electric force is not marked by a large gravitational force, then the mass of body must be small and as well as magnitude as the gravitational force it experiences, then it may be that the gravitational force will be a useful standard of comparison.

Millikan used a drop of oil as his test body. (fig12.8) It was selected from a spray produced by an ordinary atomizer. The drop was so small that its could not be measured optically. But with and optical microscope it could be seen as a bright spot because it scattered light from an intense beam like a minute dust particle in bright sun light.

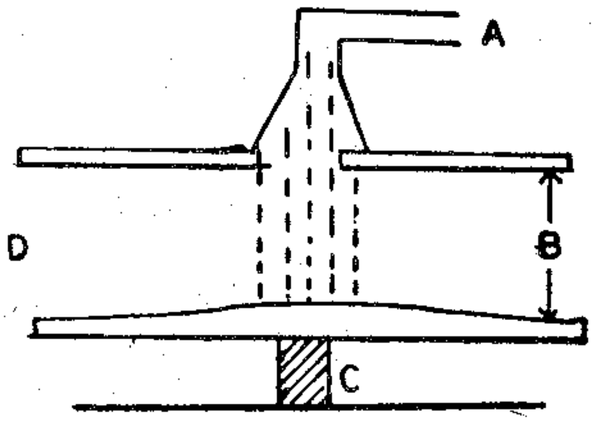


Fig. 12.8

When such a drop falls the influence of gravity, it is hindered by the air it passes through. The way in which the fall of small spherical body is hindered by air had been described by stokes, who found that such a body experiences a resisting force R proportional to its velocity.

$$R = KV \quad \ldots(12.15(a))$$

The proportionally constant K was found by Stoke's expression involving coefficient of viscosity of the resisting medium and the radius of the body.

A falling droplet of oil is acted as on by its weight W, the buoyant force F_B of the air. And the resisting force R = KV (Fig 12.8). The resultant downward force F is

$$F = W - F_B - KV \quad (12.15(b))$$

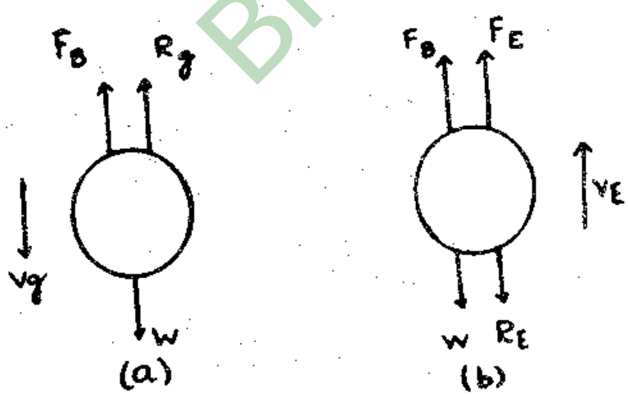


Fig 12.9 a) on falling down b) on moving up

Initially the velocity V is zero, the resisting force is zero, and the resulting downward force equals to $W - F_B$. The drop therefore has an initial downward acceleration. As its downward velocity increases the resisting force increase and eventually reaches a value such that the resultant force is zero. The drop then falls with a constant velocity called terminal Velocity. Vg since $F = 0$, $V = V_g$, we have from equation (12.8)

$$W - F_B = KV_g \quad \ldots(12.16)$$

Let ρ_o be the density of oil and ρ_a the density of air then

$$W = \frac{4}{3}\pi r^3 \rho_o g$$

$$W = \frac{4}{3} r\pi^3 \rho_a g \qquad \ldots(12.7)$$

In the experiment, the oil drop is situated between two horizontal plates where a known strong electric fiedld may be directed upward or downward as shown in Fig 12.8. the droplet has a small electric charge 'q' which may be –ve or +ve depending on whether it has excess or deficiency of electrons. The droplet gets this charge from rubbing against the nozzle of the atomizer and from encounters from stray charges left in the air by cosmic rays, or deliberately produced by X-rays etc., In the electric field a drop will experience a force, qE. which can always be directed upwards by proper choice of the direction of E. The drop must be adjusted either to rise or fall between the plates and should not touch the plates at any time.

The microscope with which the drop movements are followed is equipped with two horizontal hairlines whose separation represents a known distance along the vertical line in which the drop travels. By timing the trips of the drop over this known distance, the terminal velocities of the drop are found. The velocities of fall, Vg, are all the same, since oil does not evaporate and therefore the weight of the drop is constant. The velocity of rise, however, depends on the charge. The resultant force on the drop while it rising is

$$F = qE + FB - W - KV \qquad \ldots(12.8)$$

When the terminal velocity VE – is reached the resultant force is zero, so

$$qE = W - F_B + KV_E \qquad \ldots(12.19)$$

But from equation (12.9)

$$W - F_B = KV_g^2 \text{ so finally}$$

$$= (K/E)(V_g - V_E) \qquad \ldots(12.20)$$

since the terminal velocities are constant, they are relatively easy to measure.

In the oil drop experiment, the value of value of Vg I determined for a particular drop with the electric field off, and a whole series of V_E's for the same drop is observed with the field on.

The best determination of q is $(1.60210 + 0.00007) \times 10^{-19}$ Coulombs

Example 5:

A charged oil drop is suspended in a inform field of 300 volts/cm so that it neither falls nor rises. Find the charge on the drop, given its mass 9.75×10^{-12} g in this problem

$E = 300$ *Volts/cm*

$= 30,000$ *Volts/m*

$m = 9.75 X 10^{-12} g$

$= 9.75 X 10^{-15} kg$

and $g = 980\ m/s^2$

here $qE = mg$

$$q = \frac{mg}{E} = \frac{9.75 \times 10^{-15} \times 9.8}{30,000}$$

$= 31.85 X 10^{-16}$ Coulombs

Example2:

A charged oil drop is prevented from falling under gravity by a vertical electric field between two horizontal metal plates charged to a potential difference 6920 V, the distance between the plates being 1.3 cm. When the field is cut off the drop falls in air with uniform velocity of 1.9×10^{-11} m. Calculate (a) the radius of the drop (b) charge on the drop?

Density of oil = $0.9 X 10^3 kg/cm^3$
Coefficient of viscosity of air = $1.81 X 10^{-5}$
The density of air may be neglected in comparison to that of oil.

(a) Substituting the given values

$$a = \frac{9 \times 1.81 \times 10^{-5} \times 1.9 \times 10^{-4}}{9 \times 9.81 \times 0.9 \times 10^{3}}$$

$= 1.75 \times 10^{-6} m$

(b) $qE = (4/3)a^3$

$$\text{Hence } E = \frac{6920}{1.2 \times 10^{-2}} \text{ Volt/m}$$

$$q \times \frac{6920}{1.2 \times 10^{-2}} = 4/3\,(7.75 \times 10\text{-}6)^3 \times 0.9 \times 10^3 \times 9.81$$

solving we get

$e = 4.042 \times 10^{-19}$ Coulombs

12.7 THOMSON'S EXPERIMENT

In 1897 J.J.Thomson measured the ratio of the charge to the mass 'm' of electron by observing the deflection of electron beam in presence of electric and magnetic fields.

A modernized version of Thomson's apparatus is shown in figure 12.8

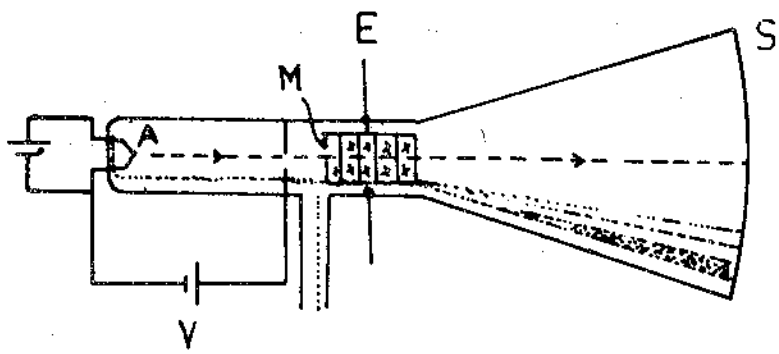


Fig 12.8 Thomson's modified apparatus

A = Hot filament *S= Fluoroscent screen*

M= Magnetic field *E= Electric filed* *V = Electrical*

The electrons enter the region in such a way. So that the motion of the electrons is perpendicular to the electric and magnetic fields. In the experimental configuration the electric and magnetic fields are arranged to be perpendicular to each other. The electron beam is allowed to strike the fluorescent screen 'S' which in there shows a illuminated spot.

In this method electrons are emitted from hot filament A and accelerated by an applied electric potential V. The entire space in which the electrons move is evacuated for avoding collisions with air molecules.

The resultant force F on a charge particle moving through an electric E and magnetic B fields is given by equation19.1

$$F = qE + q\, V x\, B \qquad ...(12.21)$$

Where q = charge and V = velocity of the particle. It can be concluded from the figure that the electric field deflects the particle upward and the magnetic field deflects it downward.

In these deflections caused by the fields, cancel each other then the net force being zero given by equation 12.21 reduces to

$$q\,E = VB = 0 \qquad ...(12.22)$$

For an electron, equation 12.22 can be written as

$$e\,E = e\,VB \qquad ...(12.23)$$

$E = vB$ ...(12.24)

e = electrical and q being designated as the charge of the electron.

Equation 12.24 shows that for a given value of V, the zero deflection can be achieved by adjusting the value of E and B. The main points in Thomson's method can therefore be summarized under.

1. The position of the undeflected beam on the screen is noted when electric and magnetic fields zero.
2. The deflection on the fluorescent screen is then noted when a fixed electric field E is applied, and
3. Finally the magnetic field B is applied and its value adjusted until deflection of the electron beam is back to zero.

12.8 SUMMARY

The energy of the charged particle increases when it is moving along the electric lines of force. When the charged particle is moving in a magnetic field the force acting on the particle is always normal to the field direction. The motion of the particle is unaffected when the charged particle is moving along the field direction.

Check your progress: Answers

1. Cyclotron.
2. $E = \frac{V_H}{B_d} = B\,v$

12.9 SAMPLE EXAMINATION QUESTIONS

I Answer the following Questions in detail

1. Describe experiment for the determination of e/m an electron.
2. Discuss the effect of electric and magnetic field o electric charge at rest and at motion.
3. Discuss the application of these effects.

II. Answer the following Questions briefly

1. What is Hall effect?
2. Describe Millikan's oil drop method.
3. Give the principle underlying the determination of e/m an electron.

III. Solve the following problems

1. A uniform field of magnetic induction B poin horizontally from South to North. Its magnitude is 1.5 Weber/m2. If 5.0 Me V protons moves vertically downward through this field, what force w act on it.

2. In a number experiment 1.0MeV protor moves in a uniform magnetic field in a circular path what energy must
   (a) an alpha particle and
   (b) a deutron have if they are circulate in the same orbit

3. A wire of 1.0 meter long carr a current of 10 amp and makes an angel 30^0 with a uniform magnetic eld with $b = 1.5$ Webers/m^2. Calculate the magnitude and direction the force on the wire.

4. Prove that the relat $T = N\ AiBSin\theta$ holds for closed loops of arbitrary shape.

5. A length L of ire carries a current 'i'. Show that if that wire is formed into a circular coi le maximum troque in a given magnetic field is developed when the coil as one turn only an the maximum troque has the value $\tau = (1/4\pi)'iB$.

UNIT – 13: ISOTOPES AND THEIR MASSES

Contents

13.1 OBJECTIVES

This unit explains the methods of isotopic separation of atomic masses and the determination of atomic masses through the use of Aston's and Dempsters mass spectrographs.

After going through this unit you should be able to calculate the isotopic content of atomic masses.

13.2 INTRODUCTION

Goldstein in 1886 observed streams coming out of perforated cathode in cathode ray tube. Since the particles associated with these rays were positive, they were called as positive rays. The particles of the positive rays are generated in the space between cathode and anode. These are essentially the positive ions which are created by the cathode particles striking the atoms and molecules of the gas in the tube. The masses of ions are the same as those of atoms and molecules and are very heavy compared to cathode rays. Thus if gas contains some impurities, the positive rays would consist of ions of all the gases unlike the cathode rays which are electrons purely.

The analysis of these ions which yields information about the actual composition of the gas is an important branch of study called the mass spectrograph.

13.3 ASTON'S MASS SPECTROGRAPH

Principle

Aston's mass spectrograph is an apparatus of high accuracy to determine the isotopic masses. The principle and working of the instrument differs from that of Thomson. In that the electrical and magnetic fields are co –terminus and do not act in the same region of space but act at different stages. First the beam is subjected to the electric field and then to a perpendicular magnetic field at such a distance away from the electric field and of such proper strength and direction of action, that the dispersion of the beam caused by the formed field is annulled by the latter so that the beam is brought back to the sharp focus on the detection system.

The positive rays emerging from the perforated cathode are made into a fine pencil by using slits. They are subjected to an electrostatic field in a direction perpendicular to the direction of the rays with the help of the electrically charged plates P_1 and P_2. The beam is not only deflected but also dispersed because the particles have different velocities. The dispersed beam is then subjected to a magnetic field, whose direction is perpendicular to the electrostatic field. Thus the magnetic field produces dispersion and deviation in an opposite direction in the same plane. If the photographic plate is held in the direction of the deflected beam, line images are obtained. Each line corresponds to a particular value of e/m. The number of lines corresponds to the number of isotopes present in the element.

Description of Apparatus

The different parts of Aston's mass spectrograph are shown in figure 13.1.

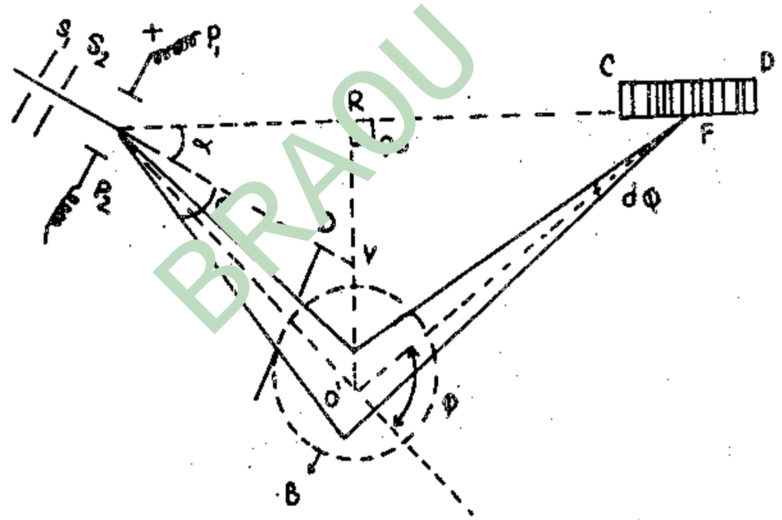


Fig. 13.1 Aston spectrograph

CD = Photographic film

B = Magnetic field

A_o is the direction of positive rays before entering the electrostatic field. S_1 and S_2 are slits, which provides a fine pencil of positive rays. The electrostatic field is maintained by the plates p_1 and P_0. The beam is deflected and dispersed downwards. Let θ and dθ be the angles of deviation and dispersion. Using a diaphragm D some of the rays are selected and are allowed to pass between the poles of an electromagnet. The magnetic field being perpendicular to the plane of the paper and inwards, according to the Fleming's left hand rule, the beam will be deflected upwards. This magnetic field annules the dispersion produced by the electric field but recombines the particles which are brought to focus in the form of sharp lines on a photographic plate (detection system) CD. The lines are similar to the lines of the spectrum.

Considering that the deflection in electrostatic field is small and near the vertex may be considered as circular for radius r, we have

$$eE = \frac{mV^2}{r}$$

$$\text{or } \frac{1}{r} = \frac{Ee}{mV^2}$$

Where E = Electric field, m = mass, V = velocity of the particle and e is the charge.

Hence the deflection θ, which is proportional to 1/r is given by;

$$\theta = C\frac{Ee}{mv^2} = C1\frac{e}{mv^2}$$

where $C_1 = c\,E$

$$\therefore \text{ dispersion } \frac{d\theta}{dv} = -2C_1\frac{e}{mv^3} = -2\frac{\theta}{v} \qquad ....(13.1)$$

if r^1 is the radius of curvature in the magnetic field then

$$Be\upsilon = \frac{m\upsilon 2}{r^1} \text{ (or) } \therefore \phi = C_1\frac{Be}{m\upsilon}$$

$$\frac{1}{r1} = \frac{Be}{mv} \quad \therefore B \text{ is constant}$$

$$= C_2\frac{e}{m\upsilon}$$

where $C_2 = C_1\,B$
again dispersion

$$\frac{d\phi}{d\upsilon} = -C_2\frac{e}{m\upsilon^2} = -\frac{\phi}{\upsilon} \qquad ....(13.2)$$

from equation 13.1 & 13.2 we have

$$\frac{d\theta}{\theta} = 2\frac{d\theta}{\theta} \qquad ...(13.3)$$

Thus for a given deflection the dispersion due to the electrical field is twice that of magnetic field. The small changes dθ and dθ refer to the particles with identical mass and charge but possessing velocities differing by dV. In the absence of magnetic field the dispersion produced in the beam for a distance (a +b) is given by $(a + b)\, d\theta$ ...(13.4)

where a is the distance O^1O

and b is the distance O^1F

The magnetic field acts in the direction perpendicular to the electric field and produces, the same dispersion in a distance b, but in opposite direction.

Dispersion produced by the magnetic field $= b\, d\phi$ ...(13.5)

As all the ions are focused to the same position

$$(a + b)\, d\theta = b\, d\phi$$

$$\text{and} \quad \frac{d\theta}{d\phi} = \frac{b}{(a+b)}$$

From equation 13.3 we have

$$\frac{d\theta}{d\phi} = \frac{2\theta}{\phi}$$

$$\frac{b}{(a+b)} = \frac{2\theta}{\phi}$$

$$\text{or } b\phi = (a + b)\, 2\theta$$

$$b(\phi - 2\theta) = 2a\theta \qquad \text{...(13.6)}$$

This is the condition of focusing.

Let $O^1 R$ be perpendicular to the line in CD

and $< ROV = \alpha$.. Then from ΔROO^1 we have

$$RO^1 = OO^1 \sin(\alpha + \theta) \qquad \text{...(13.7) (a)}$$

In $\Delta RO^1 F$,

$$RO^1 = OF \,\text{Sin}(RFO^1)$$

$$= b \sin\left[180 - (\phi - \alpha - \theta)\right]$$

$$= b \sin(\phi - \alpha - \theta)$$

$$a \sin(\alpha + \theta) = b \sin(\phi - \alpha - \theta) \qquad \text{...(13.7) (b)}$$

For small angles

$$a(\alpha + \theta) = b(\phi - \alpha - \theta) \qquad \text{...(13.8)}$$

Comparing equations 13.7(b) and 13.8 it is observed that the two equations are same when $\alpha = \theta$. This focusing condition is that the photographic plate must be placed at an angle θ with the direction of the incident positive ray beam.

Thus we find in Aston's apparatus

(i) All the particles of same e/m are brought to the same focus irrespective of their velocities.

(ii) Particles of different masses are brought to the different foci

Aston investigated positive rays from various different elements and found that the relative masses of atoms were more or less integers. Integers are found to contain atoms having two or more different masses proportional to integers and also with different abundances. Xenon with atomic weight 130.2 is found to be a mixture of atoms of atomic. Weights 128, 129, 130, 131, 132, 134 and 136, which are all isotopes of Xenon.

13.4 DEMPSTER'S MASS SPECTROGRAPH

Dempster's method is unique in the sense that in addition to the mass identification it is also possible to find the relative abundances of various isotopes of a given element. The positive ions are produced in an evacuated chamber by heating a platinum strip, coated with salt of element under observations and bombarding the vapor with electrons from a filament. All the ions are accelerated by the electrostatic field to the same energy by allowing them to pass through a constant potential difference and are then subjected to the magnetic field perpendicular to the place of the paper, where by they travel in a circular path.

The ions traversing the same circle of radius emerges out at the slits (13.2) and are collected at E, connected to the electrometer. We have the following relationship.

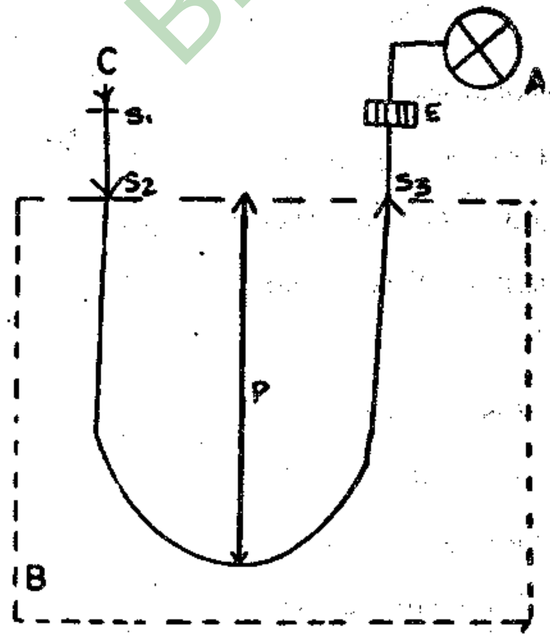


Fig 13.2 Dempster's Mass Spectrograph

A= Electrometer
B= Magnetic field
C= Positive ion

In the electrostatic field

$$V_e = \frac{1}{2} mV^2$$

In magnetic field

$$\frac{mv^2}{r} = Bev$$

Hence $e/m == 2v/ B^2 r^2$ ...(13.9)

Dempster kept H constant and current due to ions at E was measured at different values of (e/m). Ions will travel in a semicircle of fixed 'r' only at a particular value of V. Hence at the suitable value of the voltage, the current is maximum. If ions of another e/m value are present they will give maximum current at another value of the voltage. In this manner the accelerating potential corresponds to the atomic masses gives the relative abundance of isotopes as shown in Fig 13.3 for potassium element, which shows two isotopes with atomic masses 39 and 41.

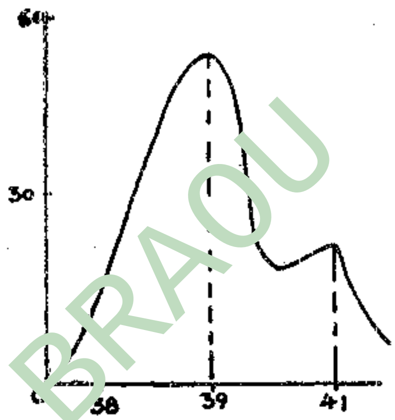


Fig 13.3 Mass spectrum of Potassium

Example – 1:

An electron beam passes through a magnetic field of 2×10^{-3} webers/m^2 and the electric field of 3.4×10^4 volts/net both acting simultaneously at the same point. If the path of the electron remains undeviated, calculate the speed o the electrons. If the electric field is removed, What will be the radius of electron path (mass of electron = 9.0×10^3 kg and charge = 1.6×10^{-19} coulombs).

In equilibrium $Bev = Ee$

$$V = E/B$$

$$= \frac{3.4 \times 10^4}{2 \times 10^{-3}} = 1.7 \times 10^7 \text{ m/s}$$

On removing the electric field

$$Bev = mv^2/r$$

$$\text{Or } r = \frac{mv}{Be}$$

$$= \frac{9.0 \times 10^{-31} \times 1.7 \times 10^{7}}{2\times10^{-3} \times 1.6\times10^{-19}}$$

$$= 4.78 \times 10^{-2}\ m$$

13.5 BAIN BRIDGE MASS SPECTROGRAPH

For determination of isotopic masses the apparatus for this purpose is shown in figure below Fig. 13.4

The beam of +ve ions produced in a discharge tube is collimated by two slits S_1 & S_2 and enter a velocity selector. The velocity selector consists of (1) a steady electric field 'X' maintained at right angles to the ion beam between two plane parallel plates P_1 & P_2 and (2) a magnetic field B.

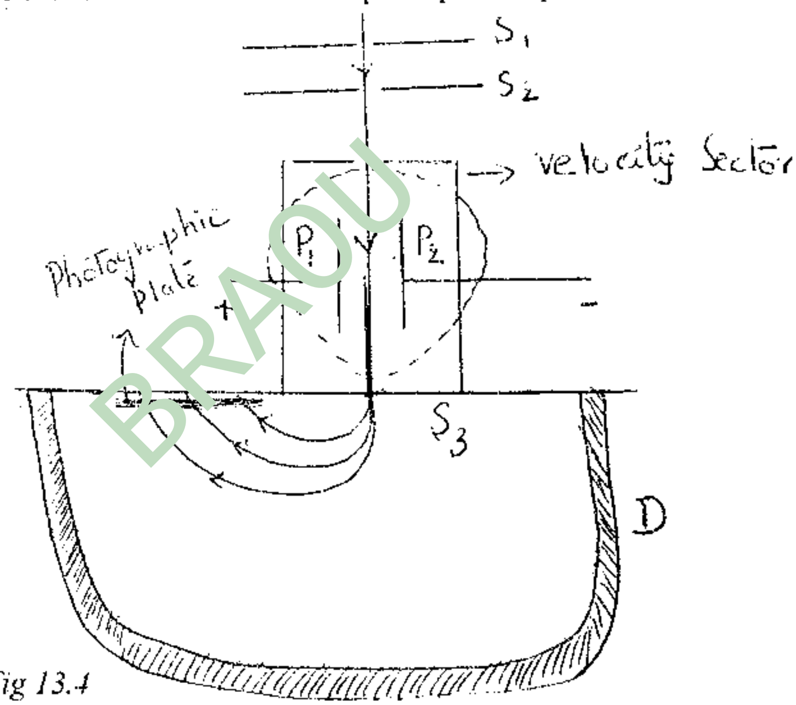


Fig 13.4

The magnetic field is produced by an electromagnet represented by the dotted circle. The magnetic field is perpendicular to X and the ion beam. The electric field and magnetic field of the velocity selector are so adjusted that the deflection produced by one is nullified by the deflection produced by the other. If X and B are the electric intensity and magnetic induction, then,

$$Xe = Bev \text{ or } V = X/B \qquad \ldots(13.10)$$

Only those ions, having this velocity V, alone pass through the entry slit S3 to enter the evacuated chamber D. Thus all ions entering D must have the same velocity. The positive, ions which enter into D are subjected to a strong uniform magnetic field of intensity B, perpendicular to its path. The force acting on each ion will be Bev. Ions with different masses trace circular paths of different radii given by

$\therefore R = \dfrac{MV}{B^1 e}$ $\left(B^1 ev = \dfrac{MV^2}{R} \right)$ ... (13.11)

$$\frac{e}{m} = \frac{V}{B^1R} = \frac{V}{BB^{11}R} \quad \left(\text{Since } v = \frac{X}{B}\right) \qquad \text{... (13.12)}$$

Since v and B^1 are constant quantities,

$$\frac{e}{m} \;\alpha\; \frac{1}{R}$$

After describing semicircles, the ions strike a photographic plate.

Now, $M = \dfrac{B^1 eR}{V}$ if 'e' is the same for all ions i.e. the charge same, then M α R

So we get a linear mass scale on the photographic plate. It will be seen that ions of different masses strike the photographic plate at different points, thus giving a typical mass spectrum.

Advantages: The advantages in this method over the other methods are that,

1. Since a linear mass scale is obtained, accuracy of measurement is increased.

2. The sensitivity depends on the strength of the deflecting magnetic field 'B' and the field area of the chambers D. Bainbridge in this actual research used a magnetic field of 1.5 webers/mt^2 over a semi circle of radius 0.2 mts. He found a definite increase in resolving power over Aston's apparatus. The ten isotopes of tin were resolved by this instrument.

Example: A beam of positively charged particles go through a velocity selector where in dielectric field of 6 x 10^5 volts / mt and magnetic induction of 2 x 10^{-2} weber/mt^2 are applied perpendicular to each other. Then the particles go through another magnetic induction of 1.25 web/mt^2 applied perpendicular to the path. If the diameters of the path taken is one meter. Calculate the specific charges of the particles.

We have, $\dfrac{e}{m} = \dfrac{V}{BB^1R}$ Here X = 6 x10^5 volts /mt

B = 2 x 10^{-2} web /mt^2, B^1 = 1.25 web /mt^2 and

r = 0.5 mts

$$\frac{e}{m} = \frac{6 \times 10^5}{(2 \times 10^{-2}\ (1.2\,5)\ 0.5} = 4.8 \times 10^7 \text{ coulomb/Kg.}$$

13.6 SUMMARY

Separation of atomic masses on the basis of specific charges were described and the isotopic abundance of atomic masses were calculated.

13.7 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1.How are isotopic masses separated?

2.Discuss the principle and application of Aston's experiment.

3.Determine the isotopic masses by using Bainbridge mass Spectrograph.

II. Answer the following questions briefly

With a neat sketch explain the principle of Dempster's mass spectrograph.

UNIT – 14: SELF INDUCTION AND MUTUAL INDUCTANCES

Contents

14.1 OBJECTIVES

This unit discusses the phenomenon of inductance. In order to make you understand the phenomenon it explains what self induction and mutual induction are.

After going though this unit you will be able to compute the coefficient of self-induction and mutual induction and evaluate the inductance of a solenoid.

14.2 INTRODUCTION

In this chapter you will know about self and Mutual induction. Know the expressions for Mutual and Self Inductance and also derive the expression for a solenoid.

14.3 SELF INDUCTION

The self-induction phenomena can easily be demonstrated with a coil of many turns wound on a laminated magnetic core. Consider two circuits as shown in Fig 14.1(a) and 14.1(b). These two circuits are similar to each other except that Fig 14.1 (a) has the coil wound on a magnetic core and Fig 14.1(b) has a resistor R which is equal to that of coil Fig.14.1 (a). When the two circuits are switched on simultaneously, the meters in their respective circuits register currents. But it is to be noted that the current in Fig 14.1(a) takes a finite time to grow to its full value while in Fig 14.1 (b) the current grows to its full value immediately. Again, When the circuits are broken the current in the 14.1(a) decreases to zero gradually in a finite time while in the circuit Fig 14.(b) it comes to zero immediately.

Why does the coil in the circuit Fig 14.1(a) cause delay in the growth and decay of current?

The growth of the current through, the coil produces a change in the magnetic flux linked with the coil. According to Lenz's law an emf gets induced in the coil, which opposes the applied emf. The induced emf lasts as long as the current is growing and decaying.

The emf induced in a closed circuit at make and the break of the circuit is called the emf due to self-induction or self-induced emf. The finite time of growth or decay depends on the flux linkage through the coil and in turn it is also dependent on coil shape. The coils of the nature and shape are known as inductors.

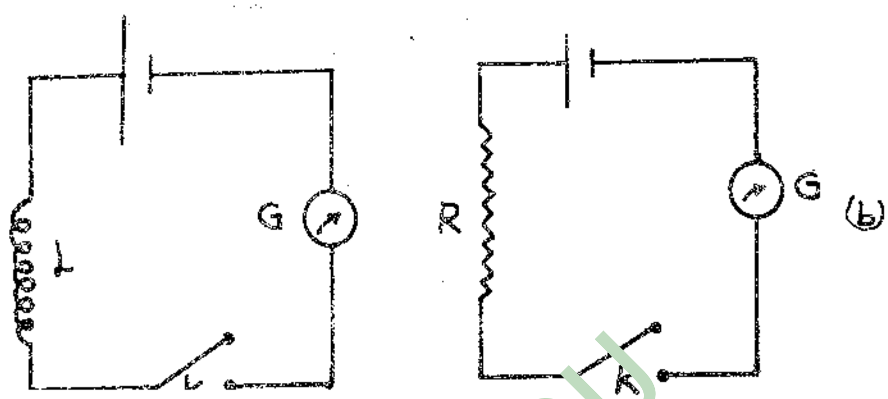


Fig 14.1 (a) and Fig 14.1 (b)

Coefficient of self induction

Consider a long solenoid having N number of turns and the flux associated with each turn is ϕ B.

According to Faradays law of induction, the induced emf is given by

$$\varepsilon = \frac{-d\,(N\phi_B)}{dt} \qquad \text{... (14.1)}$$

Where ($N\phi B$) is the total flux linkage. For a given coil the flux linkage ϕB is given as

$$N\phi B \alpha I$$

$$\therefore N\phi B = Li \qquad \text{...(14.2)}$$

Where L is the proportionality constant and is called coefficients of self-induction or inductance of the inductor.

Combining equation 14.2 and 14.1

We get that $\varepsilon = \frac{-d}{dt}\left[N\phi B\right]$

$$\varepsilon = -L\,\frac{di}{dt} \qquad \text{...(14.3)}$$

$$L = -\varepsilon \Big/ \frac{di}{dt} \qquad \ldots(14.4)$$

the relation 14.4 may be taken as the defining equation for the inductance which depends on coil shapes and sizes.

The unit of inductance, from equation 14.4 is volt-sec/amp. The unit of inductance henry

$\therefore$ 1 henry = 1 volt – sec/Amp.

The units millihenry (1mh (10^{3} h) and microhenry (1μh(– 10^{-6} henry) are also commonly used.

Check your progress – I

1. Define Self-induction.
2. Henry is the unit of .....

Note: a) You can write your answer in the space given below.
b) Compare your answer with the one given at the end of this unit.

....

....

....

14.4 MUTUAL INDCUTION

Let us consider two coils P and S which are placed, closed to each other. When a current is sent through P, a small amount of emf is generated in S, this phenomena can be understood in terms of a passage of current through P, the magnetic flux that gets developed around the coil P. A part of this magnetic flux will be linked with coil S, and therefore an emf is induced in S opposing the increase of flux linked with it. This phenomenon is called mutual induction.

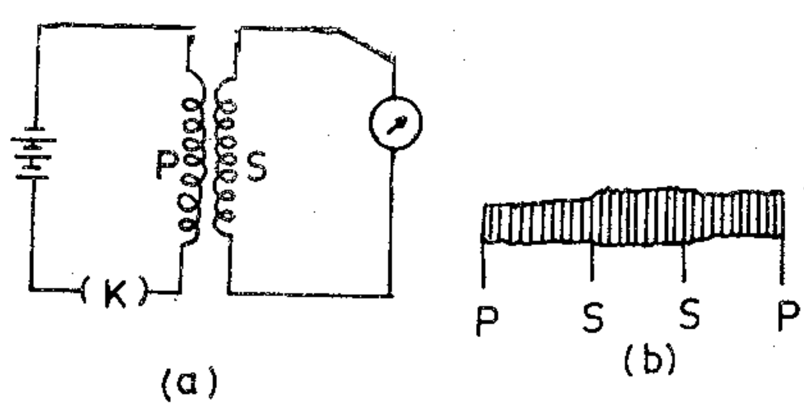


Fig 14.2 a Demonstration of mutual induction
Fig 14.2 b two coils one in another

The magnitude of this induced emf in S will depend upon the amount of flux linked in S.In turn the magnetic flux will depend upon the strength of the current in P. It is observed that $N\phi_B \alpha I$

$N\phi_B = Mi$ 14.5

Where the proportionality constant M is called coefficient of mutual induction or mutual Inductance. The value of M depends on the distance between the coils and on the magnetic property of the medium in which the coils are placed.

Differentiating equation 14.5

$$\frac{d}{dt}(N\phi_B) = M\frac{di}{dt} \quad \text{from eqn} \quad 14.1$$

$$\therefore -\varepsilon = M\frac{di}{dt}$$

$$\therefore M = -\varepsilon \Big/ \frac{di}{dt} \quad ...(14.6)$$

Thus the coefficient of mutual induction is said to be one unit, if the emf induced is one volt, when the current in the other coil is changing at the rate of 1 amp per second.

The practical unit of mutual induction is henry.

14.5 CALCULATION OF INDUCTANCE OF A SOLENOID

It is possible to calculate the self inductance for a solenoid
For an inductor

$$L = \frac{N\phi B}{} \quad ...(14.7)$$

Let us consider a solenoid having n number of turns per unit length, consisting length l and correctional area A

$\therefore$ the flux linkage $N\phi_B = n1\ BA$

Where B is the magnetic induction giving flux B
We know the magnetic induction B for a solenoid

$B = \mu o\ ni\ A$ ...(14.8)

Combining the equations

$N\phi_B = nl\mu_o niA$

$N\phi_B = n_o n^2\ liA$ ...(14.9)

The inductance of a solenoid

$$L = \frac{N\phi_B}{i} = \frac{\mu n^2 liA}{i}$$

$$L = \mu_o n^2 lA \qquad \ldots(14.10)$$

Thus the inductance of a solenoid having length l is proportional to tis volume (lA) and to the square of the number of turns per unit length. It is worth to note that inductance depends on geometrical factors.

Example:

1. A 10 henry inductor carries a steady current of 2 Amps. How can a 100 volt self induced emf be made to appear in the indicator.

$$\therefore \varepsilon = L\frac{di}{dt}$$

$$100 = 10\frac{di}{dt} \qquad \text{or} \qquad \frac{di}{dt} = 10 \text{ Amp/s}$$

1. A solenoid is wound with a single layer of 10 copper wires (diameter 0.1 in). It is 4 cm in diameter and 2 meters long. What is the inductance per/unit length for the solenoid near its center.

$$l = n^2 l A$$

$$= 1.26 \times 10^{-6}\left(\frac{10^2}{01.x2.54}\right)^2 \pi \times \left(\frac{2}{10^2}\right)^2$$

$$= 245.4 \times 10^{-6} \text{ H}$$

$$L = 0.245 \text{ mH}$$

2. A solenoid 50 cm long and of cross section 30 sq. cm has 1500 turns, on the central portion of this solenoid is closely wound another coil of 500 turns. Calculate the mutual inductance of the pair of solenoids in henrys.

$$M = 41\, n.n_1\, A \times 10^{-9} \text{ henrys (Try to derive this formula)}$$

The number of turns per unit length of the primary coil

$$n = \frac{1500}{50} = 30$$

$$n_1 = 500,\ A = 30 \text{ Sq.cm}$$

$$M = 4\pi \times 30 \times 500 \times 30 \times 10^{-9}$$
$$= 0.00566 \text{ henrys}$$

14.6 SUMMARY

The coefficient of self-conduction is ε

$$\varepsilon = -L \frac{di}{dt}$$

The coefficient of Mutual conduction is

$$M = -\varepsilon \Big/ \frac{di}{dt}$$

The coefficient of a solenoid is

$$L = -\mu_o n^2 lA$$

Check your progress: Answers

1. when a changing current flows through a conductor the changing magnetic flux surrounding the conductor should induce emf within it self ie in the conductor itself. This phenomenon is known as self induction.
2. Henry is the unit of inductance.

14.7 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Derive the expression for the coefficients of self induction and mutual induction+

II Answer the following questions briefly.

1. Derive an equation for the Calculation of inductance of solenoid.

III Solve the following Problems

1. Find the self inductance per unit length of a long solenoid of radius 'R' carrying 'N' turns per unit length.
2. Find the self inductance of a toroidal coil of rectangular cross section, inner radius 'a', outer radius 'b' and height 'h', with 'n' turns.

3. Two inductance SL_1 and L_2 are separated by a large distance. Show that the resultant inductance is $L_1 + L_2$ (When they are connected in series and (1/2) L (when they are connected in parallel and) $L_1 = L_2 = L$,

4. (a) Find the mutual inductance per unit length for two long straight wires with separation 'S' (b) If the current in one wire is i, find the emf induced per unit length in the other wire if it is at a separation S and approaching at a velocity 'V'

5. Find the mutual inductance between a long straight wire and a rectangular wire loop.

6. Find the mutual inductance between two concentric circular wire loops of radius r_1 and r_2 (a) when the radius of the inner loop is much smaller than the outer loop (b) without this restriction.

7. Show that the self inductance for a length 1 of a long wire associated with the flux inside the wire only is $\mu_0 l / 8\pi$ independent of the wire diameter.

8. A long wire carries a current of uniform density. Let i be the total current carried by the wire and show that the magnetic energy per unit length stored within the wire equals $\mu_0 I^2 / 16\pi$. Note that it does not depend on the wire diameter.

UNIT 15: FARADAY'S LAW AND LENZ'S LAW

Contents

15.1 OBJECTIVES

This unit discusses the concept of electromagnetic induction and Faraday's Laws of induction. To make you understand the concept the unit explains

1) Electromagnetic induction

2) Faraday's Laws

3) Mathematical equations for induced emf.

After going through this Unit you will be able to explain

1) What is meant by electromagnetic induction; and the principle under lying behind the working of a moving coil galvanometer.

15.2 INTRODUCTION

We have seen in the proceeding units that a current flowing through a wire produces magnetic field around its surroundings. It was observed and also investigated thoroughly by Michael Faraday in 1831 that changing magnetic lines of force through a closed loop of copper wire generated electromotive force. This electromotive force can be used as a source for production of electric power.

15.3 ELECTROMAGNETIC INDUCTION

To illustrate this empirical principle of Faraday, let us consider a surface of area bounded by a copper wire placed in a magnetic field B. The magnetic flux passing through the area A is given by definition.

$$\phi = \oint n\bar{x} . \bar{B} \, ds$$

Where n is a unit vector normal to the surface ds at the point under discussion. The flux is taken as positive when the normal points out in the direction of the field i.e., n= 0 when the normal points in the opposite direction i.e., $\theta = 180^0$, flux has negative sign (See the fig. 15.1)

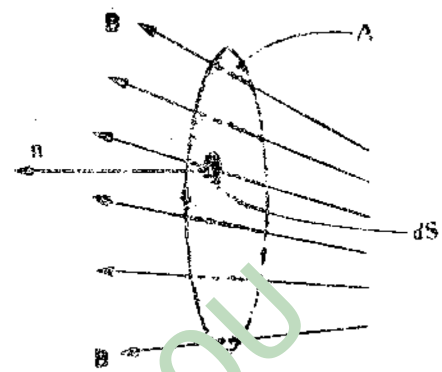


Fig 15.1.The magnetic flux through a surface ds in a magnetic field.

In the simple experiment shown in the Fig. 15.2, the two terminals of a coil C are connected to a galvanometer, G. A bar magnet is placed nearby it. In normal conditions, the galvanometer does not show any deflection. However, when the bar magnet B with its north pole is suddenly pushed into the coil the galvanometer shows a momentary deflection, indicating the presence of emf, which sends current through the coil.

If the magnet is moved away from the coil, the galvanometer again shows a momentary deflection but in the opposite direction; showing that the emf generated in the coil will be in the opposite direction. This experiment is repeated by keeping the bar magnet having its south pole facing the coil. In this trial also the galvanometer shows momentary deflections which are opposite to the results of earlier experiment. It is also observed that the galvanometer does not show any deflection if the magnet is held stationary. But deflection is observed when coil is moved either to or from towards the magnet. In each experiment it is found that the galvanometer register momentary defections when the coil and magnet are in relative motion. From these results we conclude that the emf gets induced in the coil and it ceases as soon as the relative motion ceases.

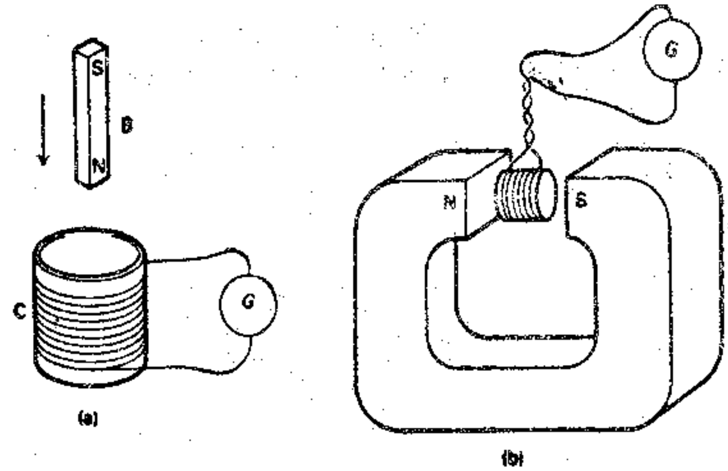


Fig 15.2 (a) Motion of a bar magnet B in a coil C.
(b) Motion of the coil C between the pole pieces of a permanent magnet.

The current that appears in the experiments is called induced current and is said to be set by an induced electromotive force. Faraday as well as Lenz were able to deduce laws governing the behaviour of these induced emfs and induced currents

15.4 FARADAY'S LAWS OF ELECTRO MAGNETIC INDUCTION

The experimental evidence of electromagnetic induction has been summed up in the form of two laws, which are stated as follows.

These laws are known as Faraday's laws of Electro magnetic induction.

(i) Whenever the magnetic flux is linked with a closed circuit changes in induced emf is set up in the circuit, whose magnet at any instant is proportional to the rate of change of magnetic flux linked with the circuit.

i.e., $e = -\frac{d\phi}{dt}$

(-ve sign because decrease influx with induced emf)

ϕ = Magnetic flux linked with the circuit at any instant.

(ii) The direction of induced emf is such that it opposes the change in flux that produces it.

This law is known as Lenz's law. Though the direction of induce emf was determined by Faraday's, but it was expressed as a law of lenz.
This law can be mathematically expressed as

$$\text{i.e., } e = -\frac{d\phi}{dt} \qquad \ldots(15.1)$$

in this e = induced emf, ϕ = instantaneous flux in Weber

To explain Faraday's laws of electro magnetic induction, we consider the coil and magnet in the experiment.

When the North Pole of the magnet is near the coil, a certain number of lines of magnetic induction originating from this pole passes through the coil, (as shown in Fig 15.3)

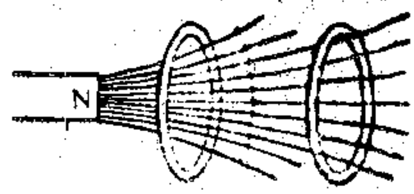


Fig. 15.3

If the magnet is moved towards the coil, the flux through the coil increases, while if the magnet is moved away from the coil, the flux decreases.

In either case an emf is induced in the coil, which changes according to the motion of the magnet. Greater the motion of the magnet greater is the rate of change of flux (may be increasing flux or decreasing flux)

Thus induced emf in the coil in either case changes according to the motion of the magnet. Greater is the rate of change of flux with more movement of the magnet.

The induced emf(ε) is directly proportional to the rate of change of magnetic flux, linked with the coil.

In both the cases work has to be done in moving the magnet. It is this mechanical work which appears as electrical energy in the coil. Thus the production of induced emf or induced current in the coil is in accordance with the law of conservation of energy.

Mathematically we may write with the above discussion

$$e = -\frac{d\phi}{dt} \quad ; \text{ Magnetic flux } \phi = + B.A \text{ (we know)}$$

$$\phi = \int \overline{B}.\overline{ds}$$

$$e = \frac{-d}{dt} \int \overline{B}.\ \overline{ds}$$

But $e = \int \overline{E}.\ \overline{dl}$; $\quad \therefore \int \overline{E}.\ \overline{dl} = \frac{-d}{dt} \int \overline{B}.\ \overline{ds}$ ...(15.2)

Using Faradays law & stokes theorem, Gives

$$\int \overline{E}.\ \overline{dl} = \int \overline{\nabla} \times \overline{E}.ds$$ ...(15.3)

$$\int \overline{\nabla} \times \overline{E}\, ds = -\frac{d}{dt} \int \overline{B} \cdot \overline{ds} \qquad ...(15.4)$$

$\rightarrow \overline{\nabla} \times \overline{E} = \frac{-dB}{dt}$. This equation is the integral form of Faradays law. ...(15.5)

15.5 LENZ'S LAW

We have seen that the Faraday's law is written as

$$\phi = -N \frac{d\phi B}{dt} \qquad ...(15.6)$$

Where –ve sign indicates the direction on which the induced emf acts. This result could also be stated in terms of Lenz's law, stating the induced emf always acts in such a direction that it opposes the charges causing it. This is a consequence of the law of conservation of energy.

To predict the direction of the induced emf, let us consider a simple experiment of single twin coil (loop) and a bar magnet as shown in Fig 15.5 Let the north pole of the

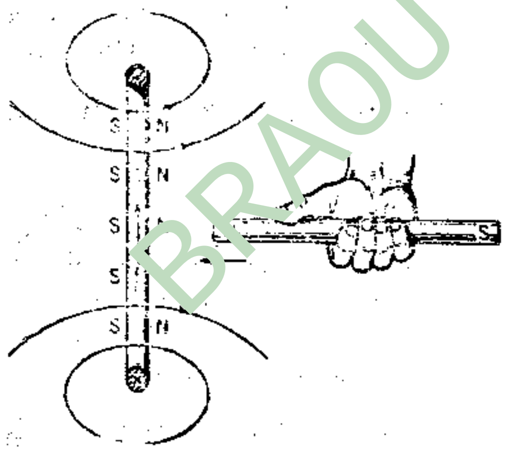


Fig 15.4 If the magnet with its north pole is moved toward the loop, the induced current develops as shown, setting up a magnetic field that opposes the movement of the magnet.

magnet is facing the conducting loop. When the bar magnet is suddenly pushed towards the loop an induced voltage will be developed in the loop. This induced emf setup its own magnetic field around it and thereby behaves like a magnetic dipole, one face of the loop, being a north pole, the opposite face being a south pole. As usual the lines of B would emerge from the North Pole. According to Lenz's law, if the loop is to oppose the motion of the magnet towards it, the face of the loop towards the bar magnet must become a north pole. Thus the two north poles, one being of the conducting loop and the other is being of the bar magnet will repel each other. The right hand rule shows that the current will be counter clock-wise as we see along the magnet towards the loop.

To make the right hand face a south pole, the current must be opposite as shown in the Fig 15.5 Whether we pull or push the magnet its motion will always be automatically opposed.

15.6 EXPRESSION FOR INDUCED EMF

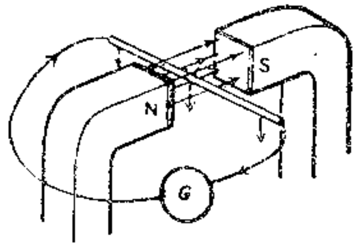


Fig 15.5

A piece of wire is connected across the terminals of a sensitive galvanometer, which gives a deflection. If this wire is moved sharply up or down between the poles of permanent magnet (as shown in fig 15.6) the galvanometer needle deflects momentarily. The direction of this induced current depends upon the direction of the magnetic field and the direction of the movement of the wire. The magnitude of the current depends on the speed with which the wire moves through the field and the strength of the magnetic field.

This induced current might have been produced from (a) the force experienced by charge moving in a magnetic field (b) the model of a current in a wire as a flow of free charges. Consider the situation as shown in figure, in which the conductor PQ of length 'L' carries free positive charges. Each of +ve PQ is part of a circuit QRS which moves from left to the right at a speed v through a uniform magnetic field of intensity B. This field, whose direction is perpendicular to the plane of the circuit, extends to the time xy as shown in Fig. 15.6

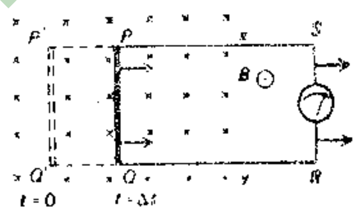


Fig 15.6 A rectangular loop PQRS is pulled out of a magnetic field

As each charge moves along with speed v, a force F will act on each charge in a direction perpendicular to B and also to v i.e, along l.

$\therefore F = Bev$ ...(15.7)

By the left hand rule, this force is directed from Q to P. This will result in a flow of positive charge in the direction Q to P. The charge will be acted on by a force F over the length of wire l, in the magnetic field and it will consequently gain energy.

$$Fl = Bevl \qquad \ldots(15.8)$$

(Now precisely the same effect could have been produced in the circuit in the absence of any motion through the magnetic field by inserting a source of emf into the circuit. The magnitude of this emf would have to be equal to the product Bev to produce the same current as we now observe).

Thus the movement of the circuit through the magnetic field generates an emf in the conductor P Q of magnitude.

$$\therefore \varepsilon = \frac{Bevl}{e}$$

$$\therefore \varepsilon = Bvl \qquad \ldots(15.9)$$

Since the flow of positive charge is in the direction PSRQ the end P of the wire l will be the conventional positive end, while Q is negative. This induced emf sets up a current given by

$$i = \frac{e}{R} = \frac{Bvl}{R} \qquad \ldots(15.10)$$

Where R is the resistance of the wire.

15.7. TIME VARYING MAGNETIC FIELD

It is also possible to have induced emfs without any physical motion of objects, i.e., magnet or the coil. However, the magnetic field should vary with time. If a conduction loop is placed in such a time varying magnetic field, the flux through the loop changes resulting in the creation of an induced emf in the loop.

Let us consider the following quantitative illustration of certain arrangement as given in the Fig 15.8 related to changing.

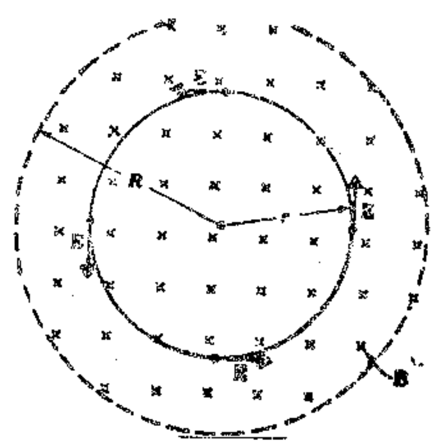


Fig 15.7 Time varying magnetic fields.

magnetic fields. The magnetic field **B** is allowed to decrease or increase steadily with time. The induced emf is still described by the same Faraday's laws of induction

$$\varepsilon = -\frac{d\phi}{dt}$$

The induced electric fields E at all points of the loop must therefore be tangential. Thus the electric lines of force that are setup by the changing magnetic fields are concentric circles.

If we consider a test charge q moving around the circle of radius r as shown in Fig. 15.8, the work done is εQ_0. However, if we look at the same problem from a different view then $(q_0E)(2\pi r)$ represent the work done where q E is force acting on the charge and $2\pi r$ is the distance over which the force acts.

$$\varepsilon q_0 = q_0 E 2\pi r$$

$$\varepsilon = E 2\pi r \qquad \ldots(15.11)$$

$$\varepsilon = \oint E.dl \qquad \ldots(15.12)$$

Therefore Faraday's law of induction can also be written as

$$\oint \overline{E}.\overline{dl} = -\frac{d\phi E}{dt} \qquad \ldots(15.13)$$

An important observation from this section is that a changing magnetic field sets up an electric field in space. There is great deal of evidence in support of view of Faraday's law in induction. One important type of particle accelerator called the Betatron depends for its operation on the existence of field quite independently of any conducting wire.

Check your progress – I

1.State Faraday's law of electromagnetic induction

Note: a. you can write your answer in the space given below.
b. Compare your answer with the one given at the end of the unit.

15.8 MOVING COIL GALVANOMETER

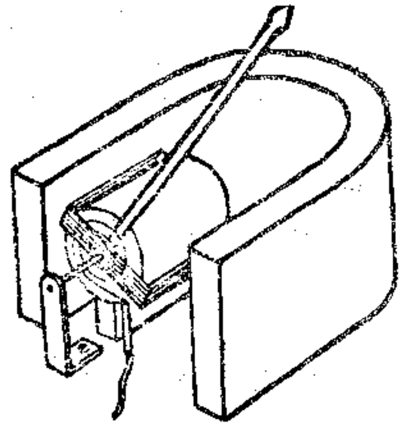


Fig 15.8 working model of a moving coil galvanometer.

It has been made use that the magnetic effect may be employed of measuring currents. The instruments in which the magnetic effect is used for measuring and detecting current or electric charge are called galvanometers. There are several types of galvanometers depending on the construction for a specific purpose. Thus, those in which a current carrying conductor moves when placed in a fixed permanent magnetic field, are called **moving coil galvanometers**. This is used to measure momentary currents.

A design formula can be derived for the moving coil galvanometer as follows.

A moving coil galvanometer consists of a flat coil of insulated wire which is suspended between the poles of permanent horse shoe type magnet. Usually the coil is suspended by a phosphor bronze wire. When a momentary current is passed through the coil, it produces its own magnetic field. There will be an interaction between this magnetic field and the field due to permanent magnet. Since the field provided by the permanent magnet is radial in the region where the coil is free to move. The plane of the coil is always parallel to the field. Hence the coil which deflected from its equilibrium or a torque is produced by the current. The kick or deflection indicated by the moving system is proportional to the strength of the momentary or steady current passing through it.

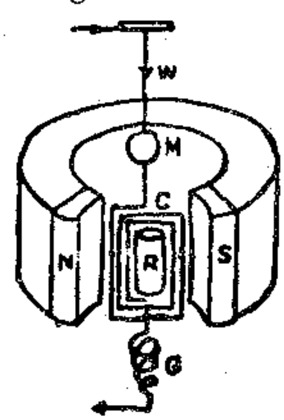


Fig 15.9 Moving coil galvanometer.

The torque on the coil is given by

$$\tau = BinA \qquad ...(15.14)$$

Where **B** is the magnetic field induction of the permanent magnet, n the number of turns having area A each and i the current.

The coil rotates against the opposing torque provided by the torsional couple of the suspension. The torque provided by the suspension can be written as Kϕ where k is the torsional constant or torsional stiffness factor of the suspension and ϕ is the angular deflection of the coil.

Hence, when current i flowing in the coil and it is in equilibrium,

$$K\phi = niBA \qquad ...(15.15)$$

$$\text{or } i = \frac{K}{Bni}\phi \qquad ...(15.16)$$

Current Sensitivity

A galvanometer is said to be sensitive if it shows a large deflection for a small current. **Hence the sensitivity can be defined as** the angular deflection of the coil per unit current.

$$\frac{\phi}{i} = \frac{BnA}{K} \qquad ...(15.17)$$

The sensitivity is large if B n or A are made large and K is made small. But there is a limit to how large any one of the parameters B n, A can be made and to how small K can be made.

Check your progress-11

1.What is moving coil galvanometer.
2. Define sensitivity of a galvanometer.
Note: a. Space is given below for your answers.
b. Compare your answers with the one given at the

....
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15.9 DAMPING CORRECTION

Galvanometers are used for quick measurement of steady currents. A moving coil galvanometer can either simply register the deflection without oscillating about its mean position (dead beat condition) or oscillate about its mean position (Ballistic condition). Under ballistic condition the vibrations are considered to be ideally simple harmonic. But in actual experiments, the amplitude of the swing progressively decreases due to several reasons such as air resistance. Thus the deflection is damped. Hence the observed deflection is less than what it would have been under ideal condition. Therefore a correction is to be added to the observed throw (Oscillation) to compensate the damping. The correction is called damping correction.

Experimentally, it is found that the ratio of any two consecutive deflections will be constant. It means that the rated of decrease of the deflection is constant.

Let $\phi_1\ \phi_2\ \phi_3\ \phi_4$ ...etc are the recorded deflections on the left and right consecutively

$$\frac{\phi_1}{\phi_2} = \frac{\phi_2}{\phi_3} = \frac{\phi_3}{\phi_4} = d \qquad ...(15.18)$$

Where the ratio d is called decrement and $\log d_e = \lambda$ called logarithmic decrement

$$\text{Hence } e^{\lambda} = d \qquad ...(15.19)$$

Therefore Eqn. (15.18) can be written as

$$\frac{\phi_1}{\phi_2} = \frac{\phi_2}{\phi_3} = \frac{\phi_3}{\phi_4} = \text{-----} = e^{\lambda} \qquad ...(15.20)$$

The deflections ϕ_1 and ϕ_2 are separated by half an oscillation, while ϕ_1 and ϕ_3 are separated by one complete oscillation

$$\frac{\phi_1}{\phi_3} = \frac{\phi_1}{\phi_2} \times \frac{\phi_2}{\phi_3} = e^{\lambda}.e^{\lambda} = e^{2\lambda} \qquad ...(15.21)$$

Similarly ϕ_1 and ϕ_2 are separated by three and half oscillations and related by

$$\therefore \frac{\phi_1}{\phi_4} = \frac{\phi_1}{\phi_2} . \frac{\phi_2}{\phi_3} . \frac{\phi_3}{\phi_4} = e^{\lambda} . e^{\lambda} . e^{\lambda} = e^{3\lambda} \qquad ...(15.22)$$

If ϕ_0 is the undamped first throw which reaches its maximum ϕ_1 after one quarter of an oscillation, from the analogy of Eqn. (15.22) one can write

$$\therefore \frac{\phi_0}{\phi_1} = e^{\lambda/2} \text{ or } \phi_0 = \phi_1 e^{\lambda/2} \qquad ...(15.23)$$

or by expression

$$\phi_0 = \phi_1 \left(1 + \frac{\lambda}{2} + \frac{\lambda^2}{4^2} + \ldots\ldots\right) \qquad \ldots(15.24)$$

when λ is small, higher powers of λ can be neglected

$$\phi_o = \phi_1 \left(1 + \frac{\lambda}{2}\right) \qquad \ldots(15.25)$$

Thus the observed throw is to be multiplied by $\left(\frac{1+\lambda}{2}\right)$ to get correct or undamped value of the deflection.

How to find λ

By noting a large number of throws the value of λ can be found more accurately. Suppose ϕ_1 and ϕ_2 are observed

$$\therefore \frac{\phi_1}{\phi_2} = e^{10\lambda}$$

$$\text{or } \log_e \frac{\phi_1}{\phi_2} = 10\lambda \qquad \ldots(15.26)$$

$$\text{or } \lambda = \frac{1}{10} \log_e (\phi_1/\phi_2)$$

After λ is found out, ϕ_0 can be calculated from expression (15.26)

Example -1:

A circular loop of wire 10 cm in diameter is placed with its normal making an angle of 30^0 with the direction of a uniform 500 gauss magnetic field. The loop is isolate, so that it normal rotates about the field direction at the constant rate of 100 rev/min, the angle between the normal and the field direction remains uncharged during the process. What emf appears in loop?

Solution:

When the coil wobbles with the normal making an angle 30^0 the induced emf in one complete rotation is zero, due to symmetry.

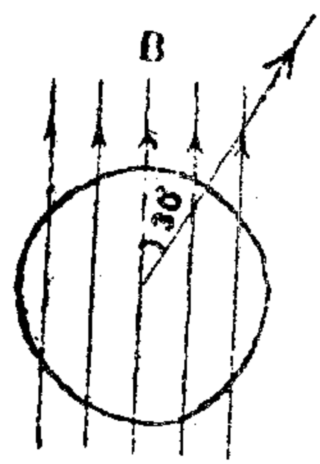

Fig 15.10

Hence the total induced emf is also zero.

Example -2:

A uniform field of induction **B** is normal to the plane of a circular ring 10 cm in diameter made of 0.1-inch diameter. At what rate **B** must change with time if an induced current of 10 Amps is to appear in the ring.

Solution:

Radius of the copper wire = 0.127×10^{-12} m.

Resistance of the wire $R = \frac{1.7 \times 10^{-8} \times \pi \times 10 \times 10^{-2}}{\pi \times (0.127)^2 \times 10^{-4}}$

$= 105.4 \times 10^{-5}$ Ohm

$\phi B = BA = 78.54 \times 10^{-4} B$

$\varepsilon = \frac{d\phi B}{dt} = 78.54 \times 10^{-4} \times \frac{dB}{dt}$

$i = 10$ Amp; $\because \frac{\phi}{R} = 10 = \frac{78.54 \times 10^{-4}}{105.5 \times 10^{-5}} \frac{dB}{dt}$

$\frac{dB}{dt} = 134$ Weber / m^2-s

Example - 3:

The figure shows a copper rod moving with a velocity. V parallel to a long straight wire carrying a current, i. Calculate the induced emf in the rod, assuming that V = 5.0 m/s

Solution

i = 100 Amp, a= 10 cm and b = 20cm

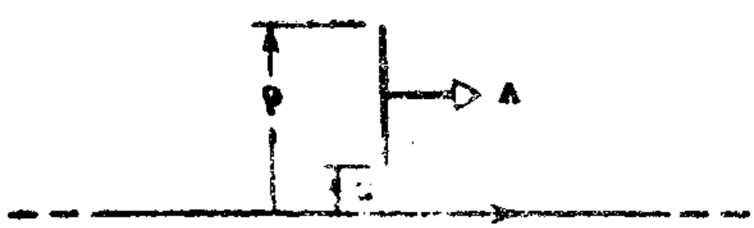

Fig 15.11

$$B = 1/10 \int_{1}^{20} \frac{\mu_0 i}{2\pi R}\, dR$$

$$= \frac{1}{19} \; \frac{4\pi \times 10^{-7} \times 100}{2\pi} \int_{1}^{20} \frac{dR}{R}$$

$$= \frac{200 \times 10^{7}}{19} \left[\log R\right]_{1}^{20}$$

$$\varepsilon = B \times V \times 19$$

$$\varepsilon = 200 \times 10^{-} \times 1.101 \times 5 \times 2.303$$

$$\varepsilon = 3 \times 10^{-4} \; Volt$$

SUMMARY

Relative motion between a conductor and a magnetic field induces an emf in the conductor. The direction of induced emf depends upon the direction of motion of the conductor with respect to the field; the magnitude of the emf produced is directly proportional to the rate at which magnetic flux lines are cut by the conductor and the number of turns of the conductor crossing the flux lines. The induced emf always acts in such a direction that opposes the change causing in the induced emf.

$$\varepsilon = \frac{d\phi B}{dt}$$

Where ϕ_B is the magnetic flux.

Check your progress : Answers.

I 1. Faraday's laws of electromagnetic induction are stated as follows:

(i) Whenever the magnetic flux is linked with a closed circuit changes in induced emf is set up in the circuit whose magnet at any instant is proportional to the rate whose magnet at any instant is proportional to the rate of change of magnet flux linked with the circuit.

$$e = -\frac{d\phi}{dt}$$

(ii) The direction of induced emf is such that it opposes the change in flux that produces it.

$$e = -\frac{d\phi}{dt}$$

e = induced emf, ϕ = instantaneous flux is Weber.

II 1. Generally the instrument in which magnetic effect is used for measuring an detecting current are electric charge are called galvanometers. But an instrument in which a current carrying conductor moves when placed in a fixed permanent magnetic field are called moving coil galvanometer. This is used to measure momentary currents.

2. Sensitivity of a galvanometer can be defined as the angular deflection of the coil per unit currents.

$$\frac{\phi}{i} = \frac{B_n A}{k}$$

The sensitivity is large if B_n or A are made large and k is made small.

15.11 SAMPLE EXAMINATION QUESTIONS

I Answer the following questions in detail

1. Describe the Faraday's law of induction and Lenz's law to explain the induced emf in a circular loop.

2. Discuss and describe the working principle of a moving coil galvanometer. How the damping correction helps to get actual value of the currents being measured by the moving coil galvanometer?

II. Answer the following questions briefly.

1. Explain about the time varying magnetic field.

2. How do you explain the induced current voltage in a loop in the basis of law of conservation of energy?

III. Solve the following problems

1. A uniform field of induction B is changing in magnitude at a constant rate dB/dt, You are given a mass m of copper which is to be drawn into a wire of radius r and formed into a circular loop of radius R. Show that the induced current in the loop does not depend on the size of the wire of the loop and assuming B perpendicular to the loop is given by $i = \frac{m}{4\pi\rho\delta}\frac{dB}{dt}$ where P is the resistivity of the copper.

2. A uniform magnetic field of induction B fills cylindrical volume of radius R, A metal rod of length l is placed as shown in the figure. If B is changing at the rate dB/dt, show that the emf that is produce by changing magnetic field and that acts between the ends of the rod is given by

$$\varepsilon = \frac{dB}{dt}\frac{l}{2}\sqrt{R^2 - (l/2)^2}$$

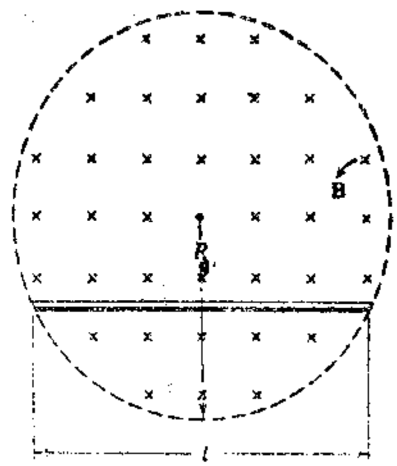

Fig III.(1)

3. Figure 15.12(b) shows a uniform field of induction B confined to a cylindrical volume of radius R. B is decreasing in magnitude at a constant rate of 100 gauss/s, What is the instantaneous acceleration (direction and magnitude) experienced by an electron placed at a, at b, and at c? Assume r =5.0 cm (The necessary fringing of the field beyond R will not change your answer as long as there is axial symmetry about a perpendicular axis through b)

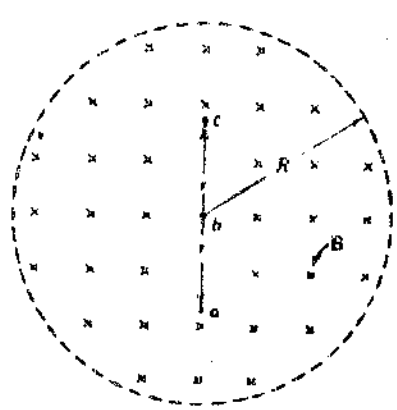

Fig III.(2)

4. In Fig 15.13(c) a conducting rod A B makes contact with the metal rails A D and b C which are 50 cm apart in a uniform magnetic field of induction 1.0 Weber/m perpendicular to the plane of the paper as shown. The total resistance of the circuit ABCD is 0.4 ohm (assumed constant). (a) What is the magnitude and direction of the emf induced in the rod when it moved to the left with a velocity of 8 m/s? (b) What force is required to keep the rod in motion? (c) Compare the rate at which mechanical work is done by the force F with the rate of development of heat in the circuit.

Fig III *(3)*

15.12 RECOMMENDED BOOKS

1	Kraus, J.D. and Carver, K.R	Electromagnetics	Mc Graw-Mill K Kogakusha, Ltd. Tokyo.
2.	Corsan, D.R. and Lorrian. P	Introduction to Electromagnetic Fields and Waves	Freeman Toppa, London
3.	Griffiths D.	Introduction to Electrodynamics	Printice-Hall of India New Delhi.
4.	Plonsey. R. And Collin, R.E	Principles and Applications of Electromagnetic Fields	Tata-Mc Graw Hill Publishing Company Ltd., New Delhi
5.	Laus'B.B	Electromagnetics	Willey Eastern Ltd. New Delhi
6.	Halliday, D and Resnick R	Physics-Part II	Wiley Eastern Ltd. New Delhi
7.	Grant. I.S. and Philips, W.R	Electromagnetism	John Wiley & Sons. Chichester
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UNIT -16: MAGNETIC ENERGY – MAXWELL'S EQUATION

Contents

16.1 OBJECTIVES

This unit explains the principle of a transformer also discusses the phenomenon of electromagnetic radiation.

To make you understand the phenomenon

1. It reviews the empirical law of electromagnetism
2. It considers Maxwell's contribution to the study of the displacement current.
3. It illustrates the transformation of energy from magnetic to electric vice versa by relating it to an L-C circuit.

After going through this unit you should be able to calculate the energy stored in a magnetic field, and able to describe the interrelations between varying electric and magnetic fields resulting in the emergence and propagation of e.m. waves in space.

16.2 INTRODUCTION

In this section we consider the energy associated with magnetic fields. As we have already seen that the work done in bringing a system of charges to a given configuration is equivalent to the energy stored in the electrostatic fields set up by the charges. Similarly, in the problems dealing with magnetic fields, the work done in setting up a current distribution is stored as energy in the magnetic field. For example, two parallel wires carrying currents in the same direction attract each other and to pull them apart work must be done. Hence, it should be thought that this expended energy is stored in the magnetic field around the wires.

Information regarding electric and magnetic fields was available to scientists about the middle of the nineteenth century. Indeed the very concept of existence of an electric or magnetic field a new idea of Faraday at that time, was considered with doubt by most scientists. Previously, fields had been looked upon as convenient means of visualizing the arrangement of force that resulted from electric and magnetic action. But to Faraday the magnetic field was the actual means by which magnetic force was exerted.

From the time of Ampere's work (1820 to 1825) it has been considered that one wire carrying current exerted a force on another wire carrying current and no intermediate agency for exerting force was taken into account. This was the action-at-a-distance theory, and it followed logically Newton's Universal law of Gravitation. Newton's Law assumed action at a distance, for it did not consider any medium necessary for the transmission of gravitational force. However, Faraday conceived the physical reality of electric and magnetic fields, and Maxwell undertook to express the mathematical relations involved.

16.3 ENERGY STORED IN A MAGNETIC FIELD

In a similar way, an inductor stores energy in its surroundings when the current is passing through it. This may be demonstrated with the help of the circuit shown in the Fig 16.1 When the switch S is closed the lamp glows. But, when the switch is opened the brightness of the lamp increases momentarily. This increase in its brightness is due to the magnetic. Energy stored in the magnetic field of the induction, as the magnetic field of the inductor induces a large current momentarily when the field collapses.

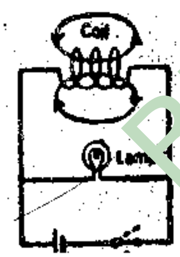


Fig 16.1 Circuit for demonstrating energy stored in a magnetic field.

To calculate the work in establishing the magnetic field, We shall calculate the energy supplied by a source to an isolated circuit (Fig 16.2) when the current increases from zero to some value. The circuit shown in the figure 16.2 consists of R the resistance, inductive coil with self-inductance L and E and emf of the battery.

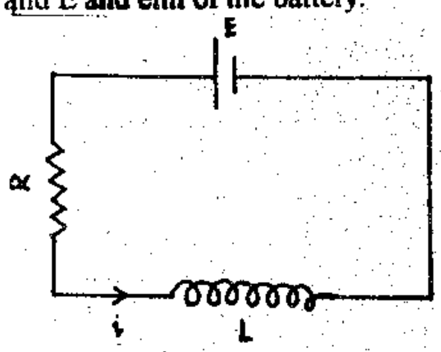


Fig 16.2

Let i be the current flowing through the circuit, and the voltage drop across the coil is L(di/dt), the net forward emf is $\varepsilon - L\frac{di}{dt}$

$\therefore$ By ohms law

$$\varepsilon - L\frac{di}{dt} = iR \qquad \text{... (16.1)}$$

the work done by the emf ε in moving a small amount of charge dq through the circuit is

$$dw = \varepsilon\, dq$$

$$= idt \qquad \text{...(16.2)}$$

$$\therefore \frac{dw}{dt} = \varepsilon i$$

$$\therefore \frac{\varepsilon i}{i} = Li\frac{di}{dt} + i^2 R \qquad \text{...(16.3)}$$

This equation has the following physical interpretation in terms of work and energy. The left term of the equation 16.3 is the rate at which the battery delivers energy to the external circuit, where as the i^2R term on the right hand side is the energy dissipated as Joule heat in the resistance.

Hence the energy that does not appear as Joule heat must be stored in the magnetic field associated with the inductor. Thus the term Li (di/dt) represents the rate at which the energy is stored in the magnetic field.

$$\therefore Li\frac{di}{dt} = \frac{dU_B}{dt}$$

$$\text{or} \quad dU_B = Lidi \qquad \text{...(16.4)}$$

$$\therefore U_B = \int dU_B = \int_0 Lidi$$

$$U_B = \tfrac{1}{2} Li^2 \qquad \text{...(16.5)}$$

This equation 16.5) represents the magnetic energy associated with the inductance L carrying current i. The magnetic energy has the dimension of inductance times of the squared current.

16.4 ENERGY DENSITY IN A MAGNETIC FIELD

It has been seen through the equation 16.5 that the energy associated within inductor carrying current is stored in its surroundings in the form of magnetic field. Let us find the density of this energy in terms of the magnetic induction. To make the calculation simplified, the magnetic field is considered to be uniform inside the inductor, there by the stored energy must also be distributed uniformly through out the volume of the solenoid.

Consider a solenoid consisting of length l and cross sectional area A. Then lA is the volume associated with the length of the solenoid.

∴ The magnetic field energy density $v_B = \dfrac{U_B}{Al}$

Since $U_B = \frac{1}{2} Li^2$

$$U_B = \frac{\frac{1}{2} Li^2}{lA} \qquad \text{....(16.6)}$$

But the coefficient of inductance of a solenoid is $\mu_o n^2 Al$ and the magnetic induction $B = \mu_o ni$ Substituting L and i values in the equation

$$\text{We get } U_B = \frac{1}{2}\frac{B^2}{\mu_o} \qquad \text{...(16.7)}$$

Though this magnetic field density equation is derived for the special case of a solenoid, the formula of equation 16.7 holds good for all magnetic fields.

16.5 PRINCIPLE OF A TRANSFORMER

A transformer is an electrical device by which a low alternating voltage can be converted into a high alternating voltage, however the initial current gets reduced proportionately or vice versa with the increase in voltage. The same device can also be employed for converting high voltages to low voltages.

The transformer consists of two coils wound on a common core. The core is made of high permeability material such as soft iron. The function of this core is to increase the magnetic flux linking the coil, hence increasing the mutual inductance between each pair of them. The coil P connected to the voltage to be changed is called the primary and the other coil S between the ends of which the alternate voltage is obtained is called secondary. The core carrying the coils P and S are usually made up of thin layers of magnetic materials, also known as laminations, which are electrically insulated from each other by layers of laminating paper. The figure 16.3 shows a cross sectional view of a transformer and conventional circuit symbol for a transformer.

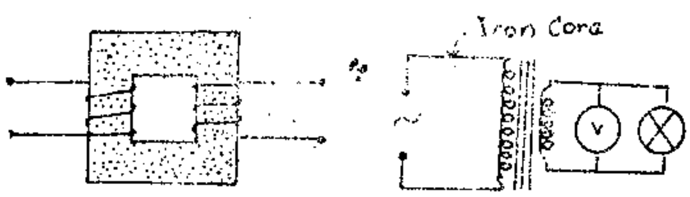


Fig 16.3 (a) (b)

16.3a a model transformer — *16.3b - Transformer in a circuit*

When an alternating voltage is applied across the ends of the primary coils, it causes a varying magnetic flux in the core. By Faraday's law of induction an alternating induced emf ε_1 is developed in the primary coil, This induced emf ε_1 is almost equal to the applied voltage in the primary. Let the number of turns in the primary be N and the rate of change of flux be $\frac{d\phi}{dt}$

$$\text{By Lennz's Law } \varepsilon_1 = -N1 \frac{(d\phi)}{dt} \qquad \text{...(16.8)}$$

As the secondary coil is also wound on the same core, the entire flux and corresponding flux changes are also linked with the secondary coil. If N_2 are the number of turns of the secondary coil and $(d\phi / dt)$ is the rate of change of flux, then the induced emf ε_2 in the secondary coil is

$$\varepsilon 2 = -N_2 \frac{(d\phi)}{dt} \qquad \text{...(16.9)}$$

comparing the equations 16.8 and 16.9 we obtain.

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2} = t \qquad \text{... (16.10)}$$

The equation 16.10 is called the transformer equation. The factor t is known as transformer ratio. In step up transformer $\varepsilon_2 > \varepsilon_1$ therefore N_2 must be greater than N_1 while in step down transformer $\varepsilon_2 < \varepsilon_1$ ie $N_2 < N_1$ Hence

$$\frac{\text{emf induced in secondary}}{\text{emf induced in primary}} = \frac{\text{No. of turns in secondary}}{\text{No. of turns in primary}}$$

or

$$\frac{\text{Output voltage}}{\text{Input voltage}} = \frac{\text{No. of turns in Secondary}}{\text{No. of turns in primary}} \qquad \text{...(16.11)}$$

An ideal transformer is one which has perfect coupling between its primary and secondary coils having no dissipation of energy. The coils, therefore, have zero resistance and thus there is no heat loss within the core. The input and output power, under the approximation that no net dissipation, must be equal.

$$\text{Thus } \varepsilon_2 i_2 = \varepsilon_1 i_1 \qquad \text{...(16.12)}$$

or

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1} \qquad \text{...(16.13)}$$

In a step up transformer the primary coil carries a stronger current than the secondary while in the step down transformer the secondary carries a stronger current than the primary.

In Practical transformer, the output power is always less than the input power because of energy losses due to the imperfections in the design etc., The contributing factors for the power losses are eddy currents, hysterisis and magnetic leakage. In large transformers, which are widely used, the efficiency is 98% whereas in small transformers it is about 90%.

Example:

1. A coil with an inductance 2 henrys and a resistance of 10 ohms is suddenly connected to a resistance less battery of 100 volts. (a) what is the equilibrium current (b) How much energy is stored in the magnetic field when this current exists in the coil ?

   (a) $\frac{E}{R} = \frac{100}{10} = 10$ amps (equilibrium current)

   (b) Stored energy = $\frac{1}{2} Li^2$

   $= \frac{1}{2} \times 2 \times 10^2$

   $= 100$ joules

2. A circular loop of wire 5 cm in radius carries a current of 100 amps what is the energy density

   $$vB = \frac{1}{2} \frac{B^2}{\mu o} \text{ and } B = \frac{\mu oi}{2R}$$

   $$vB = \frac{1}{2} \frac{\mu_o^2 i^2}{4 R^2}$$

   $$= \frac{1.26 \times 10^{-6} \times (100)^2 \times 10^4}{8 \times 5 \times 5}$$

   $= 0.63$ Joule / m^3

3. An ideal transformer has a turns ratio of 2, i.e, N2/N1 = 2 An ac emf of 10 V is applied to the primary. Find the emf or voltage appearing in the secondary terminals.

   $$\varepsilon_2 = \frac{N_2}{N_1} \varepsilon_1$$

   $= 2 \chi 10 = 20$ V

16.6 MAXWELL'S EQUATIONS

These equations are based on the several crucial experimental results obtained as mentioned below:

(a) An electric field is found to exist and is defined.

(b) Divergence of electrostatic field is proportional to charge density.

(c) A magnetic field is found to exist and is defined. But it is impossible to create an isolated magnetic pole.

(d) A changing magnetic field is found to induce an electric field (Faraday's Law)

(e) The magneto static field is a solenoidal (without divergence)

(f) The curl of the magneto static field is proportional to the current density.

Maxwell assumed that (1) the dynamic electric field has divergence proportional to charge density and (2) that the dynamic magnetic field has no divergence since there is no magnetic counter part of the electric charge. It was Maxwell who noticed certain inconsistencies in experimental laws and pointed out the existence of displacement current. Then this remarkable proposal was that displacement current also produces magnetic field just as conduction current since total current (for mathematical purpose) is the sum of conduction current and the displacement current. This hypothesis led to the formulation of a theory, which predicted the existence of electromagnetic waves, which transport electromagnetic energy in free space.

The action at a distance theory assumed electrical action appeared simultaneously at all points however remote. Maxwell's theory on the other hand, showed that energy be transmitted by waves traveling at a finite speed. In general, when there are varying electric fields, a magnetic field is produced by the sum of the conduction current (i) and the displacement current i.e.

$$\nabla \times \bar{H} = J + \frac{d\bar{D}}{dt} \quad \ldots(16.14)$$

($\nabla = I\frac{d}{dx} + j\frac{d}{dy}\ \hat{k} + \frac{d}{dz}$ called 'del' operator) the other fundamental field equations

$$\nabla \times \bar{E} = -\frac{d\bar{B}}{dt} \quad \text{(Faraday's law of induction)} \quad \ldots(16.15)$$

$$\nabla \times \bar{B} = \rho/\varepsilon \quad \text{(Gauss's law of electricity)} \quad \ldots(16.16)$$

$$\nabla \times \bar{B} = 0 \quad \text{(Gauss's law of magnetism)} \quad \ldots(16.17)$$

Equations 16.14 to 16.17 are the basic equations of electromagnetic theory known as Maxwell's equations. It will be noted that Maxwell's equations become simple and symmetrical when applied to a homogeneous medium in which there is no charge and no conductivity.

$$\nabla \times \bar{H} = \varepsilon \frac{dE}{dt} \quad \ldots(16.18)$$

$$\nabla \times E = -\mu \frac{d\bar{H}}{dt} \quad \ldots(16.19)$$

$$\nabla \times \bar{B} = 0 \quad \ldots(16.20)$$

$$\nabla \times \bar{H} = 0 \quad \ldots(16.21)$$

Once again, the physical meaning of these equations can be inferred as follows:

Equation 16.18 says that a changing electric field will produce a magnetic field and equation 16.19 says that a changing magnetic field will produce an electric field. These equations are particularly interesting to consider relative to the propagation of electric waves. It will be seen at once that, if a changing electric field produces a magnetic field and that in turns produces an electric field which produces a magnetic field and so on, some kind of series of energy transfers is started whenever any electric or magnetic disturbance takes place. While propagating energy will be transformed form the electric to the magnetic and back to the electric and so on indefinitely (this illustrated by figures 16.4 & 16.5).

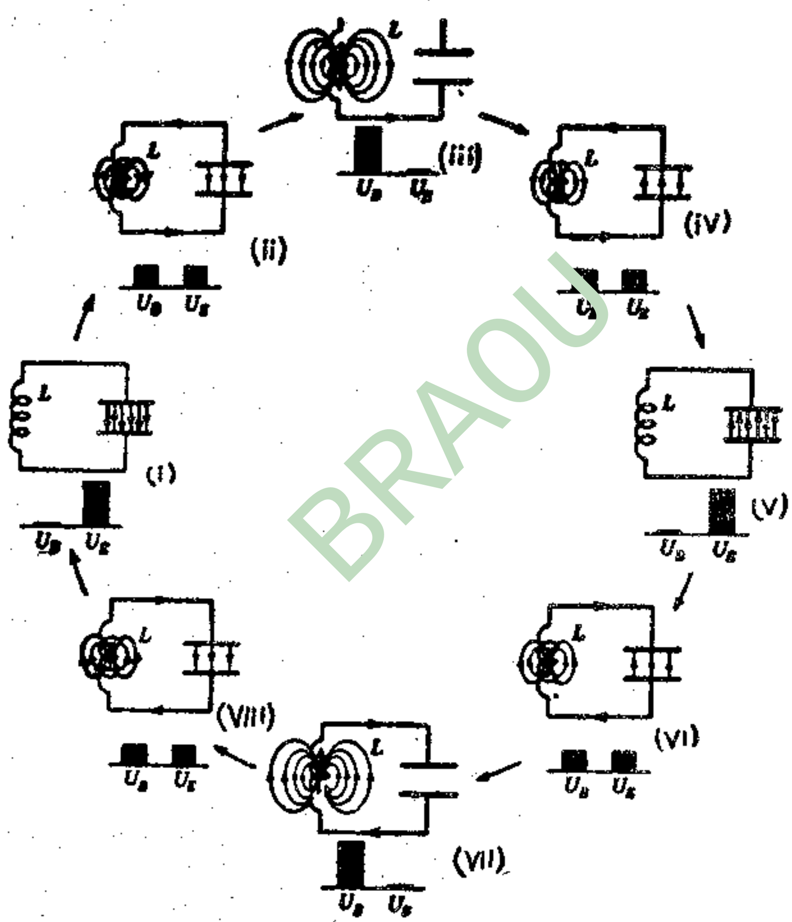


Fig 16.4

If (as is actually true) the magnetic energy is not confirmed to precisely the same location in space as the electric energy from which it is derived, but extends a little beyond, and, if the electric energy derived from that magnetic energy is again a little farther advanced in space, and so on, so that the energy is changing form to form and is also being propagated through space, the result may quite reasonably be a traveling wave of electromagnetic energy. To understand this more clearly an analogous situation as described below:

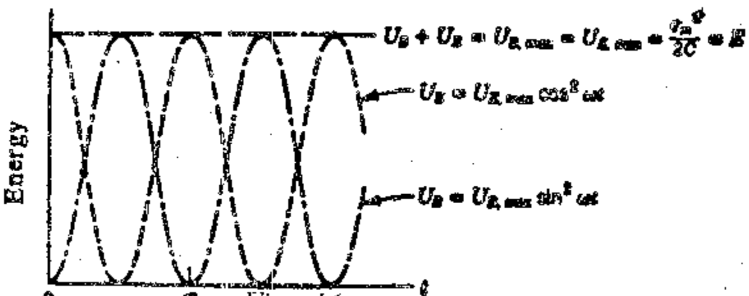


Fig : 16.5

By some means a small volume of water in the middle of a lake is artificially set into vertical oscillatory motion for example, by suddenly dumping a bucketful of water into the lake. The surface of the water at that point rises and falls-in an oscillatory manner. But it is not possible for the bucketful of water to oscillate independently of the water surrounding it Its periodic excesses and deficiencies of pressure are transmitted to the surrounding water, which thereby energy received and is, in turn, put into motion. In its resulting undulation it also transfers energy to the next outer region. By this process a wave is propagated across the surface of the lake Therefore, the fundamental reason for the existence of water wave is, the motion and pressure of a given volume of water are not independent of the motion and pressure of the water surrounding the volume and as the given volume of water is distributed, it transmits energy to the water next to it.

The fundamental reason for the existence of an electromagnetic wave is similar. A changing magnetic field induces an electric field both in the region in which the magnetic field is changing and also in the surrounding region; like –wise, a changing electric field produces a magnetic field in the region in which the change takes place and also in the surrounding region. Consequently when there is a disturbance of either the electric or magnetic induction in a given region of space, the disturbance cannot be confined to that space. The changing fields within that region will induce field in the surrounding region also, and those in turn, in the next surrounding space, and energy is propagated outward, As this action-continues a wave of electromagnetic energy transmitted. When there is excess of electromagnetic energy in unbounded space, it can neither stand still nor merely subside. It can only travel as a wave until the energy is dissipated.

Although Maxwell accepted these conclusions, many other scientists did not until Hertz and later Bose proved the physical existence of electromagnetic waves.

Check Your Progress-I

1. What is a Transformer and what is its efficiency?
2. Write down the marwell's equations of electromagnetic induction.

Note: a) Space is given below for your answer

b) Compare your answers with the one given at the end of this unit

....

....

....

16.7 THE POYNTING VECTOR

A very important aspect of wave propagation is the flow of energy through space. As the wave passes through a surface in space the energy get transported too. At any instant there will be a flow of power (watt / meter^2) through each unit area of the surface and is denoted by the symbol $\vec{P}$. The product $\vec{P}$. A is the power passing through in area. A. Thus The rate of energy flow per unit are in an electromagnetic wave can be described by a vector P. known as the Poynting vector after John Henry Poynting who first pointed out its properties. When flux line (vector P) are drawn they show the flow of electromagnetic energy.

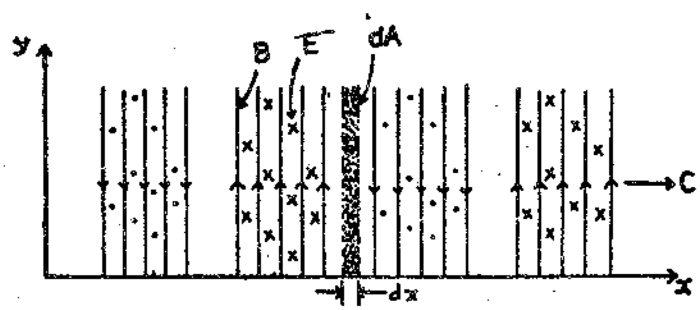


Fig. 16.6

Consider a region of space, enclosed within an imaginary surface. The rate at which electromagnetic energy flows out of this region is found by integration P. over the enclosed surface. The outward flow of power = $\oint \vec{P}.d\vec{A}$. But, if the energy is flowing out of the region, there must be a corresponding loss of electromagnetic energy stored within that region. This is the sum of electric and magnetic energies given by the volume integrals:

Electric energy $\frac{1}{2}\int \vec{D}.\vec{E}\, dv$ ...(16.22)

Magnetic energy = $\frac{1}{2}\int \vec{B}.\vec{H}.\, dv$ ...(16.23)

Total energy = $\frac{1}{2}\int (\vec{B}.\vec{H} + \vec{D}.\vec{E})\, dv$ ...(16.24)

The rate at which this stored energy diminishes is obtained by differentiation i.e.., the rate of decrease of

$$\text{Stored energy} = \frac{d}{dt}\frac{1}{2}\int (\vec{B}.\vec{H}+\vec{D}.\vec{E})\, dv \quad ...(16.25)$$

Assuming that there is no loss of energy by other means, we have

$$\oint \vec{P}.d\vec{A} = -\frac{1}{2}\frac{d}{dt}\int (\vec{B}.\vec{H} + \vec{D}.\vec{E})\, dv$$

since $\vec{B} = \mu \vec{H}$ and $\vec{D} = \epsilon \vec{E}$ we have

$$\oint \vec{P}.d\vec{A} = \frac{1}{2}\int \frac{d}{dt}(\mu \vec{H}.\vec{H} + \vec{E}.\vec{E})\, dv$$

$$= -\frac{1}{2}\int \left(\mu \vec{H}.\frac{d\vec{H}}{dt} + \epsilon - \frac{d\vec{E}}{dt}\right) dv$$

$$= -\frac{1}{2}\int \left(\vec{H}.\frac{d\vec{B}}{dt} + \vec{E}d\vec{D}\right) dv$$

Now using maxwell's equations to substitute for the time derivative and considering the instantaneous values of B and D, we get

$$\oint \vec{P}.dA = -\int \left(\vec{H}.\frac{d\vec{B}}{dt} + \vec{E}.\frac{d\vec{b}}{dt}\right) dv \qquad \ldots(16.26)$$

$$= \int (\vec{H}.\nabla \times \vec{E} - \vec{E}.\nabla \times \vec{H})\, dv \qquad \ldots(16.25)$$

From the mathematical theorem applicable to any vector field we have

$$\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\nabla \times \vec{B}) \qquad \ldots(16.27)$$

equal to $\int \nabla.(E \times H)\, dv$ ...(16.28)

i.e the divergence is here integrated through a volume and by using Gauss's theorem we may substitute for this an integration over the surface enclosing the volume. Therefore equation 16.28 reduces to

$$\oint \vec{P}.d\vec{A} = \oint (\vec{E} \times \vec{H}).d\vec{A} \qquad \ldots(16.29)$$

since both sides of equation 16.29 are surface integrals integrated over the same surface enclosing an arbitrary region of space we have the pointing vector.

$$\vec{P} = \vec{E} \times \vec{H}$$

$$(\text{or } \vec{P} = 1/2\mu\,(\vec{E}_m \times \vec{B}_m)) \qquad \ldots(16.30)$$

and thus the flow of power in wave motion is obtained and is generally valid for the flow of electromagnetic energy in either conducing or non – conducting region.

As a simple example of the Poynting vector field, consider a long cylindrical conductor carrying current. As shown in figure 16.7 a steady current.Is flowing upward in a cylindrical conductor, the front half of the conductor being cut away in the diagram. The electric field with the conductor is correspondingly uniform and upward. The electric field outside the conductor is much stronger, having a tangential component that terminates on some other part of the circuit. The magnetic field within the conductor is circular, and its' strength is proportional to the radius.

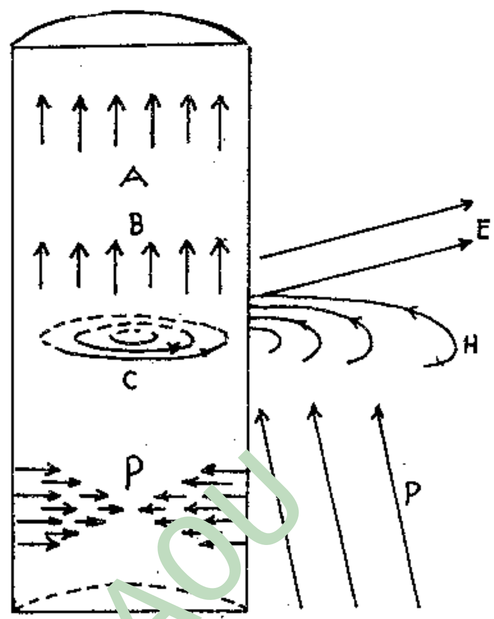


Fig 16.7

A – Current, B – Electric field, C – Magnetic Field, P – Poynting Field.

The Poynting field within the conductor, being $\vec{E} \times \vec{H}$, is radially inward, growing weaker as it penetrates the conductor.

The increasing weakness of the pointing field indicates the consumption of energy. Energy enters the surface of the conductor and flows towards the center. This is used to supply resistance loss in the conductor and the inward flow of energy decreases to zero, as the center of the conductor is approached. The Poynting field outside the conductor is Primarily parallel to the conductor, showing that the energy is being carried in the direction of the conductor (which serves as a guide for energy).But the external field has a sufficient radial component to give an inward flow of energy to provide for the loss of energy in the conductor. Only around a conductor of perfect conductivity would the pointing field the wholly parallel to the conductor.

Note particularly the Poynting vector and therefore the direction of travel of a wave. If the fingers of the right hand curve from $\bar{E}$ to $\bar{H}$, the thumb shows the direction of travel of the wave. This very important relation is easily remembered if $\bar{E} \times \bar{H}$ is firmly impressed on the mind. Obviously a reversal of the order (i.e $\bar{H} \times \bar{E}$) of these vectors would be ruinous, but the memory can be helped by noting that $\bar{E}$ precedes $\bar{H}$ as in the alphabet.

Example 1

Analyze energy flow in the cavity of figure 16.8 using Poynting vector.

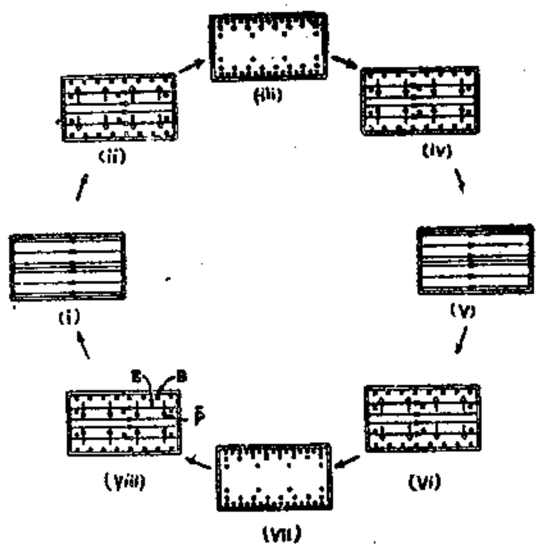


Fig. 16.8

Ans: Study of figure 16.8 shows that when the energy is all electric (figure 16.8 I and v) it is concentrated along axis, because this is the region in which E has its' maximum value. When the energy is all magnetic (fig 16.8iii and viii) it is concentrated near the wall. Thus the energy surges back and forth periodically between the central region and the region near the walls. The open arrows in the figure show the direction of P at various point in the cavity and at various times in the cycle. Note that P equals to zero for figures 16.8 iiii,v,vii because at these instants of time the field configurations are momentarily stationary and energy is not flowing.

16.8 SUMMARY

The energy stored in the electro static field set by the charges is

$$vB = ½ Li^2$$

and the energy density in a magnetic field is

$$vB = \frac{½ Li^2}{lA}$$

In step up transformer number of turns in secondary is greater than number of turns in primary where as in step down transformer the number of turns in secondary are less than the number of turns in the primary.

Maxwell's equation of electromagnetic induction are.

$$\nabla \times H = I + \frac{\partial \bar{D}}{dt} \quad (1)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{dt} \qquad (2)$$

$$\nabla \times \bar{E} = 0 \qquad (3)$$

$$\nabla \times \bar{B} = 0 \qquad (4)$$

The electric and magnetic field in the magnetic equation vectors represent the wave amplitudes traveling with the finite velocity in space these waves do not require a medium. These waves, are transverse in character and they transport energy from one place to other place. Poynting vector represents the energy flow.

Check your progress :Answers

1. A Transformer is an electrical device by which a low alternating voltage can be converted into a high alternating voltage and vice versa.
   The efficiency of a large transformer is 98% where as the efficiency of a small transformer is only about 90%.
2. Maxwell's equations of electromagnetic induction are
   (i) $\nabla \times H = i + \frac{\partial D}{dt}$
   (ii) $\nabla \times E = 0$
   (iii) $\nabla \times E = -\frac{\partial B}{dt}$
   (iv) $\nabla \times \bar{B} = 0$

16.9 SAMPLE EXAMINATION QUESTION

I Answer the following question in detail.

1. Discuss in detail the principle of a transformer.
2. Discuss about the construction and working of electromagnetic cavity oscillator.
3. show that electric and magnetic field vectors represent the wave amplitude, traveling with finite velocity in space.

II. Answer the following question briefly.

1. Obtain the expression for the energy in a magnetic field.
2. What is Maxwell's equation?
3. What is pointing vector.
4. What are traveling waves.

III. Solve the following problems

1. What is the magnetic energy density at the center of a circulating electron in the hydrogen atom.

2. The coaxial has a = lmn; b = 4 mm and l = 50. It carries a current of 10amp in the inner conductor and an equal but opposite directed return current in the outer conductor. Calculate and compare the stored magnetic energy per meter of cable length (a) within the central conductor (b) in the space between the conductors (c) within the outer conductor.

3. What is the mutual inductance of an ideal transformer if a 60 Hz. Current of 2 amp applied to the primary induces 6 V rms (The ratio is the same) at the secondary terminals.

BLOCK – 5: VARYING CURRENTS

Also, at any time 't' the orientation θ of the coil is given by

$$\theta = \omega\tau + \delta \qquad \text{...(17.5)}$$

Where δ represents the value of θ at t = 0. When equations 17.4 and 17.5 are substituted into equation 17.3, the expression for an instantaneous value of emf induced in the rotating loop becomes.

$$E = E_{max} \sin(\omega t + \delta) \qquad \text{...(17.6)}$$

Thus as asserted above, the induced emf behaves like harmonic oscillator.

17.4 GRAPHICAL REPRESENTAION OF ALTERNATING EMF AND CURRENTS

Equation 17.6 is represented by figure 17.2 as a solid curve with E as the ordinate and ωt as the abscissa.

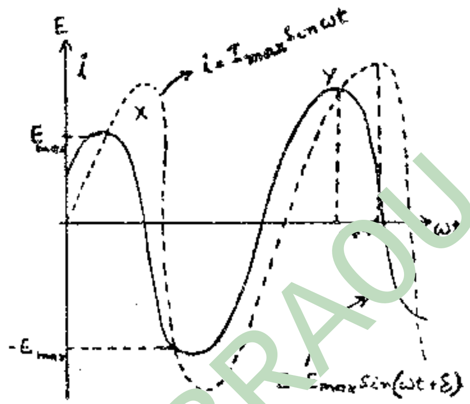


Fig. 17.2 Graphical representation of alternating EMF and currents.

As shown by the solid curve in figure 17.2 the instantaneous emf varies between the limits ± where E_{max} is known as the amplitude of the emf. The quantity appearing in equation 17.6 is the initial phase angle; thus the initial value of E (that is at t =0) is given by $E_{max} \sin\delta$. The phase of E at any subsequent time is determined by the instantaneous phase angle, ωt + δ; one complete alternation of emf is called one cycle and is accomplished in a time known as one period T.

When AC emf of the type described by equation 17.6 drives the current in circuit, the resulting current is also harmonic although not necessarily in phase with the emf. Let us suppose that the initial conditions and circuit parameters are such that the instantaneous current and emf may be written as

$$i = I_{max} \sin(\omega t + \delta_1) \qquad \text{...(17.7)}$$

$$E = E_{max} \sin(\omega t + \delta_2) \qquad \text{...(17.8)}$$

Where δ_1 and δ_2 are the initial phases. The emf reaches maximum at an earlier time than does the current, then the emf is said to lead the current by the phase angle.

$$\delta = \delta_2 + \delta_1 \quad ...(17.9)$$

It is this quantity, the relative phase angle between the emf and current, which is of practical importance, For this reason, it is customary to chose the zero of time so as to make either δ_2 or δ_1 equal to zero. If by this choice $\delta_1 = 0$, the instantaneous current and emf could be written as

$$i(t) = I_{max} \sin \omega t \quad ...(17.10)$$

$$E(t) = E_{max} \sin(\omega t + \delta) \quad ...(17.11)$$

The equation 17.10 representing current is shown in Fig 17.2 as a dotted curve. As required by the equation 17.10 & 17.11, it "lags" the emf curve by the phase angle δ (measured in radians)

17.5 EFFECTIVE OR RMS VALUE OF AN AC CURRENT

We shall calculate the relationship between the effective value of an AC current and its maximum value. This is done by computing the average heating effect of the alternating current as it is a measure through instruments. The r.m.s value of an alternating current is the value of the direct current, which produces the same amount of heat during the same time in the same resistor

The rate of production of heat in a current carrying resistor is given at any instant, by heat generated = i^2R

The average or mean value of this quantity is

$$\overline{i^2} R = i^2_{eff} . R \quad ...(17.12)$$

$$\text{or } I_{eff} = \sqrt{\overline{i^2}} \quad ...(17.13)$$

Here the effective value of an AC current is the mean squares of the instantaneous values taken over the entire time period. For this reason, the effective current is frequently termed as the rms current. The value of $\overline{i^2}$ may be calculated as

$$\overline{i^2} = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{\int_0^T i^2 dt}{T} \quad .....(17.4)$$

It is be noted that integration over one period is sufficient to obtain the mean square to average. Similar definition is applicable to emf also.

$$\overline{i^2} = \frac{I^2 max \int_0^T \sin^2 \omega t dt}{T} \quad ...(17.15)$$

Making use of the identity $\sin^2 \omega t = ½ (1 - \cos 2\omega t)$ ...(17.16)

UNIT 17: - LR AND CR CIRCUITS

Contents

17.1 OBJECTIVES

This Unit discusses the passage of electricity through electrical elements. In order to make you understand the effects of current passed through different combinations of electrical elements are set forth.

After going through this unit you should be able to understand the principle of operation of the alternating current generator; and the relation between the instantaneous and the effective values of current and emf for different combinations of electrical elements.

17.2 INTRODUCTION

In this unit the principle of operation of the alternating current generator is discussed also you will know about the alternating currents. You will be knowing about the power factor and energy stored in a LR circuit.

17.3 SIMPLE A.C GENERATOR

When a loop of wire rotates with constant angular velocity in a uniform magnetic field as in Fig 17.1 the e.m.f induced in the loop follows a harmonic law. Let us analyze the relation between the induced e.m.f and the angular velocity of the rotating loop.

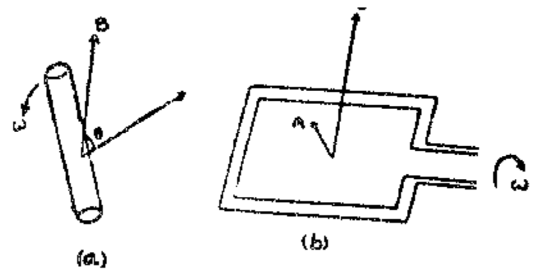


Fig17.1 (a) End view (b) Top view

The total magnetic flux passing through the loop of wire, at any stage during the rotation, is given by

$$= \vec{B}.\vec{A} \qquad ...(17.1)$$

Where $\vec{B}$ is the magnetic induction associated with inform magnetic field and A is unit vector representing the area enclosed by the loop. Thus

$$r = BA\cos\theta \qquad ...(17.2)$$

Where θ is the angle between $\vec{B}$ and $\vec{A}$

The instantaneous value of the induced e.m.f is given by Faraday's Law, viz.

$E = -\frac{dr}{dt}$ which, upon substitution of equation17.2 becomes

$$E = -\frac{d}{dt} - (BA\cos\theta) = \frac{d\theta B}{dt}A\sin\theta$$

$$= BA\omega\sin\theta \qquad ...(17.3)$$

Where $\omega = \frac{d\theta}{dt}$ is the angular velocity of the coil.

We notice from equation 17.3, that maximum value of the induced emf denoted as E_{max} occurs when $\theta = \pi/2$ and is given by

$$E_{max} = BA\omega \qquad ...(17.4)$$

We can evaluate equation 17.15 viz.,

$$\overline{i^2} = \frac{I^2max}{2T}\left[\left(t - \frac{\sin 2\omega t}{2\omega}\right)\right]_0^T \qquad = \frac{I^2_{max}T}{2T}$$

$$\text{or } \overline{i^2} = \frac{I^2max}{2} \qquad \ldots\textbf{(17.17)}$$

$$\text{Thus } I_{eff} = \sqrt{\overline{i^2}} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max} \qquad \ldots\textbf{(17.18)}$$

Where the effective value of AC current is 0.707 times **the amplitude or its maximum** value. Following the same arguments as given above, we have

$$E_{rms} = 0.707\, E_{max} \qquad \ldots\textbf{(17.19)}$$

AC measuring instruments like ammeters and voltmeters, **unless it is specifically stated to** the contrary, read effective or rms values. As a result, effective values are **ones normally** dealt with

Example 1:

The equation for an alternating current is $i = 42.42 \sin 314t$. **Determine (i) its maximum** value (ii) its frequency (iii) its RMS value.

We know

$i = I_{max} \sin \omega t$

(a) $I_{max} = 42.42$

(b) $\omega t = 314t$

$\omega = 314$

Now $\omega = 2\pi f$

$f = 314/2\pi = 50Hz$

(c) $I_{rms} = I_{max} / \sqrt{2}$

$= 42.42 / 2 = 30$ Ampers

Example 2:

An AC voltage E =200 sin 628 t is applied to a device which offers ohmic resistance of 10 to the flow of current in one direction while entirely prevents the flow in the opposite direction. Calculate the rms value?

The standard voltage equation of an alternating Quantity is given by

$E = I_{max} \sin \omega t$

Now equating the given equation we have

E = 200 sin 628 t we have

$E_{max} = 200, \omega = 628, \& R = 10$

Then $I_{max} = E_{max} / R = 200 / 10 = 20$

Since the current flows in one direction

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$= 20 / \sqrt{2}$ Amp.

Example 3:

If the main AC supply is 220 volts, find the maximum voltage of the supply?

Here

$E_{rms} = 220$

$$E_{rms} = \frac{E_{max}}{\sqrt{2}}$$

$E_{max} = \sqrt{2} \text{ X } E_{rms}$

$= \sqrt{2} \text{ X } 220$

$= 1.414 \text{ X } 220$

$= 311.1$ Volts

Check your progress –I

1. Define RMS value of alternating current.
2. Why effective current is termed as RMS current ?

Note: a. Space is given below for your answer.
b. Compare your answers with those given at the end the unit

....
....
....
....

17.6 POWER FACTOR OF AN AC CIRCUIT

At any instant the rate of energy consumed in a circuit is given by the product of the current through it and the potential difference across it

Instantaneous power dissipation $= \overline{iV}$ ...(17.20)

Average power dissipation $= \overline{iV}$ ...(17.21)

$$= \frac{1}{T}\int_0^T ivdt$$

$$= \frac{1}{T}\int_0^T I_{max} \sin \omega t \, V_{max} \sin(\omega t + \delta)dt$$

$$= \frac{I_{max}}{T} V_{max} \int_0^T \sin \omega t(\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= \frac{I_{max} V_{max}}{T} \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) \cos \delta + \sin \omega t \cos \omega t \sin \delta) dt$$

$$= \frac{I_{max} V_{max}}{T} \left[\frac{t}{2} \cos \delta \quad \frac{\sin^2 \omega t \cos\delta}{4\omega} + \frac{\sin^2 \omega \sin\delta}{2}\right]_0^T$$

$$= \frac{I_{max} V_{max}}{2} \cos \delta$$

$$= IV \cos \delta$$ ...(17.22)

Cos δ, which appears in equation 17.22 is known as the POWER FACTOR. For purely resistive circuits, in which δ = 0, the power factor is unity and the power is given by the product of the effective current and the effective voltage.

17.7 ENERGY STORED IN A LR CIRCUIT

Consider a circuit containing a pure inductive coil of inductance L Henry and pure resistance of R ohm connected in series as shown in Fig 17.3

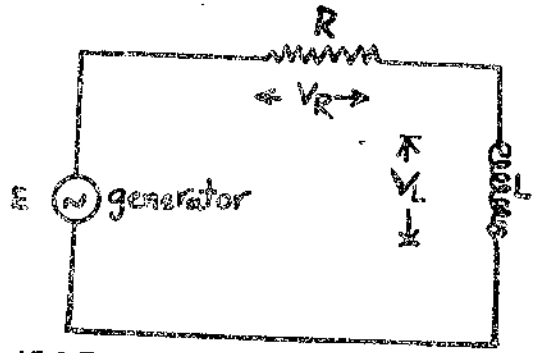


Fig 17.3 Energy stored in LR circuit

When an alternating voltage is applied in this circuit the voltage drops on the resistance say V_R and on the L say V_L. If 'i' is the current in the circuit then

$V_R = iR$ in phase with current

$V_L = i\omega L$ leading the current by 90^0

These voltages are shown by the vector diagram Fig 17.4 since V_L is in quarature ($\pi/2$) with V_R

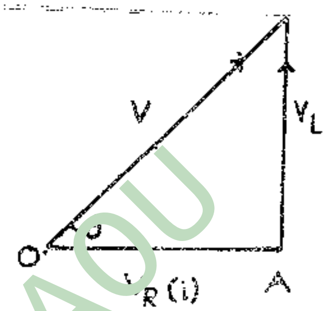


Fig 17.4 Vectorial representation of circuit in Fig 17.3

$$V^2 = V^2L + V^2/R$$

$$= (\omega Li)^2 + (Ri)^2$$

$$= i^2 [R^2 + (\omega L)^2]$$

$$V = i\sqrt{R^2 + (\omega L)^2}$$

$$i = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$$

$$= V / \sqrt{R^2 + X^2L} \quad \ldots(17.23)$$

The term $\sqrt{R^2 + X^2L}$ is called impendence of the circuit and is denoted by z. Its units are Ohm

$$Z = \sqrt{R^2 + X^2L}$$

Thus the relation shown in equation 17.23 can be written as ...(17.24)

$$i = V / Z$$

or $$V = iZ$$ ...(17.25)

A triangle whose sides are proportional to the voltage V, V_L and V_R is called voltage triangle. A triangle whose sides are proportional to R, X_L and Z is called impudence triangle Fig. 17.5

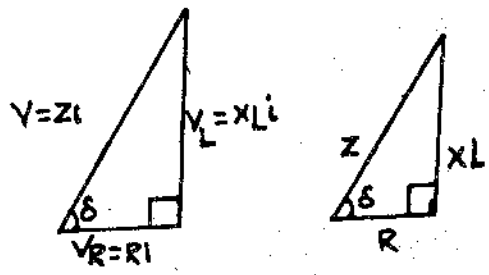


Fig 17.5 Impedance triangle

It is clear that the vector V leads the vector the 'i' by an angle. The cosine of this angel i.e. cos δ is called the power factor.

Power factor = $\cos\delta = R / Z$ ...(17.26)

Assuming voltage leads current by an angle δ the current and voltage can be shown as

$$V = V_{max} \operatorname{Sin} \omega t$$

$$i = I_{max} \sin (\omega t - \delta)$$

We know that power is equal to the product of current and voltage

$$P = Vi$$

$$= V_{max} \sin \omega t \, I_{max} \sin (\omega t - \delta)$$

$$= V_{max} I_{max} \sin \omega t \sin (\omega t - \delta)$$

$$= \tfrac{1}{2} V_{max} I_{max} [\cos (2\omega t - \delta)]$$

Now the value of cos δ is constant in a given circuit where as cos (2ωt - δ) is a variable quantity and its value is zero. Thus the average power.

$$P_{av} = \tfrac{1}{2} V_{max} I_{max} \cos\delta$$

$$= \frac{V_{max}}{\sqrt{2}} X \frac{I_{max}}{\sqrt{2}} . \operatorname{Cos}\delta$$ ...(17.27)

Thus power factor (p.f.) cos δ of a circuit is the factor by which the product of rms current and voltage must be multiplied in order to have the true factor in Watt.

17.8 RESISTANCE AND CAPACITANCE IN SERIES A.C CIRCUIT

Fig 17.6 shows a R and C in series circuit. When an alternate current is passed, let V be the applied emf, e be the charge on the capacitor of capacity 'C' at any instant and R be the ohmic resistance in the circuit. In this case the resistance drop $V_R = I R$, is in phase with current $Vc = I / \omega C$ is in quadratine with current. The capacitative reactance X_C being regarded as negative, we have from voltage triangle Fig 17.7

Fig 17.6 R, C series circuits

$$V^2 = V_R^2 + V_C^2$$

$$V^2 = (iR)^2 + (iX_c)^2$$

$$= i \sqrt{R^2 + (X_c)^2}$$

$$V = iZ$$

$$i = V / Z$$

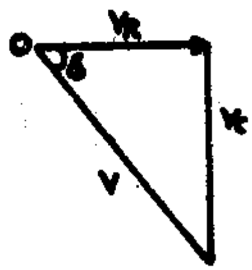


Fig 17.7 Voltage triangle

Where $Z = \sqrt{R^2 + X_c^2}$ is called impedence of the circuit. Thus in this case the circuit leads the voltage V by an angle and the two are represented as

$V = V_{max} \sin \omega t$

$i = I_{max} \sin (\omega t + \delta)$

or $i = I_{max} \sin \omega t$

$V = V_{max} (\sin \omega t - \delta)$ ...(17.28)

In figure 17.8 the impedence triangle is shown

The power consumed $P = Vi$

$P = V_{max} I_{rms} \cos \delta$ ...(17.29)

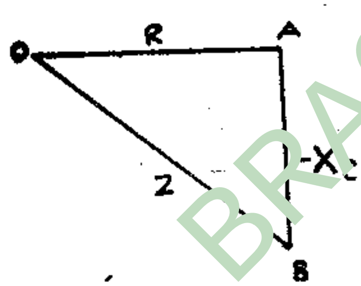


Fig 17.8 Impedence triangle

17.9.L.C.CIRCUIT

At resonance in a series circuit the inductance & capacitance voltage are equal and opposite i.e. $V_L = -V_c$ while in parallel resonant circuit the inductive & capacitative currents are equal and opposite.

i.e. $I_L = -I_C$

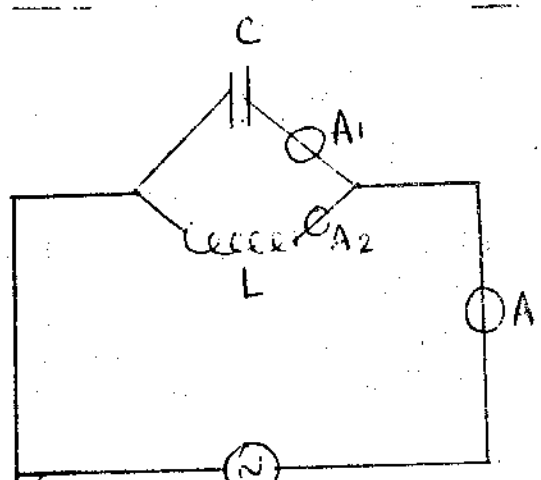


Fig 17.9

in series same voltage but opposite, in parallel the same current but opposite.

Thus at resonant frequency & when R=0, the line current would be zero but there is an oscillatory current in L & C. The applied emf merely supplies the energy to compensate for any circuit losses

One finds that, for fixed values of L,C & R, the impedance of an L.C & R circuit depends on $(\omega = 2\pi f)$. Hence it varies with frequency. In such a circuit, $W_L = 1/\omega c$ or $2\pi fL = 1/2\pi fc$ i.e. If the inductive reactance = capacitative reactance, or at a particular frequency of Ac, f is such that

$$f^2 = \frac{1}{4\pi^2 LC} \qquad \text{or } f = \frac{1}{2\pi\sqrt{LC}} \qquad \ldots(17.30)$$

Then in equation $Z = \sqrt{R^2 + (\omega^2 + (1/\omega^2)^2}$

Maximum current will be admitted through the circuit and the circuit will behave as if the coal and condenser were absent. The circuit is said to be in time or Resonance with applied emf (i.e. the value of the frequency of applied emf) for which $W_L = 1/\omega c$, is called resonant frequency (f_0) and is given as so many cycles /sec. When L is in henries & C is in Farads. This gives condition for Resonance.

In a circuit like this

$$L\frac{di}{dr} + \frac{q}{c} = 0 \qquad \ldots(17.31(a))$$

$$\& \frac{di}{dr} + \frac{q}{CL} = 0 \qquad \ldots(17.31(b))$$

$$\& \frac{d^2i}{dr^2} + K^2q = 0 \qquad \ldots(17.31(c))$$

$$\& \; W^2 = K^2, W = K = \frac{1}{\sqrt{LC}} \qquad \ldots(17.31(d))$$

$$\text{or } 2\pi f = \frac{1}{\sqrt{LC}} \quad , f = \frac{1}{2\pi\sqrt{LC}}$$

Which is the same as equ.17.30 above. This is the frequency of the oscillatory discharge in a circuit of inductance L and capacitance C when the resistance is low.

Thus the strength of AC in any series circuit due to a given applied emf is greatest when the frequency of applied emf is equal to the natural frequency of the circuit, a condition analogous to that necessary for resistance in the case of sound. The above circuit is then called a series resonate resonance circuit with R, &

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad ...(17.32)$$

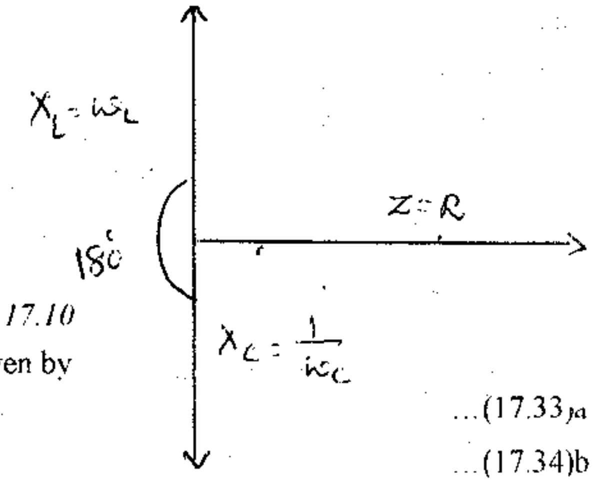


Fig 17.10

the maximum current in the circuit is given by

$$I_0 = E_0 / R \qquad ...(17.33)a$$

$$\& \; E_{rms} = I_{rms} R \qquad ...(17.34)b$$

Also at resonance the alternating P.D the inductance (L) & the capacity (c) are equal & 180^0 out of phase, or in antiphase

Thus the ratio of the voltage across (across the condenser) to the voltage across resistance or applied voltage at resonance is called the Q or voltage magnification factor of the circuit also the voltage across inductance or (capacitance)

$=WLI_{emf}$

Voltage magnification or factor

$$= \frac{WLI_{emf}}{RI^{rms}} = \frac{WL}{R}$$

If the frequency of a C supply is varied or it contains a no. of frequency components (as in a receiving serial) a series LCR circuit across its supply will or accept a maximum current, i.e. a maximum response for only that component of a supply which has a frequency

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad ...(17.37)$$

provided the resistance R is very low or negligible.

It is also some times called an acceptor circuit. It is used as a tuning circuit with the arial of a radio receiving station. Also when R =0, the impedance of the circuit becomes zero at the resonant frequency i.e. in LC circuit in other words.

Check your progress- II

1. Define Power factor
2. What is resonance frequency?
3. If R = O. what happens to the impendance in L C circuit!

Note: a. Space is given below for your answers.

b. Compare your answers with those given at the end of the unit.

....

....

....

....

17.10. SUMMARY

An alternating current is one that changes the direction periodically. The effective value of AC current is 0.707 times the maximum of the AC current. Power factor of an AC circuit is IV cosδ

Check your progress – answers

I 1. The RMS value of an alternating current is the value of the direct current. which, produces the same amount of heat during the same time in the same resistor.

2. The effective value of an AC current is the mean squares of the instantaneous values taken over the entire time period. Ie $I_{eff} = \sqrt{i^2}$ Hence the effective current is termed as the rms current.

II 1. At any instant the rate of energy consumed in a circuit is given by the product of the current through it and the potential across it.
P=Vi.

2. When the circuit is said to be in Resonance with applied emf (ie the value of the frequency of applied emf) for which WL=1/wc is called resonant frequency.

3. When R=o the impedence of the circuit becomes zero at the resonant frequency in LC circuit.

17.11 SAMPLE EXAMINATION QUESTIONS

I. Answer each of the following questions in detail.

1. Calculate the relationship between the effective value of an AC current and its maximum value.
2. What is the amount of Energy stored in LR circuit.

II. Answer each of the following questions briefly.

1. Describe a simple AC generator.
2. What is a power factor of an AC circuit?
3. Discuss the application of AC voltage to a resistor.
4. Explain in detail about LC circuit.

UNIT-18: TRANSIENT RESPONSE IN CIRCUITS

Contents

18.1 OBJECTIVES

This Unit discusses the phenomenon of growth and decay of current in LR, CR and LCR series circuits.

To make you understand the phenomenon, circuits containing inductances and capacitances are explained.

After going through this Unit you should be able to evaluate the growth and decay of current in LR, CR and LCR circuits

18.2 INTRODUCTION

In this Unit we will discuss the currents in a LR circuit and charges in a CR Circuit.

Here in this Chapter we will also discuss about the transient behaviour of LR, CR and series LCR circuits. LCR series behaviour is analogous to that of a damped mechanical oscillator. The behaviour is analogous to what happens in a simple pendulum if it is displaced from its equilibrium position and then released. Depending on the amount of friction, or damping, the pendulum will either oscillate back and forth, with a gradually decreasing amplitude, or will move towards its equilibrium position without oscillations. We discuss the electrical analogue shown in Fig 18.1

18.3.1 Resistors

Fig 18.1 Shows a resistor connected to a source of sinusoidal voltage, with ac meters for measuring current and voltage. We assume that the meters are ideal, that is the current meter has negligible resistance and that voltmeter passes negligible current. We also assume that the meters can tell the instantaneous phases of current and voltage.

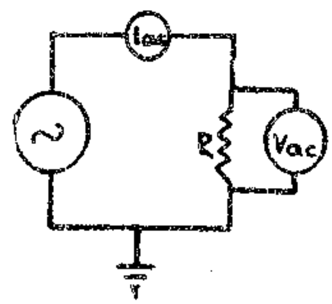


Fig 18.1 Resistor connected to Ac source

In order to determine what happens in this simple case, we examine circuit equations: The current conservation equation, Ei = 0 is easy to apply in this circuit.

The time average current at any point in the circuit can be represented as:

$$i(t) = 0 \qquad \text{...(18.1)}$$

Similarly second circuit equation for voltage can be written as

$$E(t) = i(t)\,R \text{ (Ohm's Law)} \qquad \text{...(18.2)}$$

Since R is a constant, the time variation of i must be proportional at every instant to the time variation of the source E. Thus if E (t) is written as

$$E(t) = V_0 \cos \omega t$$

Where V_0 is sinusoidal voltage amplitude of the generator,

We find: $i(t)\,R = V_0 \cos \omega t$

$$\text{or } i(t) = V_0 / R \cos \omega t$$

The only way equation can be satisfied is to get

$$i(t) = I_0 \cos \omega t$$

Where I_0 is the sinusoidal amplitude of the current, this gives

$$I_0\, \omega t = V_0 / R \cos \omega t \qquad \text{..........18.3}$$

From this result the amplitude of ac current follows as

$$I_0 = V_0 / R \qquad \text{...(18.4)}$$

The result is just the Ohms law which works equally well for both ac and steady voltages. The nomenclature for R can be generalized as resistive impedance.

18.3.2 Inductance

Fig 18.2 shows a simple inductance connected to a source of sinusoidal EMF and the coil is assumed to be of pure inductive nature.

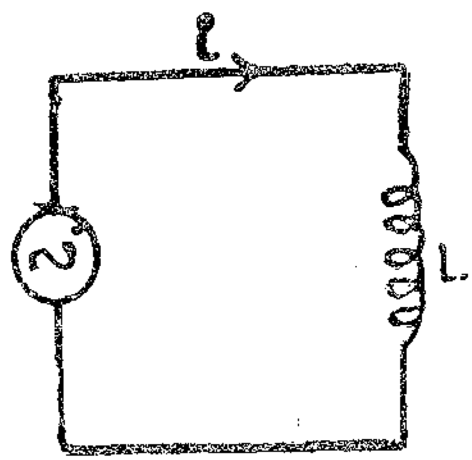


Fig 18.2 Inductance connected AC circuit.

As per the circuit equation we know that

$$E + E_l = 0$$

Where E_L is induced voltage across the inductance coil. The induced EMF E_L is thus

$$E_l\ (t) = \overline{L}\ (di / dt) \qquad ...(18.5)$$

Where L is the self inductance of the coil. The negative sing denotes the effect of EL in the circuit which tends to decrease the magnitude of current.

Combining the last two equations we get

$$E = V\ (t) = V_0 \cos \omega t \qquad ...(18.6)$$

Since $E_L = -E$

$$L\ (di / dt) = V_0 \cos \omega t \qquad ...(18.6(a))$$

The interpretation of the i (t) is such that the slope *(dω / dt)* is also a varying function of time having the same phase as the generating voltage and amplitude is given by V_0 / L. An expression for the current (i) t can be obtained by integrating the expression.

$$\frac{di(t)}{dt}$$

Thus

$$i(t) = \frac{V_0}{\omega L} \int \omega \cos \omega t \, dt$$

$$= \frac{V_0}{\omega L} \sin \omega t$$

$$= I_0 \sin \omega t \qquad \ldots(18.7)$$

Where I_0 is the amplitude of the time varying current shown in fig 18.3

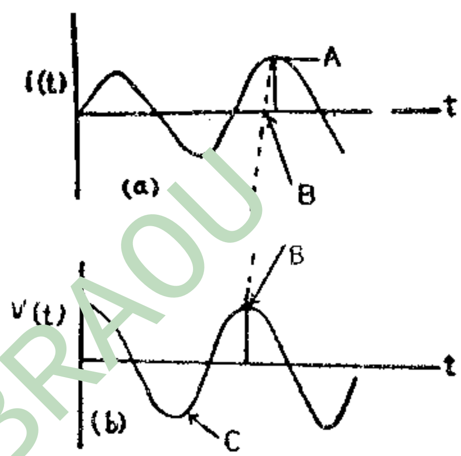


Fig 18.3 Voltage and current across a pure inductance

A. $i(t) = \frac{V_0}{\omega L} \sin \omega t$

B. This current maximum occurs later than the point c, which is $V(t) = V_0 \cos \omega t$

From these, we learn that the current passing through a pure inductance when connected to a sinusoidal voltage is also sinusoidal, with the same frequency, but $\pi / 2$ or 90^0, behind the voltage in phase. In addition the amplitude of the current.

$$I_0 = \frac{V_0}{\omega L} \qquad \ldots(18.8)$$

depends not only on L but also on the frequency of the driving voltage v (t); the higher the ac frequency the smaller is the ac current.

The 90^0 phase lag of the current becomes obvious if we use the well known trigonometric transformation:

$$i(t) = \frac{V_0}{\omega L} \sin \omega t$$

$$= \frac{V_0}{\omega L} \cos(\omega t - \pi/2) \qquad \ldots(18.9)$$

Equation 18.8 manifests that the quantity (ωL) plays same role as does R in resistive case and ωL is known as the inductive reactance.

18.3.3 Capacitance

The instantaneous voltage V across a capacitor 'C' is shown in Fig 18.4

$$V(t) = \frac{q(t)}{C} \qquad \ldots(18.10)$$

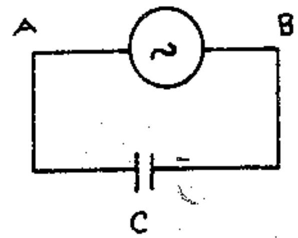


Fig.18.4 Circuit consisting of a capacitor connected to a AC source

The circuit equations for this case has to be modified to accommodate the voltage across the capacitor. caused by the charge q on it at any instant of time. Choosing zero time when the voltage at A caused by the generator is positive and say, increasing the charge on the capacitor starting from an instant when its charge was zero, is

$$q(t) = \int_0^t i.dt \qquad \ldots(18.11)$$

The situation at time t is described by the circuit equation, i.e.

$$v(t) = + E(t) = 0 \qquad \ldots(18.12)$$

In order to express this result in terms of i(t), since $i = dq/dt$ we differentiate equation 18.12 to find

$$\omega C V_0 \sin \omega t dt = dq$$

$$\text{or} \quad -\omega C V_0 \sin \omega t = dq/dt = i(t) \qquad \ldots(18.13)$$

We can now write the current as $I_0 \sin \omega t$ with I_0 being equal to $C\omega V_0$

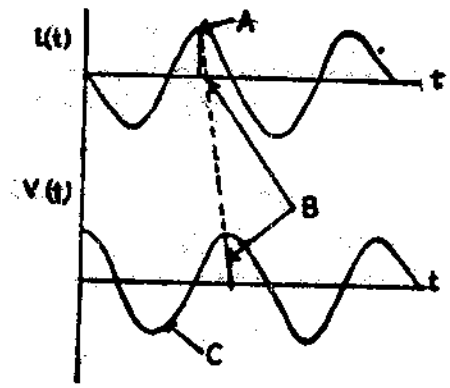


Fig 18.5 Voltage and current across a capacitor.

(A) $i(t) = -\omega C V_0 \sin \omega t$

(B) = This current maximum occurs before the voltage maximum

(C) $V(t) = \cos \omega t$

Fig 18.5 is a graph of the source potential, V (t) and the resulting current through the capacitor, i (t) showing that the current leads the sinusoidal voltage by 90^0. The current amplitude I_0 is given by

$$I_o = \frac{V_o}{(1/c\omega)} \qquad \ldots(18.14)$$

Eqn. 18.14 shows that the term $1/\omega C$ plays a role of R in resistance case except for a sign. The $1/\omega C$ is known as the comparative reactance.

Use of the complex algebra

The use of complex numbers in ac circuit elements greatly simplifies the analysis. In appendix II it is shown that the term number in the complex plane can be characterized by the vector sum of two directed physical quantities, as shown Fig 18.6 one the real number A and the other imaginary number JB.

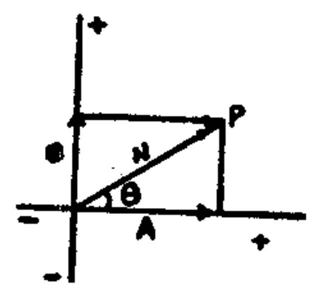


Fig 18.6 Point 'p' in the complex plane represents the complex number.

Real numbers multiplied by $j = (\sqrt{-1}$. are imaginary the complex number) P can then be represented as

$$P = A + jB$$

Another way to characterize the complex number is to write

$$P = Ne^{j\theta}$$

where $e^{j\theta} = \cos\theta + j\sin\theta$ ...(18.16)

Where N is a real number, called the modules, and θ an angle, in radians. Together N and θ describe the magnitude and direction of a vector in the complex plane, as shown in Fig. 18.6 (e is the base of the natural logarithms). This result depends on the identity shown in appendix II.

The particular advantage of complex numbers in a circuit is that they provide a two dimensional name with appropriate vectors arising in the a.c. circuit analysis. In particular let the angle θ be a function of time using θ = ωt, then equation (18.16) becomes.

$$e^{j\omega t} = \cos\omega t + j\sin\omega t \quad ...(18.17)$$

$e^{j\omega t}$ now represents a unit vector rotating with constant angular velocity ω, and is thus suited for representing any rotating vector in an ac circuit. The two terms cos ωt and j sin ωt give the real and imaginary components of the rotating vector. Either one can be used to represent a sinusoidal functions of time.

Resistor

We replace sinusoidal voltage E(t) = Vo cos ωt by the expression

$$E(t) = V_0 e^{j\omega t} \quad ...(18.18)$$

This gives the vector amplitude V_0 rotating at an angular frequency ω, As per the eqn. 18.17 one can get either cosine or sine function of the time variation of voltage. The circuit equation now gives

$$i(t)R = V_0 e^{j\omega t} = V_0[\cos\omega t + j\sin\omega t]$$

$$i(t) = [V_0/R]\,e^{j\omega t} = V_0/R\,[\cos\omega t + j\sin\omega t] \quad ...(18.19)$$

The ac resistive impedance of the resistance is the real number R.

Inductor

From equation 18.6a the generator voltage in a complex numbers notation is given by

$$\frac{di}{dt} = \frac{Vo}{L}e^{j\omega t} \quad ...(18.20)$$

This may be written in integral form

$$di = \frac{Vo}{L}e^{j\omega t}dt \quad ...(18.21)$$

Which immediately integrates to

$$i(t) = \frac{V_0}{j\omega L} e^{j\omega t} \qquad \ldots(18.22)$$

One of the charms of the exponential form is the ease with which it can be integrated or differentiated. By comparing with the resistive case we see that the complex form of the inductive reactance is

$$X_L = j\omega L \qquad \ldots(18.23)$$

If the term $V_0 / j\omega L$ were real, the current would have the same phase as the voltage across the inductor, since this term is a real number.

$\frac{V_0}{\omega L}$ multiplied by

[illegible] $\ldots(18.24)$

It is a pure imaginary number. As shown in Fig 18.7 multiplying a real number $V_0 / \omega L$ by $-j$ has the effect of rotating the vector -90^0 into the pure imaginary axis. In other words, the current lags the voltage by 90^0. Thus the complex notation, phase information is automatically contained in the expression for the reactance vector.

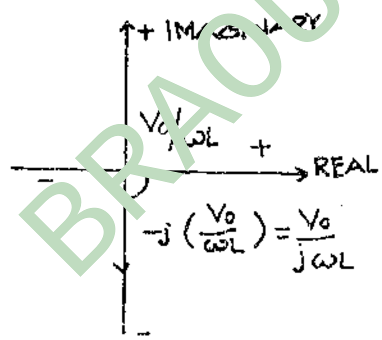


Fig 18.7

Capacitor.

Starting with eqn 18.11 the circuit for a capacitor connected to ac voltage source, we write:

$$q(t) = CV_0 e^{j\omega t} \qquad \ldots(18.25)$$

Differentiation with respect to time gives

$$i(t) = \frac{dq}{dt} = j\,\omega . CV_0 e^{j\omega t} \qquad \ldots(18.26)$$

Thus $1/j\omega c$ is the complex form of the capacitve reactance.

$$X_c = \frac{1}{j\omega c} = -j/\omega C \qquad \ldots(18.27)$$

In this case ac current vector, according to eqn. 18.26 is real number $j\omega V_0$ multiplied by j. As shown in Fig 18.8 this rotates the current vector 90°, ahead of the voltage to lead it by 90°

To summarize the current amptitude

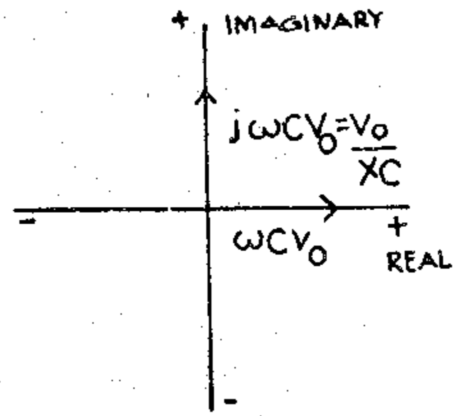


Fig 18.8

each kind of element R, L and C can be written in terms of the ac impedance Z, using the equation.

$$I_0 = \frac{V_0}{Z} \qquad \ldots(18.28)$$

Where $Z_R = R$ resistive inpedence I_0 in phase with V_0

$Z_L = j\omega L = X_L$ inductive resistance I_0 lags V_0 by 90^0

$Z_C = \frac{1}{j\omega C} = X_C$ capacitive resistance I_0 leads V_0 by 90^0

All three terms may contribute to the total ac impedance of a current in general.

18.4 GROWTH AND DECAY OF THE CURRENT IN LR CIRCUITS

In circuit where the energy can be stored as in a charged capacitor or in an inductance while it is carrying current, the sudden application or removal of an applied voltage causes a momentary changing response in the circuit while it adjusts to the new conditions. Here in this we discuss few cases.

To begin with we discuss the case of an inductor and a resistance in series to which a voltage source is connected with a closing and opening switch.

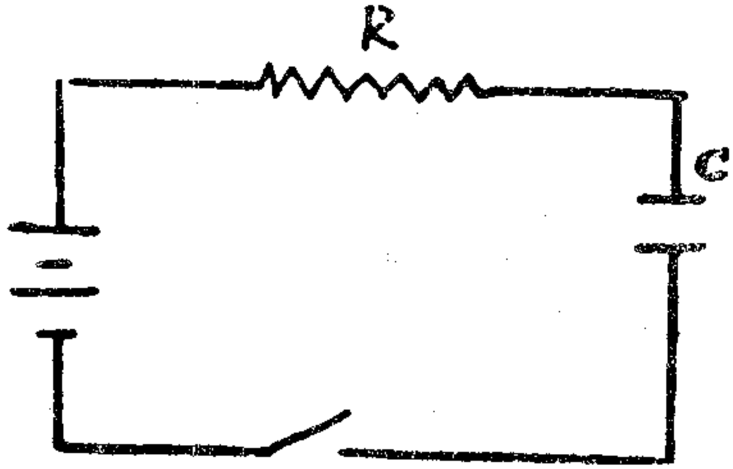


Fig 18.9

as shown in fig 18.9. The resistance might be simply the minimum unavoidable resistance in the circuit, or it might be a resistor put in with a purpose. As soon as the switch is closed the situation is described by

$$V = Ri + L\frac{di}{dt} \qquad \text{...(18.29)}$$

This differential equation can be solved when we put in the required initial and final boundary conditions. But before solving it formally we may see by inspection the kind of time variation of current is expected. The instant the switch is closed, since the current is zero, the entire voltage drop in the circuit must be across the inductance. Thus the initial value of di/dt is V/L. After a long time the current will become steady so the inductive voltage term goes to zero. The steady state value of current I_0, will thus be

$$I_0 = V / R$$

The initial rate increase of current as well as its final steady value, have now been established as displayed in Fig 18.10. All that is needed is to establish the details of the transition region, as shown by the dotted part of the curve.

Now formally we show that the current after starting from zero, approaches its final value exponentially, that is according to the equation:

$$i(t) = I_0[1-\exp(-t/\tau)] \qquad \text{...(18.30)}$$

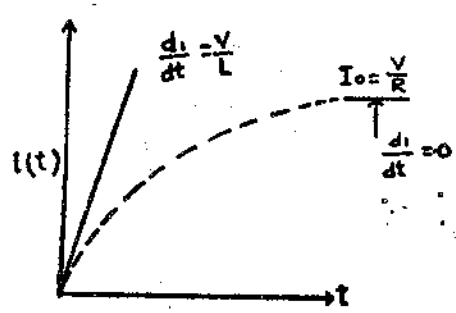


Fig 18.10

Where τ is the relaxation time of the circuit. Now on separating the variables i and t in equation 18.29 putting it in the form

$$\int -\frac{R\,di}{V-Ri} = -\frac{R}{L}\,dt$$

(Where we have multiplied both sides by −R to make left–hand side a perfect differential and integrating we get

$$\log(V-Ri) = \frac{Rt}{L} + C \qquad \ldots(18.31)$$

Where the integration constant C can be evaluated from the condition that at t=0, (when the switch is closed) i=0. This gives C =log V. We put this in above equation and convert it to be exponential form

$$V - Ri = Ve^{(R/L)t} \qquad \ldots(18.32)$$

The current is given by

$$i = \frac{V}{R}\left[1 - e^{-(R/L)t}\right]$$

$$= I_0\left(1 - e^{-(R/L)t}\right) \qquad \ldots(18.33)$$

A plot of this solution is given in Fig 18.11. The behaviour of the circuit is completely determined by the value of R/L, or by its reciprocal L/R = τ the relaxation time of the circuit, with the use of τ as defined the equation becomes

$$i = I_0\left(1 - e^{-t/\tau}\right) \qquad \ldots(18.34)$$

τ is the time for the current to build up to $\left[1 - \frac{1}{e}\right]$ or 0.632 of its final value. We see this by putting $t = \tau = \frac{L}{R}$

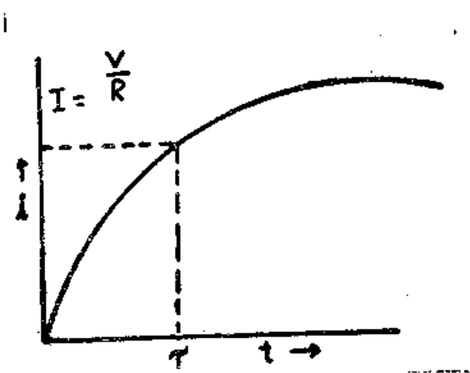


Fig 18.11

$$i = I_o\left(1 - \frac{1}{e}\right) = I_o(1 - 0.318) = 0.632\,I \qquad \ldots(18.35)$$

There are many physical situations which lead to equations like V = Ri + L (di/dt) and hence to solutions of the form of equation (18.33). These can be identified as giving rise to an exponential approach of variable to a final value. When this identification can be made it is possible to omit the formal steps to the solution to the differential equation, write a solution which has the correct exponential form, and evaluate the required constants. Now let us illustrate this procedure, using equation

$$i(t) = I_o(1 - e^{-t/T}) \quad \ldots(18.36)$$

Let us differentiate equation 18.36

to get $\frac{di}{dt} = \frac{I_o e^{-t/T}}{T}$

But for $t = 0$, $e^{-t/T} = 1$

So $\left(\frac{di}{dt}\right)_0 = I_o/T$

We have already seen that the initial slope of the current curve is V/L, so we have

$$\left(\frac{di}{dt}\right)_0 = \frac{I_o}{T} = \frac{V}{L} = \frac{V}{RT} \quad \ldots(18.37)$$

This gives, for the relaxation time,

$$T = \frac{L}{R} \quad \ldots(18.38)$$

In agreement with result of equation 18.33

Now let us take the same circuit elements but this time suddenly change the applied voltage from V to zero by switching from A to B in Fig 18.12

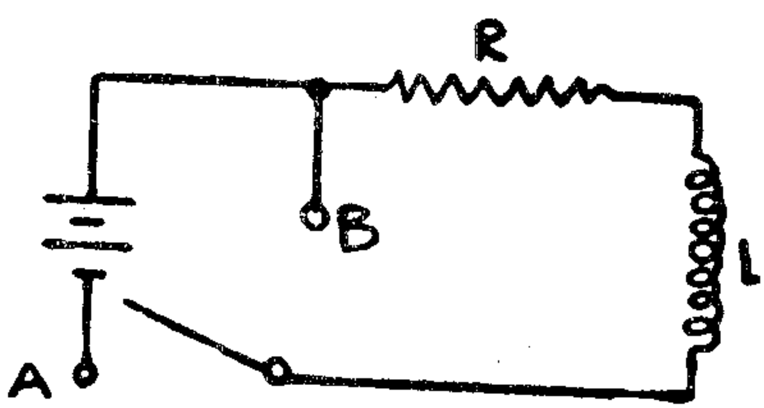


Fig 18.12

This removes battery from the circuit, while allowing the current to flow until decays exponentially to zero. Letting t = 0 at the moment the switch is changed, the equation describing the circuit becomes.

$$0 = Ri - L\frac{di}{dt} \text{ (because of } \frac{di}{dt} \text{ - vc)} \qquad \ldots(18.39)$$

This problem could be solved by separation of variables; but instead we go directly to the solution by applying the boundary condition of the problem to an assumed exponential solution. In this case at t = 0, V --Ri = I_0 since there is no voltage drop across the inductance when the current is steady. Also after a long time the current will go to zero, So the obvious solution to equation 18.39 is of the form

$$I(t) = I_0 e^{-t/\tau} = \frac{V}{R} e^{-t/\tau} \qquad \ldots(18.40)$$

it is easy to verify that τ has the same value L/R, A plot of the current against time is shown in Fig 18.13. Here τ is the decay of current in RL circuit after the voltage source is removed, time for the current to fall to the fraction i/e of its original steady state.

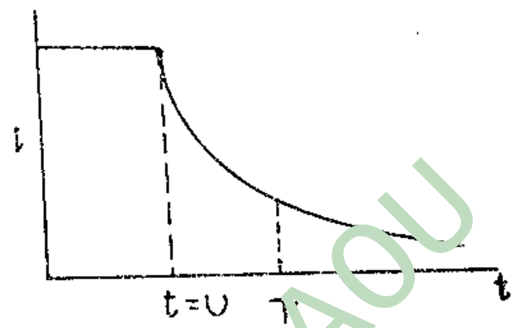


Fig 18.13

18.5 GROWTH AND DECAY OF CURRENT IN CR CIRCUITS

b The RC circuit is shown in Fig 18.14

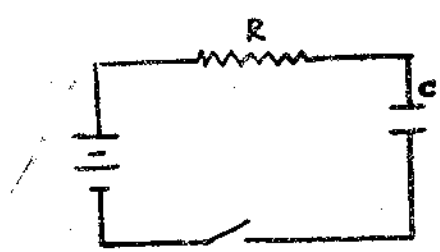


Fig 18.14

The equation that applies when the switch is closed is

$$V_0 = R\frac{dq}{dt} + \frac{q}{C} \qquad \ldots(18.41)$$

Using dq/dt for the current so as to limit the variables to q and t, where q is the charge on the capacitor

At t = 0, q = 0, and after a long time, when flow of charge ceases the charge on the capacitor is q

At t = 0, q = 0, After a long time t = α, when flow of charge to the capacitor has ceased,

$$CV_0 = Q_0 \qquad ...(18.42)$$

Therefore we guess the solution to equation 18.41 to be of the form

$$q(t) = Q_0 (1 - e^{-t/\tau}) \qquad ...(18.43)$$

Since this gives a function which has the correct value for q at t = 0 and at t = α. The form of this function is shown by dotted curve in Fig 18.15. Substitution to equation 18.43 and its time derivative, dq/dt in the circuit equation 18.41 it gives

$$V_0 = \frac{R}{\tau} Q_0 e^{-t/\tau} + \frac{Q_0}{C} [1 - e^{-t/\tau}] \qquad ...(18.44)$$

At t = 0 this becomes

$$V_0 = \frac{RQ_0}{\tau}$$

Which, with the use of equation 18.42 leads to

$$\tau = RC \qquad ...(18.45)$$

With this equation of constant τ, equation (18.41) or (18.44) is satisfied for all times, showing that equation 18.43 is indeed the solution needed.

The expression for the current as a function of time is obtained by differentiating equation 18.43

$$\frac{dq}{dt} = i = \frac{Q_0}{RC} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau} \qquad ...(18.46)$$

the current decay curve is shown as the solid curve in Fig 18.15

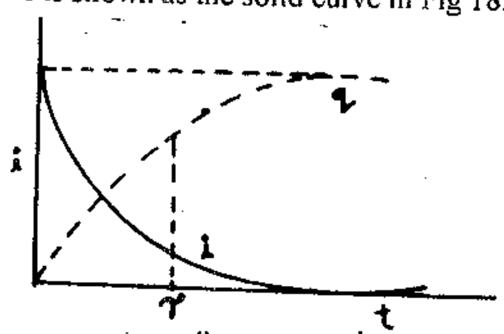


Fig 18.15

Finally, we study the case of the discharge of the capacitor as shown in Fig 18.16. We start with the capacitor charged to a potential $V_0 = Q_0/C$ when the switch is closed, the situation is described by

$$\frac{Q}{C} + R\frac{dq}{dt} = 0$$

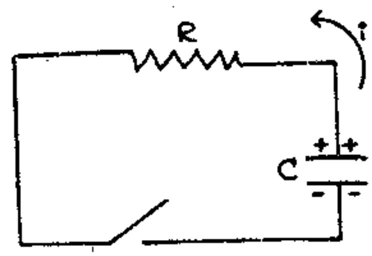


Fig.24.16 18·16

Fig 18.16

The solution of the above equation

$$q = Q_0 e^{-t/RC} \qquad ...(18.47)$$

$$i = -\frac{dq}{dt} = \frac{Q_0}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/\tau} \qquad ...(18.48)$$

This result is identical with equation 18.40. However, the current is flowing in a direction to discharge the capacitor in the later case, where as in the former case the current was in the direction to charge the capacitor.

18.6 TRANSIENT BEHAVIOUR OF SERIES LCR CIRCUIT

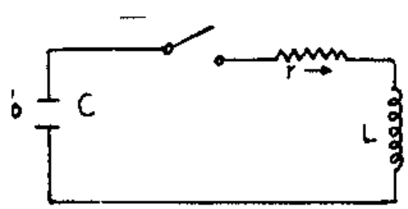


Fig 18.17 LCR Circuit

Initially, we charge the capacitor with a charge 'q' producing a voltage across the plates

$$V_0 = \frac{q_0}{C} \qquad ...(18.49)$$

At t = 0, the switch is closed and the charge q begins to leak off around the circuit through the inductance L, and the resistor R. Positive current will be defined clockwise as shown. Since a positive current **leads** to a decreasing positive charge on the capacitor, we have

$$i(t) = -\frac{dq}{dt} \qquad \ldots(18.50)$$

The current has been written as i (t) to emphasize that i varies with time. The potential across the capacitor also changes with time and can be written.

$$V_c(t) = \frac{q(t)}{C} \qquad \ldots(18.51)$$

The time rate of change of V_c is related to the current by differentiating eqn. (18.51) and substituting the result in eqn. (18.50)

$$\frac{dV_c}{dt} = \frac{1}{C}\frac{dq}{dt} = -\frac{1}{C}i(t) \qquad \ldots(18.52)$$

According to the circuit equation

$$V_c(t) - L\frac{di}{dt} = i(t)R \qquad \ldots(18.53)$$

The sign for L di/dt has been chosen correctly, because, when the switch was closed, the current was zero and increasing. Thus di/dt is initially positive. From the negative sign of Faraday induction it is clear that the voltage across the inductance will be in the direction to tend to decrease di/dt thus forming equation 18.53

Substituting equation 18.52 and its derivative in equation 18.53 we get

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0 \qquad \ldots(18.54)$$

This is a second order differential equation with constant coefficients. We are in search of an expression for V_c, the voltage across the capacitor, as a function of time. It is reasonable guess to expect damped oscillatory variation V_c, and we try such a solution. The work can be simplified if the oscillator term is expressed in complex notion. Thus we write for the trial solution

$$V_C(t) = V_0 e^{-\alpha t} e^{j\omega t} \qquad \ldots(18.55)$$

Where both the constants α and ω are to be real numbers. We have to find the value of these constants for which equation 18.55 satisfies the differential equation

The solution is certainly correct at t = 0 since the exponential terms become equal to unity; So $V_C(t) = V_0$ the correct starting condition.

And after a long time, the first factor $V_0e^{-\alpha t}$, goes to zero, giving $V_C \to 0$ as we expect this factor is the exponential damping term.

The second term in sinusoidal term expressed in complex –number notation.

We can rewrite equation 18.55 in a more concise form and take its derivatives as required for substitution in 18.54

$$V_c(t) = V_0 e^{(j\omega-\alpha)t}$$

$$\frac{dV_0}{dt} = (j\omega - \alpha)\ V_0 e^{(j\omega - \alpha)t}$$

$$\frac{d^2V_0}{dt^2} = (j\omega - \alpha)^2\ V_0 e^{(j\omega - \alpha)t}$$

Substitution in equation 18.54 and simplification gives

$$-\ \omega^2 - 2\,j\omega\alpha + \alpha^2 + \frac{R}{L}(j\omega - \alpha) + \frac{1}{LC} = 0$$

This equation has both real and imaginary terms. But a complex number can equal zero only if both real and imaginary parts are zero. We therefore separate the equation with real and imaginary parts. The imaginary part gives.

$$-2j\omega\alpha + \frac{L}{R}\,j\omega = 0$$

$$\text{or } \alpha = \frac{R}{2L}$$

the real part gives

$$-\omega^2 + \alpha^2 - \alpha\frac{R}{L} + \frac{1}{LC} = 0$$

Replacing α by R/L leads to

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

These values of α and ω make proposed solution satisfy the differential equation.

We now see the condition for critical damping of the LCR circuit. Since oscillatory motion requires ω to be real, oscillations occur only if

$$\frac{1}{LC} > \frac{R^2}{4L^2}$$

Otherwise ω in equation 18.55 becomes imaginary, the exponent in the term $e^{j\omega t}$ becomes real and negative and the term become a camping term.

Whenever I/LC exceeds $R^2/4L^2$ the voltage oscillates according to

$$V_c(t) = V_0 e^{-Rt/2L} e^{j\omega t}$$

Where $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

giving a voltage which varies with time, as shown in Fig 18.8

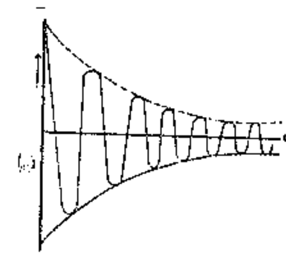


Fig 18.8

In extreme cases in which the resistance term is negligible compared with 1/LC, The solution reduces to

$$V_c(t) \quad V_0 e^{j\omega_0 t}$$

Where $\omega_0 = \frac{1}{\sqrt{LC}}$ and the circuit oscillates without damping as its resonance frequency ω_0, the effect of the resistance is to damp the oscillations and shift the frequency to the lower value.

If 1/LC just equals $R^2/4L^2$ is zero, and the circuit is critically damped, according to

$$V_c(t) \quad V_0 e^{-Rt/2L}$$

If the resistance term is so large that

$$\frac{R^2}{4L^2} > \frac{1}{LC}$$

ω becomes and imaginary quantity which we call ω^1

$$\omega^1 = j\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = j\sqrt{A}$$

Where $\sqrt{A}$ is a real positive number. The voltage equation, then becomes

$$V_o(t) = V_0 e^{-(\alpha - j\omega)t}$$

$$= V_0 e^{(\alpha - \sqrt{A})}$$

$$= V_0 \exp - \left[\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \right] t$$

It the terms 1/LC can be neglected in comparison with the resistive term, the solution becomes

$$V_0(t) = V_0 e^{-Rt/L}$$

SUMMARY

AC voltages are applied to the resistors, inductors and capacitors. Current passing through the resistive impedance inductive resistance and capacitive will be sinusoidal and in phase, lags by 90^0 and leads by 90^0 with respect to the applied voltage.

The transient behaviour of series LCR circuit is analogous to that of a damped mechanical oscillator

18.7 SAMPLE EXAMINATION QUESTIONS

I. Answer each of the following questions in detail.

1. Derive an expression for the energy stored in LR circuit.
2. Discuss the growth and decay of current in LR Circuits
3. Discuss about parallel LCR circuit and series response circuit
4. Derive an expression for the quality factor of series response LCR circuit.

II. Answer each of the following briefly

1. Give a graphical representation instantaneous voltage of a capacitor 'C'
2. Discuss the growth of current in CR Circuits
3. Write notes on i) Parallel LCR circuit ii) resonance circuit

UNIT – 19: SERIES AND PARALLEL RESONANCE CIRUCIT

Contents

19.1 OBJECTIVES

This Unit discusses the series and parallel resonance circuits by applying the rotating amplitude vector to the problems of simple LCR circuit.

After going through this Unit you should be able to evaluate the quality factor concerning Series Resonance LCR circuit

19.2 INTRODUCTION

In this Unit we will discuss the series and parallel resonance circuit. We will also evaluate the quality factor of series resonance LCR Circuit. We will also study the usage of these circuits in radios & others.

19.3 SERIES LCR CIRCUIT

So far we have discussed about the AC current – voltage relations in individual R, L and C elements. We have seen that with a sinusoidal voltage applied across these elements, there is a sinusoidal current each one which is either in phase (R), 90^0 behind the voltage (L), or 90^0 ahead (c) . We now show the application of the idea of rotating amplitude vectors to the problems of simple LCR series circuit shows in Fig 19.1 below:

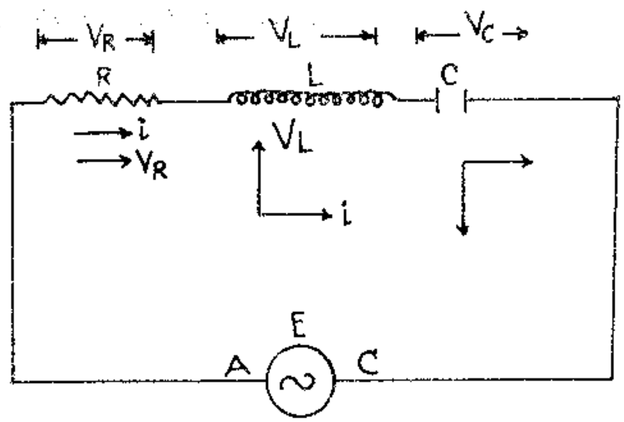


Fig 19.1 LCR Series Circuit

In a series circuit with no branches, the current – continuity equation $\Sigma i = 0$, leads to the result that everywhere in the circuit the current is the same. Thus for example, at a point A in the figure, the current coming from the left equals that going on to the right. This principle applies to all points in the circuit, such as B and C.

The AC current everywhere in the circuit thus has the same amplitude and phase. It can therefore be written in the form

$$i = I_0 \cos \omega t \qquad \ldots(19.1)$$

$$\text{Or } i = I_0 \quad e^{j\omega t} \qquad \ldots(19.2)$$

But if the currents are everywhere the same, and if the phase and amplitude relations we have just developed for individual elements are to be obeyed. It follows that the AC voltage vectors for each elements must have different amplitudes and phases. In addition, the instantaneous values of voltages across each element must always add to equal the voltage of the generator. This satisfies the second circuit equations, as modified by the inclusion of voltages across the capacitor.

$$\Sigma E = \Sigma iR + \sum q/C \qquad \ldots(19.3)$$

The effect of inductance is contained in the left hand term. That is $\sum E$ contains not only the generator emf but also the term – L (di/dt)

Since the voltage across each element is sinusoidal and has the same frequency, each voltage can be represented by rotating amplitude vector moving at the same frequency. Thus if the relative phase and amplitude of each voltage vector can be found and if the vector sum is made equal to and in phase with voltage source, the circuit equation will be satisfied at all times, Fig 19.2 reviews the current voltage relations for each circuit element at the same time, 't' the relative phase of V_R, V_L and V_C can be found by rotating.

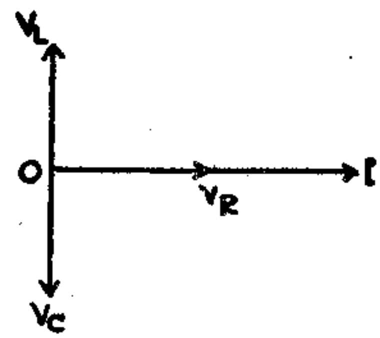


Fig 19.2

These figures illustrate the current generating vectors are all in phase. The resulting directions of the voltage vectors are shown in fig 19.3 (A) (B) along with current vector I.

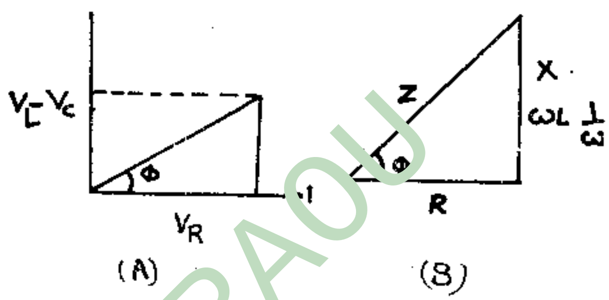


Fig 19.3

The vector sum of V_R, V_L and V_C is the voltage between points A and C in Fig 19.1.This sum can be obtained by redrawing the voltage vectors as in Fig 19.4.

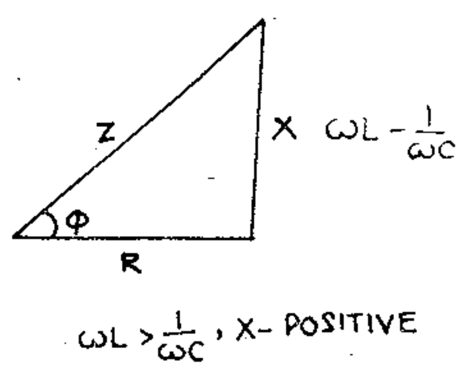


Fig 19.4

In neither case the vector sum is given by the dotted vector labeled V_{AC}. The angle θ is the resultant phase difference between the current in any of the elements and the voltage across the combination V_{AC}.

According to the circuit equation this voltage V_{AC} must be just equal in phase and amplitude to the driving emf E. Therefore we may write

$$E = V_{AC} = V_R + V_L + V_C \qquad ...(19.4)$$

The magnitudes of these separate voltages are given by

$$V_R = RI_R$$

$$V_L = \omega LI_L$$

$$V_C = [1/\omega C] I \qquad ...(19.5)$$

But since

$$I_R = I_L = I_C = I_0 \qquad ...(19.6)$$

Equation 19.4 can be written as

$$E = I_0 [R + j\omega L + j \ (1/\omega C) = I_0 Z \qquad ...(19.7)$$

The impedance terms have been written as vectors to allow for the different phases of the vectors. The meaning of this vector equation is shown in fig 19.5 (complex notation has been used). The diagram is identical with fig 19.3 (a) except that each voltage vector is expressed as the current times the appropriate impedance term. It is apparent from the figure that the magnitude Z in equation 19.6 is given by

$$Z = \sqrt{R^2 + (\omega L - 1/\omega c)^2} \qquad ...(19.7)$$

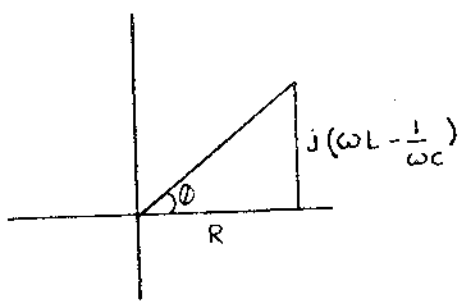


Fig 19.5

This is the AC impedance of three elements in series. The phase angle between the driving voltage and the current in the circuit is given by

$$\tan \phi = \frac{\omega L - 1/\omega C}{R}$$ ...(19.8)

As defined here. Ø is the angle by which the AC current lags the driving voltage E.

If the common factor Io is removed in the vector diagram of Fig 19.5 it becomes the impedance diagram (Fig 19.6.)

In the usual terms for inductive and capacitive reactance. the impedance may be written as

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{R^2 + (X_L - X_C)^2}$$ ... (19.9)

and $$\tan \phi = \frac{X_L - X_C}{R}$$

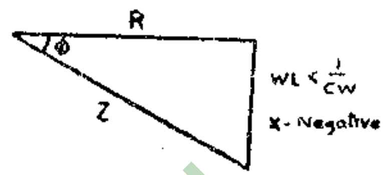


Fig 19.6 (a)

The instantaneous current is given by

$$i(t) = \frac{Vo \cos(\omega t - \phi)}{Z}$$ ...(19.10) (a)

Using complex notation. the impedance becomes

$$Z = R + (J\omega L - 1/\omega C)$$

Which has the same meaning as equation 19.9 (tan ϕ). This notation gives for the current in an LCR series circuit.

$$i(t) = \frac{Voe^{J\omega t}}{R + j(\omega L - 1/\omega C)}$$ ...(19.10) (b)

19.4 PARALLEL LCR CIRCUIT

The parallel LCR circuit is shown in Fig 19.7 provides a contrast to the series circuit

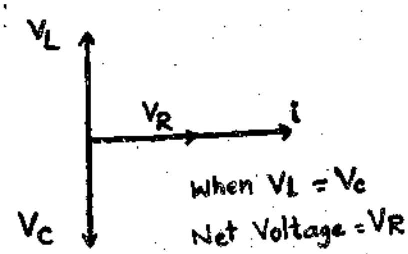


Fig 19.6(b)

In the case of this parallel circuit, the voltage rather than the current, which is the same on each element. As a result, the currents in three branches have different amplitudes and phase. If the amplitude applied voltage to the circuit is V_o the current amplitude in R,C and L will be

$$I_R = \frac{V_0}{R}$$

$$I_C = \frac{V_0}{Xc} = \frac{Vo}{1/jwc} \qquad \ldots(19.11)$$

$$I_L = \frac{V_0}{jwL}$$

The requirements of the three currents at any instant is that they add to give the total current passing through the source of emf. This requirement is satisfied if the generating vectors of the three AC currents add vector ally equal the total current vector.

As in the last section, the relative phases of vectors can be obtained by rotating the current voltage diagrams of Fig 19.4 until the circuit requirements are met. In this case the requirement is that the phases of the voltage vectors of the three elements are same. Fig.19.7 shows the resulting current phase diagram, which is more convenient to use in the form or Fig 19.8

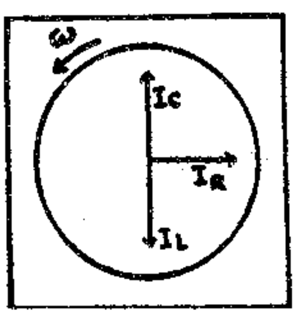


Fig 19.7

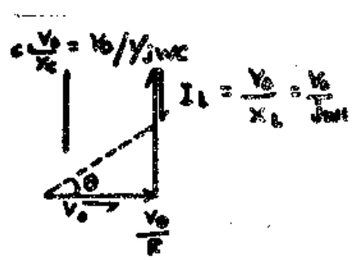


Fig 19.8

This diagram gives the basis of calculation of the total current in the circuit as follows. Using Io for the amplitude of the total current we write.

$$Io = I_R + I_C + I_L$$

$$= Vo\left(\frac{1}{R} + \frac{1}{X_C} + \frac{1}{X_L}\right)$$

$$= Vo\,\frac{1}{Z} \qquad \ldots(19.12)$$

The impedance terms are written as vectors to allow the phase information of Fig 19.4 to be induced, using this diagram: the reciprocal impedance can be evaluated as

$$\frac{1}{Z} = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2} \qquad \ldots (19.13)$$

Here in comparison with the series current the sings of X_C and X_L are reversed. The full expression for the current becomes.

$$Io = \left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2\right]^{1/2} Vo\,Cos(\omega t + \phi) \qquad \ldots (19.14)$$

Where

$$\tan\theta = \frac{1/X_C - 1/X_L}{1/R} = \frac{\omega C - 1/\omega L}{1/R} \qquad \ldots (19.15)$$

Here ϕ is the phase angle by which the current leads the voltage, as can be seen from fig 19.8.

The same problem can be handled in complex form for impedance terms.

$Z_R = R$

$Z_L = j\omega L = X_L$

$Z_C = 1/j\omega = X_C$ $\ldots (19.16)$

Substitution of these in equation 19.12 gives

$$Io = \frac{Vo}{Z} = Vo\left(\frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}\right) \qquad \ldots (19.17)$$

$$i(t) = \left(\frac{1}{R} + \frac{1}{j\omega c} + \frac{1}{1/j\omega c}\right) Voe^{j\omega t} \qquad \ldots (19.18)$$

The reciprocal of the impedance can be written as

$$\frac{1}{Z} = \frac{1}{R} + J\left(\omega C - \frac{1}{\omega L}\right) \qquad \ldots (19.19)$$

Evaluation of these complex numbers result in equation 19.13 directly

19.5 SERIES RESONANCE CIRCUIT

In an AC circuit consisting of an inductance, capacitance, and resistance, if the inductive resistance is equal to the capacitive reactance, the current at each instant is in phase with the applied voltage and the circuit is purely resistive effect.

Since $L\omega - \frac{1}{C\omega} = 0$, the magnitude of impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

The effective reactance or capacitive reactance depends upon

$X_L > X_C$ or $X_L < X_C$

The inductive reactance X_C is directly proportional to the frequency and increase as the frequency increases from zero on wards. The capacitive reactance is inversely proportional to the frequency, decrease from an infinite value downwards. At certain frequency both reactance's become equal and this frequency is called resonant frequency (f_r) At resonant frequency the two reactances are equal ie. $X_L = X_C$ or $X_L - X_C = 0$ then

$V_L = V_C$ (See Fig 19.9)

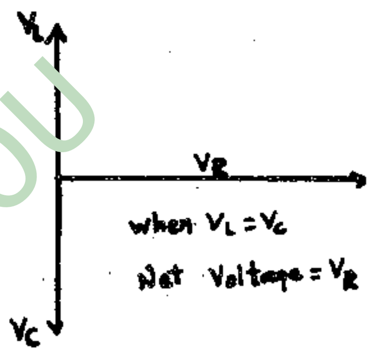


Fig 19.9

$$\omega L = \frac{1}{\omega C} \text{ or } \omega^2 = \frac{1}{LC}$$

$$\text{or } 2\pi f_r = \frac{1}{\sqrt{LC}} \quad (\text{as } \omega = 2\pi f) \qquad \ldots(19.20)$$

$$f_r = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \text{ Hz}$$

Where $X_L = X_C$ at resonant frequency. The impedance is minimum and equal to the resistance i.e $Z = R$.

Hence current in the circuit under these conditions is given by

$$i = \frac{V}{R}$$

$$\text{Since } \cos \varnothing = \frac{R}{Z} = 1$$

$$\varnothing = 0$$

This shows current and voltage are in phase. Such a circuit is also called ACCEPTOR CIRCUIT

19.6 QUALITY FACTOR OF SERIES RESONANT LCR CIRCUIT

The resistive part of the LCR circuit keeps on consuming energy and hence natural oscillations of such a circuit (not fed with an external a-c source) will be damped and will progressively die out. A parameter that measures damping in an oscillatory system is the 'Q' of the system. Let us first discuss the meaning of the Q value.

A curve where the amplitude does not change with time represents an un-damped system. The curves where the amplitude falls off with time represent damped systems. The less the damping, the large is the number of Q. For an oscillator with frequency ω, Q is dimension less ratio, defined as follows.

$$Q = \frac{\text{energy stored}}{\text{Average power dissipated}} \qquad \ldots(19.21)$$

Or Q, may be defined as 2 times number of cycles required for the energy in the oscillator to diminish by the factor 1/e.

In an LCR circuit the energy is stored alternatively in the capacitor and the inductor and the oscillator involve a transfer of this energy back and forth from one to the other. During the course of this transfer, the circuit resistance R keeps on dissipating energy and as the oscillations go on, the energy in the system gradually diminishes.

The ratio of V_L to V_R at the resonating frequency is called the voltage – magnification or Q-factor.

$$Q = \frac{V_l}{V_R} = \frac{j\omega L}{iR} \text{ (as } Z = R)$$

Where $\omega = \frac{1}{\sqrt{LC}}$ and i = the resonate current. Thus we see that at resonance $V_L = V_C = QV$

$$\text{And } fr = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$Q = 2\pi \frac{1}{2\pi} \frac{L}{R} \sqrt{\frac{1}{LC}}$$

$$= \frac{1}{R}\sqrt{\frac{1}{C}} \qquad \ldots(19.22)$$

Generally, speaking the higher the Q value of the circuit, higher is the peak of its response as a function of driving frequency ω

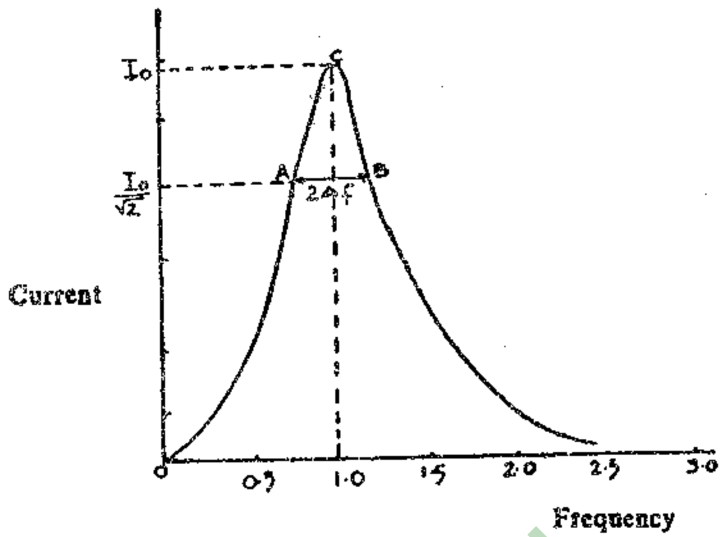


Fig 19.10

Figure 19.10 is a plot of the response (current) in case of an LCR circuit as a function of the frequency of the applied emf. It can be seen that the current and, hence, the power dissipation in the circuit is maximum at the point of resonance (C). Referred to this curve Δω is the full width of the resonance peak at A,B where the power dissipation drops to half its peak value (or, where the current drops to $1/\sqrt{2}$ of its peak value.

19.7 PARALLEL RESONANCE CIRCUIT

A parallel resonance circuit is shown in Fig 19.11 It is assumed that the resistance of the inductance coil is negligible. The current in the inductance L will lag in phase by 90^0 to the applied voltage and is given by

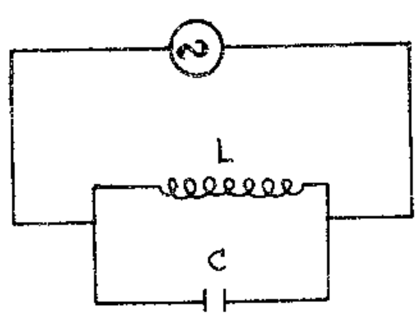


Fig 19.11

$$i_L = \frac{V_{max} \sin(\omega t - 90^0)}{\omega L}$$ while the current in the capacitor C will lead the phase by 90^0

the applied voltage is given by

$$V_{max} \frac{\sin(\omega t + 90)}{\omega C}$$

These currents being out of phase can be considered equivalent to an a.c. of the supply current.

$$i = i_L + i_C$$

$$= \frac{V_{max}}{\omega L} \sin(\omega t - 90^0) + V_{max}\, \omega\, C \sin(\omega t + 90^0)$$

$$= V_{max} \left(\omega C - \frac{1}{\omega L}\right) \cos \omega t$$

When $\omega C = \frac{1}{\omega L}$ the current is zero

i.e when $X_C = X_L$ hence

$$\omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi fr$$

$$fr = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \text{ Hz}$$

For this frequency i.e at resonance frequency this circuit does not allow the current to flow and works as a perfect choke for A.C. Such circuits are called rejecters.

Check Your Progress:

1. Define Quality factor.
2. What are acceptor and rejector circuits.

Note: a) Space is given below for your answers.
b) Compare your answers with those given at the end of the chapter.

....
....
....
....

Example – 1:

In an LCR circuit if L = .02 Henries, C = 0.5 microfarads and R = 10 ohms and an alternating voltage of 200 volts is applied, find the frequency of the applied voltage to produce resonance. Find also the P.D. across the inductor and the capacitor.

Let fr be the frequency of the applied voltage at resonance.

$$fr \quad \frac{1}{2\pi\sqrt{LC}} \quad \frac{1}{2\pi \times 0.2 \times 0.5 \times 10^{-6}}$$

$$= 1.59 \times 10^{3} \text{ c.p.s or Hz}$$

The current flowing through the circuit at resonance

$$ir = \frac{200}{10} = 20 \text{ Amp}$$

P.D. across the inductor

$$= L\omega \times i_r$$

$$= 0.02 \times 2\pi f_r \times 20$$

$$= 0.02 \times 2\pi \times 1.59 \times 10^{3} \times 20 = 4000 \text{ volts.}$$

P.D across the condenser plates

$$\frac{1}{C\omega} \times ir$$

$$= \frac{20 \times \sqrt{0.02 \times 0.5 \times 10^{-6}}}{0.5 \times 10^{-6}} = 4000 \text{ volts}$$

19.8 SUMMARY

A parameter that measured damping in an oscillatory system is called the quality factor Q of the system. Series resonance circuit is an acceptor circuit where as parallel resonance circuit is a rejecter circuit.

Check your progress-Answers.

1. Quality factor 'Q' may be defined as 2 times number of cycles required for the energy in the oscillator to diminish by the factor i.e.

$$Q = \frac{\text{energy stored}}{\text{Average power dissipated}}$$

A curve where the amplitude does not change with time represents an undamped system. The curves. where the amplitude falls off with time represent damped systems. The less damping the a large is the value of Q.

2. LCR series resonance circuits are called 'Acceptor circuits' where current and voltage are in phase. Parallel circuits are called 'Rejector circuits'. At resonance frequency parallel resonance circuit does not allow the current to flow and work as a perfect choke for A. C.

19.9 SAMPLE EXAMINATION QUESTIONS

I Answer each of the following questions in detail

1. Applying the rotating amplitude vector, find current in

   (a) Series resonance circuit

   (b) Parallel resonance circuit

2. Find the quality factor for series resonance LCR circuit.

BLOCK – 6: LAWS OF THERMODYNAMICS

UNIT 20: ZEROTH AND FIRST LAW OF THERMO DYNAMICS

Contents

20.1 OBJECTIVES

This Unit discusses the basic concepts of thermodynamics leading to the formulation of zeroth and first laws of thermodynamics. To make you understand the concepts the unit explains

1) the conditions under which a thermodynamic system will be in equilibrium;

2) the terms adiabatic and diathermic walls; and

3) concept of temperature.

After going through this Unit you should be able to

1) measure the temperatures using various methods; and

2) evaluate the amount of work done by a gas while expanding

20.2 INTRODUCTION

In recent times we have studied, in case of a flow of energy into a system, when system is working, that means heat energy converting into mechanical energy by means of this a thermal phenomena is taking place . In daily life we come across so many thermal phenomena. In modern tome due to usage of coal heat energy is seen. Not only steam engine but also petrol & diesel engine's produce energy. All these led to the devolopment of thermodynamics. About all these we will learn in this chapter.

Conversion of heat energy into mechanical energy is dome following some rules. To know the working principle of heat engines we have to study the laws pertaining to thermodynamics By studying these laws we know about the phenomena taking place in the Universe in which we are living. That is why by studying about the analyzing the and applying them becomes necessary.

20.3 THERMAL EQUILIBRIUM OF A SYSTEM

If the thermodynamic variables which specify a system are constant in time then the system is said to be in a state of thermal equilibrium. For a thermodynamic system of constant mass like an amount of gas, there are two thermodynamic variables, say, pressure and volume, which specify the thermodynamic state of the system. If the pressure and volume of a gas remain constant in time then the gas is said to be in a state of thermal equilibrium

Whether a system will be in thermal equilibrium or not will depend upon the proximity of other systems and the nature of the wall between this system and the other systems.

20.4 ADIABATIC AND DIATHERMICAL WALLS

Let us suppose that we have system A (which is a certain amount of some gas) with thermodynamic variables P, V. Let us suppose that we have another system B (which is again a different amount of some other gas) with thermodynamic variables P^1 V^1. Now let us suppose that they are brought into contact with a wall in between them (Fig20.1) Now let us suppose we go on changing of P, V of system A. If this produces changes in P^1, V^1 of system B, we say that the wall between them is diathermic. or in other words the values of P^1,V^1 of system B are determined by the P,V values of system A and Vice-versa if the wall is diathermic. If on the other hand changes in the values of P, V of system A do not produce any changes in the values of P^1, V^1 of system B, then the wall of contact is said to be an adiabatic. Thus with an adiabatic wall in between them the values of P V of system A and the values of P^1, V^1 of system B are independent of each other. Thin metal sheets are examples of diathermic wall. Thick layers of wood, concrete asbestos etc. are examples of adiabatic wall

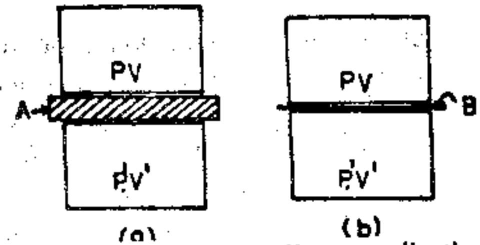


Fig 20.1

If two systems are brought into contact through a diathermic wall, normally the thermodynamic variables of the two systems are not compatiable and they will go on changing till they become compatiable. After compatiable values are reached there will be no further change and the values of the thermodynamic variables will remain constant in time. The two systems will be in thermal equilibrium with each other.

20.5 THERMAL EQUILIBRIUM BETWEEN TWO SYSTEMS

Two systems are said to be in thermal equilibrium with each other when they are in contact with diathermic wall in between them and their thermodynamic variables are constant in time

20.6 ZEROTH LAW OF THERMODYNAMICS

Let us suppose that we have two systems A and B with an adiabatic wall in between them. Let A and B are in contact with system C through a diathermic wall. The whole assembly is surrounded by an adiabatic wall Fig (20.2). After sometime systems A and B will come to thermal equilibrium with C. Now if we remove the adiabatic wall between A and B and put a diathermic one, there will be no further change. In other words A and B will be in thermal equilibrium with each other even if we first allow A and C to come to thermal equilibrium with each other and then allow B and C to come to thermal equilibrium with each other (Provided the state of C is unaltered through out)

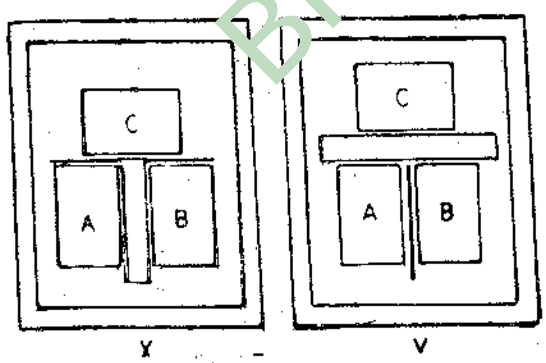


Fig 20.2

These results are stated in the form of a law which says 'Two systems in thermal equilibrium with a third one are in thermal equilibrium with each other". R.H. Fowler called this as the Zeroth law of thermodynamics.

20.7 CONCEPT OF TEMPARATURE

When two systems come into thermal equilibrium with each other, some property of both the systems attains the same value. Thus when A is in thermal equilibrium with

C and C is in turn in thermal equilibrium with B, the said property of three system will have the same value so that when A and B are brought into contact they will be in thermal equilibrium

That property of the thermodynamic system which determines whether two systems are in thermal equilibrium or not, is called the temperature. Therefore equality of temperature will be a necessary and sufficient condition for two systems to be in thermal equilibrium with each other.

This identification of temperature is in agreement with our experience in daily life. We know that two systems which have the same degree of hotness when touched, will be in thermal equilibrium with each other when they are brought into diathermic contact with each other. If the two systems have different degrees of hotness to our sense of touch they will not be in thermal equilibrium with each other when they are brought into diathermic contact. The degree of hotness to sense of touch is the origin and basis to the scientific concept of temperature.

20.8 MEASUREMENT OF TEMPERATURE

Systems possess many physical Properties which vary as our physiological feeling of temperature varies. Therefore the variation of any one of these properties can be used to measure temperature of a system. Such a property of a system, which varies with out physiological feeling of temperature is called a thermometric property and the system is called a thermometer.

Let X be the thermometric property with which we wish to set up a temperature scale. Let us suppose temperature T is a linear function of X

$$T(X) = aX \qquad ...(20.1)$$

Where a is a constant. The linear relationship between T and X implies equal changes in X for equal temperature differences. It also means that the ratio of two temperatures of the system is equal to the ratio of the values of X at those temperatures.

$$\frac{T(X_1)}{T(X_2)} = \frac{X_1}{X_2} \qquad ...(20.2)$$

In order to determine the constant 'a' we specify a standard fixed point at which all the thermometers must give the same temperature T. This point is chosen as the triple point of water. (The point where water vapour, water and ice coexist). The temperature of this point is arbitrarily assigned as 273.16 degrees Kelvin (written as 273.16^0K). Then for any thermometer.

$$\frac{T(X)}{T(X_{tr})} = \frac{X}{X_{tr}} \qquad ...(20.3)$$

Where subscript tr indicates that, the values are at the triple point. Now since for all thermometers.

$$T(X_{tr}) = 273.16^0 K \qquad ...(20.4)$$

We get

$$T(X) = 273.16^0 K \frac{X}{X_{tr}} \qquad ...(20.5)$$

20.9 DIFFERENT TYPES OF THERMOMETERS

If we use as a thermometer a system with two thermodynamic variables, for example, an amount of gas, it will be necessary to keep one of them constant and only allow the other to vary. Therefore x in case of a gas we can construct two types of thermometers. The first is the constant volume thermometer in which only pressure varies and we can measure temperature by the equation

$$T(P) = 273.16^0 K \frac{p}{p_{tr}} \qquad ...(20.6)$$

The second is the constant pressure thermometer in which only the volume is allowed to vary and we get temperature from the equation

$$T(V) = 273.16^0 K \frac{V}{V_{tr}} \qquad ...(20.7)$$

In addition to these there are systems in which the variation of a single physical property can be used to measure the temperature. Thus the volume expansion of a liquid is used to measure the temperature in the case of liquid in glass thermometers like the mercury thermometer. The change in electrical resistance of a wire is used to measure temperature in the platinum resistance thermometer. The increase in the length of rod can be used to measure temperature. The thermo e.m.f generated by a thermocouple is used to measure temperature in thermocouples. The variation in spectral composition (colour) of the light emitted by a body is used to measure temperature in pyrometers.

20.10 NATURE OF HEAT

We assume that when two systems at different temperatures are brought together, something flows from the system at higher temperature to the system at a lower temperature. This flow will continue till thermal equilibrium is established. We call the thing that flows from one system to the other because of temperature difference as heat.

In the early days people thought that heat was some sort of fluid. They called this fluid as calorie. A body which is at a higher temperature has this fluid at a higher pressure, and a body which is at lower temperature has this fluid at a lower pressure. Thus when two bodies which are at different temperatures are brought into diathermic contact, the calorie fluid flows from the body in which it is at higher pressure to the body in which it is at lower pressure till the pressures equalise.

In order to explain the production of heat by friction they said when bodies are rubbed together the caloric fluid contained in them is squeezed out and we feel therefore heat to be generated.

But this theory did not hold water for a long time. Count Rumford who was the founder of the Royal Society of London, showed that inexhaustible amount of heat can be produced by continuously turning a blunt drill bit in the bore hole of a canon. This is not in agreement with the caloric theory. According to caloric theory any body can have only a finite amount of caloric fluid. Once this amount is squeezed out by friction there should be no further production of heat. Thus the generation of inexhaustible amount of heat by friction does not support caloric theory.

We know that we have to supply heat in order to convert ice into water. Thus water should contain more calorie than ice. Sir Humphrey Davy showed that two blocks of ice when rubbed together are melted to produce water. If caloric theory were correct by rubbing we will be squeezing out calorie out of ice and the product which is produced in this process must contain less calorie than ice. Thus water must contain less calorie than ice. This is against the fact that we mentioned earlier.

Count Rumford has shown further that, there is a relationship between the amount of work done by the drill bit and the quantity of hat generated. From this he came to the conclusion that heat is a form of motion and whenever motion disappears heat is generated. This leads us to the conclusion that heat is a form of energy because whenever motion disappears it is the kinetic energy of the body that disappears. Since heat appears in its place this is a case of conversion of mechanical energy to heat. Hence heat is a form of energy.

There must be a rate of conversion between mechanical and heat energy. If W is the mechanical energy in joules and H is the heat energy in calories, then there must be an equation.

$$H = W / J \qquad \ldots(20.8)$$

Where J is the conversion factor called the mechanical equivalent of heat. Its value was first determined by joule. The present value is

J = 4.186 joules / calorie.

We have already said that when we add heat to a system we are actually adding energy to it. This must be stored in the form of internal energy of the system. Let us now suppose that we have an amount of gas in a cylinder with a movable piston. Let us suppose that we surround this system by adiabatic walls on all sides so that no heat can enter or leave the system. Let us now suppose that some work is done on the system by moving the piston down. Let us suppose as a result of this the system goes from some initial state i to some final state. It has been found experimentally that as long as the initial-final states of the gas remain the same, you have to do the same amount of work irrespective of the path by which you go from the initial state to the final state. Since the amount of work we have done must be stored, as the internal energy of the system, the amount of work we have done must be equal to the increase in the internal energy in the system. Since we have found that this is independent of the path by which we reach the final state from the initial state, we conclude that the internal energy of the system is a function of the state of the system, so that the difference between the internal energies in the initial and final states remain constant as long as the initial and final states are the same. If $dw\ i \rightarrow$ represents the work done by the system as the system goes from the initial state to the final state in a process no heat supplied to or taken out from the system and U_f and U_i are the internal energies of the system in the final and initial states respectively, we have

$$dU = U_f - U_i = -dW_{i\rightarrow f} \qquad \text{...(20.9)}$$

where dU is increase in the internal energy of the system.

First Law of thermodynamics

Now let us suppose that we do not surround the system by adiabatic walls but permit heat to enter or leave the system. Let us suppose an amount of heat dQ enters the system. Then dQ must be equal to the increase in the internal energy of the system plus the external work done by the system.

$$dQ = U_f - U_i + dW_{i\rightarrow f} \qquad \text{...(20.10)}$$

or

$$dQ = dU + dW \qquad \text{...(20.11)}$$

This is called the First law of thermodynamics

20.11 WORK DONE BY A GAS AS IT EXPANDS

Let us suppose that we have a cylinder containing a gas with a piston which can move freely (fig20.3). The work done by the gas when the piston is pushed up by a distance dx is equal to

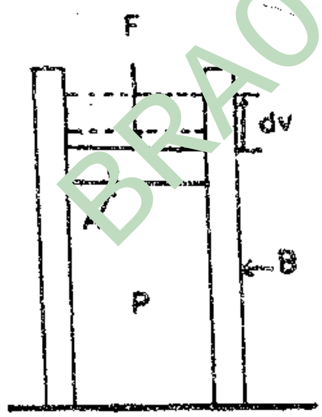


Fig 20.3

$$dW = F.dx = P\,Adx = PdV \qquad \text{...(20.12)}$$

where A is the area of cross section of the cylinder and dv is the increase in volume. In general, as the volume of a gas increases its pressure does not remain constant. Therefore in order to calculate the total work done by a gas as it expands we have to integrate Pdv.

$$W = \int_{V_i}^{V_f} Pdv \qquad \text{...(20.13)}$$

This integral is graphically the area under the curve along which the system moves in the P-V diagram between the limits V_i and V_f (Fig 20.4) limits V_i and V_f (Fig. 31.4).

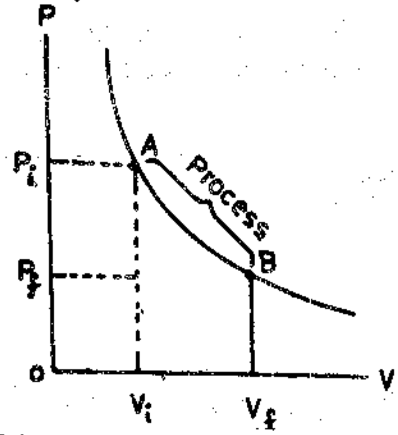


Fig 20.4

If the pressure acting on the piston remains constant as the gas expands, the work done by the gas is easy to find and is equal to

$$W = \int_{V_i}^{V_f} P\,dv = P\,(V_f - V_i) \qquad \ldots(20.14)$$

If the gas in the cylinder is expanding at constant temperature (we call such a process isothermal process) using ideal gas equation we can substitute $p = \frac{\mu RT}{V}$ where μ is the number of the moles of gas present, and we get

$$W = \int_{V_i}^{V_f} P\,dV = \mu RT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$= \mu RT \ln \frac{V_f}{V_i} \qquad \ldots(20.15)$$

20.12 SUMMARY

A system will be in thermal equilibrium if the thermodynamic variables of the system are constant in time. When two systems are in contact through a wall and if altering the thermodynamic variables of one-system results in the change of thermodynamic variables of the second system the wall is said to be diathermic. If there is no such change in the thermodynamic variables of the second system when the thermodynamic variables of the first system are changed, the wall is said to be adiabatic. The Zeroth law of Thermodynamics states that "two systems in thermal equilibrium with a third one are in thermal equilibrium with each other". Temperature is the property of a thermodynamic system whose equality determines the thermal equilibrium. Temperature can be determined with the help of other different properties of the system, like volume, length, electrical resistance and thermo emf.

The first law of thermodynamics states

$$dQ = dU + dW$$

Where dQ is the heat energy supplied to the system, dU is the increase in the internal energy of the system and dW is the work done by the system. work done by the gas as it expands at pressure is $W = PC\ V_f - V_i$

20.13 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail

1. State Zeroth law of thermodynamics. Explain how the temperature of a body is related to its physical properties.
2. Write a brief note on the measurement of temperature of a body.
3. State first laws of thermodynamic and discuss some of its applications.

UNIT- 21: REVERSIBLE AND IRREVERSIBLE PROCESSES

Contents

21.1 OBJECTIVES

This Unit explains the reversible and irreversible processes.

After going through this unit you should be able to make out what reversible and irreversible processes are.

21.2 INTRODUCTION

A change in the physical or chemical state of a system can be brought about by a variety of processes. If the system does not interact with the surroundings, it is called a closed system. A process is said to be cyclic if the system returns to its original state after undergoing through a series of operations. Any actual process however complicated it is, may be considered to be equivalent to a sequence of simple processes. Simple process may be classified as (1) isothermal (change in temperature is zero) (2) isobaric (change in pressure is zero) (3) isochoric (change in volume is zero) and (4) adiabatic (no heat transfer between the system and its surroundings). These simple processes may be reversible or irreversible. In this Unit we study in detail what is meant by reversible and irreversible processes.

21.3 REVERSIBLE AND IRREVERSIBLE PROCESS

The characteristics of reversible and irreversible processes can be understood well by considering a typical thermodynamic system. Let us consider a real gas of mass M-confined in a cylinder-piston arrangement of volume V. Let the pressure and temperature of the gas be P and T respectively. When this system is in an equilibrium state, the thermodynamic. variables namely P,T, V n remain constant with time. Let the cylinder m , whose walls are ideal heat insulators with the base being a good conductor be placed or a large heat reservoir maintained at temperature T. as illustrated in Fig 21.1. Let us now try to change the system to another equilibrium state in which the temperature T is the same as that of the initial state but the volume is reduced to half of its original volume. Out of many ways of achieving this two ways are quite important.

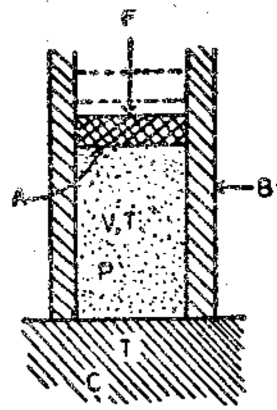


Fig.21.1

Process I: The piston is moved very rapidly. After some time the system comes into equilibrium with the reservoir. During the process the gas is turbulent and hence we can not well define the pressure and temperature. The process can not be represented as a continuous line on a P – V diagram since we do not know what pressure the system would have had at a given volume. The system passes from one equilibrium state i to another equilibrium state f as illustrated in Fig 21.2 through a series of new equilibrium states.

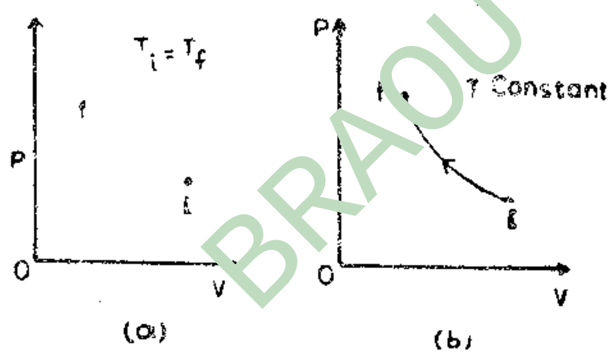


Fig 21.2

Process II. The piston is moved very slowly say by adding sand to the top of the piston. In this process the pressure, volume and temperature at all times could be well defined. Let a few grains of sand be added on the piston which is considered to be friction less. As a result the volume of the system reduces by a small amount leading to a small raise in temperature. The system departs from equilibrium state slightly. There will be transfer of heat from the system to the reservoir but with in a short period the system will reach a new equilibrium state, its temperature being of the reservoir. Let a few grains of sand be again added on to the piston. The volume of the gas in the system reduces. After some time the system comes to a new equilibrium state. By repeating the process in succession we can reduce the volume to half its value at the initial state. During this entire process the system is always in equilibrium with the reservoir. If the entire process is carried out with elemental changes in pressure, the intermediate state will depart from equilibrium even less. By indefinitely increasing the number of elemental changes, the size of each elemental change can be reduced correspondingly. This will lead to an ideal process in which the system passes through a continuous succession of equilibrium states.

These changes can be represented as a continuous line on a P-V diagram, shown in Fig 21.2b. In this process a certain amount of heat Q will be transferred from the system to the reservoir.

Process I is called irreversible process. This is because it is difficult to trace back the process. To illustrate this aspect more clearly consider a gas confined at pressure P_0 with in a volume V_0 of a thermally insulated enclosure as shown in Fig 21.3. The gas is confined to a volume V_0 by means of the diaphragm D. The rest of the volume in the system (V_1-V_0) is evacuated. When the diaphragm is broken the gas undergoes free expansion. When equilibrium is established its volume will be V_1. This is a spontaneous process. It is irreversible since the gas would never return to its original volume V_0 by itself. All processes, which do occur in nature, are called spontaneous, irreversible or natural processes. Spontaneous process always proceed towards equilibrium, terminating when equilibrium is established. Referring to Fig 21.1, suppose the piston is suddenly brought to its original position. The process rapidly runs its course terminating with the establishment of equilibrium. Even though the initial and final states are equilibrium states and hence the process even if it be cyclic is called an irreversible process. The following spontaneous processes are irreversible processes.

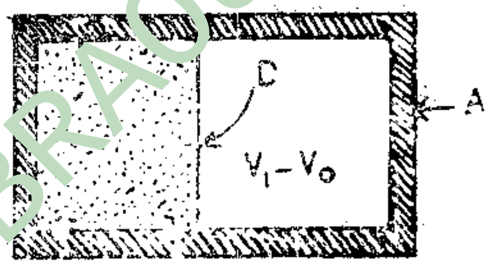


Fig 21.3

1. Degradation of mechanical energy into heat by friction
2. Flow of heat from a warm body to a cooler body.
3. Inter diffusion of two gases
4. Freezing of super cooled liquid
5. Reaction of a mixture of H_2 and F_2
6. Atomic explosion

It is impossible to think of the above processes to occur spontaneously in reverse. Betrand Russel's statement "you cannot unscramble eggs" is the best example of an irreversible process

Process II illustrated in Fig 21.2b. is called a reversible process. A thermodynamically reversible process is defined as a process whose direction can be reversed by an infinitesimal change in one of the properties of the system. No actual process is reversible, but reversibility is a limit which actual processes can be made to approach more or less closely by a choice of experimental conditions. Thus in the process II let the piston be moved slowly downward by removing a few grains of sand on the piston. The external pressure will then be less than the internal pressure by dP. The gas expands and the system goes to an equilibrium state, which it had earlier while contracting. By successive elemental decrement of external pressure on the system by dP, we can trace back the equilibrium states through which the system has passed through earlier.

Process II is not only reversible but also isothermal because the temperature of the gas differs at all times by only a differential amount dT from the temperature of the reservoir on which the cylinder rests. In other words we can say that the process takes place at constant temperature.

The volume of the gas in the system can also be reduced adiabatically by keeping the cylinders on a non-conducting platform say sand. In an adiabatic process no heat is allowed to enter or leave the system. An adiabatic process can be either reversible or irreversible. In a reversible adiabatic process the piston is moved exceedingly slowly by employing the sand-loading technique. In the irreversible adiabatic process the piston is pushed down quickly.

In an adiabatic compression the temperature of the gas increases, because as per the first law of thermodynamics, the work W done in pushing down the piston must appear as an increase dU in the internal energy of the system. W will have different values for different rates of pushing down the piston. Hence dU and the corresponding change dT will not be the same for reversible and irreversible adiabatic processes.

A process whether reversible or irreversible depends upon the state of the system and the nature of the process. Only reversible processes can be mathematically described in thermodynamics or represented by means of graphs like Fig 21.2b. It is a matter of concern to know why all naturally occurring processes in thermodynamics are irreversible, particularly when in dynamical processes reversibility can be at least progressively attained as a final limit by eliminating friction in elasticity etc. As per Boltzmann, irreversibility is confined to the behavoiur of the complex structure like a gas treated as a whole and is not to be expected in the behaviour of the individual molecules and arises from our inability to deal with individual molecules. It is also worth while to note here that in the irreversible process, reversibility is not impossible, but is almost infinitely improbable.

21.4 SUMMARY

A change in the physical or chemical state of a system can be brought about by a variety of processes. If the system does not interact with the surroundings it is called a closed system. A process is cyclic if the system returns to its original state after undergoing through a series of operations. The direction of thermodynamically reversible process can be reversed by an infinite change in one of the properties of the system.

21.5 SAMPLE EXAMINATION QUESTIONS

I. **Answer the following questions in detail.**

1. Explain with suitable examples the reversible and irreversible processes

UNIT-22: CARNOT'S ENGINE

Contents

22 OBJECTIVES

This Unit discusses how the efficiency of a heat engine can be estimated.

In order to make you understand it explains what is Carnot Cycle, and

1) how it is useful in converting heat into energy; and

2) the factors that control the efficiency of heat engines.

After going through this Unit you should be able to understand

1) on what principle a refrigerator works; and

2) the efficiency of heat engine depends on the temperatures of hot and cold bodies and not on the working substances.

2.2 INTRODUCTION

The laws of thermodynamics have a negative quality, which distinguishes them from other laws of physics. The first law of thermodynamics may be stated as that energy cannot be destroyed. The second law of thermodynamics also has this negative aspect where in we can state the law that the spontaneous tendency of a system to go towards thermodynamic equilibrium *can not be reversed* at the same time without changing some organised energy say work, into disordered energy say heat. The discovery of heat engines and their application to various industrial processes enabled the formulation of second law of thermodynamics. Heat engines function on the principle of conversion of heat energy into mechanical energy. Modern steam engines used in locomotive, gas turbines employed in large electric power stations and in big ships and internal combustion engines used in motor cars and aeroplanes all have the common feature of conversion of heat energy into mechanical energy.

A heat engine generally consists of a hot body called source which can supply heat at a higher temperature, a cold body called sink into which heat is rejected at a lower temperature and working substance which can absorb heat from the source convert some of it into external work by its expansion and reject the rest to the cold body. It is not possible to extract work from heat without a source and a sink at different temperatures. The process occurring in the heat engine is cyclic. These cyclic operations where heat energy is converted into mechanical energy are called Carnot Cycles, after the name of the scientist Sadi Carnot whose theoretical researches on the ideal Reversible heat engine led to the recognition of the second law of thermodynamics as a law of nature. In this Unit we shall study about the various processes involved in the Carnot Cycle, factors that decide the maximum efficiency of heat engines and the Carnot's theorem.

22.3 CARNOT'S CYCLE

A gas confined to a cylinder –piston arrangement undergoes expansion and compression and if the process involved is reversible and cyclic then certain amount of work will be done by or on the system. As shown in fig 22.1

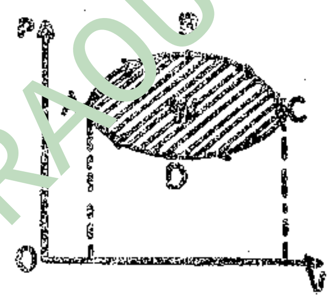


Fig 22.1

If the cyclic process takes the path ABCDA the work W done by the system is positive. If the cyclic process takes the path ADCB A the work W done by the system would be the negative of the work done.

In order to have a system do a net amount of work a more complicated cycle must be used. The simplest of these cycles, which is the most ideal one, is the Carnot Cycle. Here the system consists of a working substance such as a gas. The cycle consists of two isothermal and two adiabatic reversible processes. The working substance, which is an ideal gas, is contained in a cylinder with a heat conducting base and non-conducting walls and piston. The environment also consists of two heat reservoirs of large heat capacity kept at temperatures T_1 and T_2 and two non-conducting stands. The Carnot Cycle is effected in four steps as detailed below and schematically shown in Fig 22.2

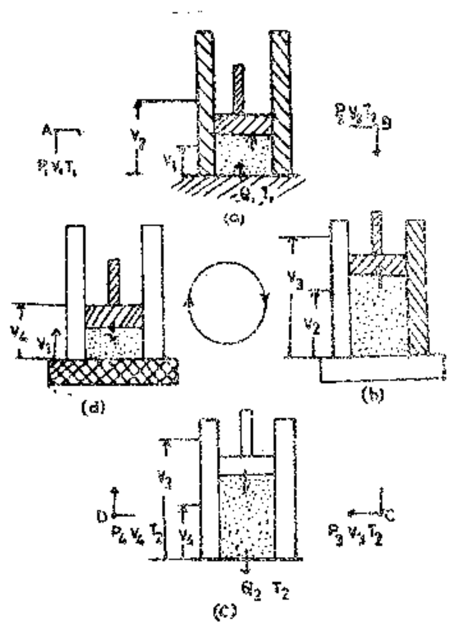


Fig 22.2

Step 1: To start with let the gas inside the cylinder be in an equilibrium state whose pressure, volume and temperature represented are P_1, V_1 and T_1 respectively. This state is represented by the point A in the P-V diagram illustrated in Fig 22.3. Let the system be place on the heat reservoir at temperature T_1 as shown in Fig 22.2a and the gas be allowed to expand slowly. The process is reversible isothermal expansion. Let the expansion take place for some time when the pressure volume and temperature attain the values P_2, V_2 and T_1. This state is represented by the point B in Fig 22.3. During this process heat energy Q_1 is absorbed by the gas by conduction through the base. Since the expansion is isothermal at T_1 the gas does work in raising the piston and its load.

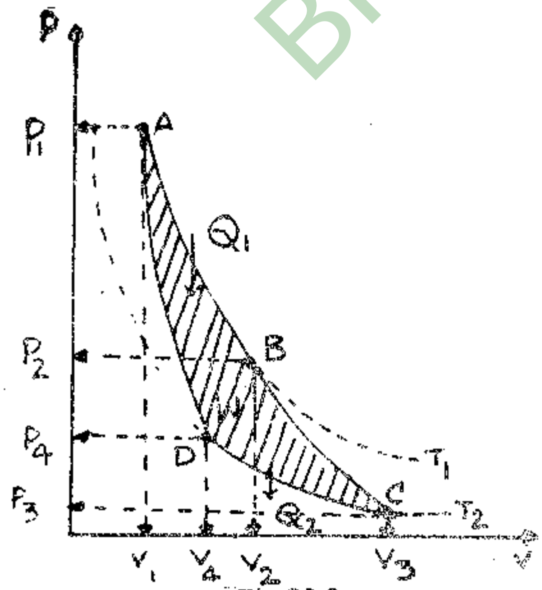


Fig 22.3.

Step 2: The system is put on a non-conducting stand and the gas is allowed to expand slowly to $P_3V_3T_2$. This state is represented by the point C in Fig 22.3. The expansion is reversible adiabatic process. The gas does work in raising the piston and hence its temperature falls from T_1 to T_2

Step 3: Now we put the system on a heat reservoir at temperature T_2 and the gas is compressed slowly so that the pressure, volume, and temperature of the gas are P_4V_4 and T_2. This state is represented by the point D in Fig 22.3. During the process heat energy Q_2 is transferred from the gas to the reservoir by conduction through the base. The compression is isothermal at T_2. In this process work is done on the gas by the piston and its load.

Step 4: The system is placed on a non-conducting stand and the gas is compressed slowly to the initial condition P_1, V_1 and T_1. The compression is adiabatic since no heat can enter or leave the system. Work is done on the gas and its temperature rises to T_1

The net work W done by the system during the cycle is given by the shaded area in Fig 22.3 enclosed by the path ABCD. Since Q_1 is the heat energy absorbed by the system isothermal expansion (step 1) and Q_2 is the heat energy given out by the system in the isothermal compression (step 3), the net amount of heat energy gained by the system in the cycle is given by Q_1-Q_2. Since that initial and final states of the system are identical there is no change in the internal energy of the system. Hence the net heat energy gained by the system is converted into work. Hence as per first law of thermodynamics

$$W = Q_1 - Q_2 \qquad \ldots(22.1)$$

Where Q_1 and Q_2 are taken as positive quantities. The net effect of the Carnot cycle on the system is that heat has been converted into work by the system. Any amount of required work can be done by the system by repeating the Carnot Cycle. Thus the system works like a heat engine.

In the ideal Carnot engine the working substance is an ideal gas. The working substance can be anything and in such cases the P-V diagrams will be different. Common heat engines use steam or a mixture of fuels and or fuel and oxygen as their working substance. Heat may be obtained from the combustion of a fuel such as gasoline or coal or from annihilation of mass in nuclear fission process in nuclear reactors. Heat may be discharged at the exhaust or to a condenser. Even though real heat engines do not operate on a reversible cycle the Carnot Cycle which is reversible gives useful information about the behaviour of any heat engine.

22.4 REVERSIBILITY OF CARNOT CYCLE

Since the processes, isothermal expansion, adiabatic expansion, isothermal compression and adiabatic compression are reversible processes because of total absence of friction between the piston and the cylinder, the Carnot Cycle can be made perfectly reversible. Starting from the point A on the P-V diagram shown in Fig 22.3 the Carnot Cycle can be traced back in succession along the line A D C B A. The sequence of processes are (1) adiabatic expansion (A to D) (2) isothermal compression (D to C) (3) adiabatic compression (C to B) and (4) isothermal compression (B to A). In this process an amount of heat Q_2 is removed from the reservoir kept at lower temperature T_2 and an amount of heat Q_1 is delivered to the reservoir kept at higher temperature T_1. In the reversed Carnot Cycle work must be done on the system which extracts heat from the reservoir at low temperature. Any amount of heat can be removed from the lower temperature reservoir by repeating the reverse cycle. Thus the system functions as a refrigerator transferring from a body at a lower temperature that is freezing compartment to one at high temperature that is the room by means of work supplied to it in the form of electrical energy.

22.5 EFFICIENCY OF HEAT ENGINES

The efficiency of a heat engine η can be defined as the ratio of the net work done by the engine during one cycle to the heat taken in from the high temperature source in one cycle.

For the ideal Carnot engine we have

$$\eta = \frac{W}{Q_1} = \frac{Q1 - Q2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \ldots(22.2)$$

Since $Q_1 > Q_2$ the efficiency of a heat engine is less than 1 so long as the heat Q_2 delivered to the exhaust is not zero. A study of all practical heat engine indicates that energy heat engine rejects some heat during the exhaust stroke. This is the amount of heat absorbed by the engine, which is not converted into work in the process

The efficiency of Carnot engine can also be expressed in terms of the temperatures T_1 and T_2 of the source and sink.

Referring to Fig 22.3 when the system is taken through the path A B by reversible isothermal expansion process, the temperature and internal energy of the ideal gas remain constant. As per the first law of thermodynamics the heat Q_1, absorbed by the gas in its expansion must be equal to the work W_1, done in this expansion. Hence,

$$Q_1 = W_1 = MRT_1 \log\left(\frac{V_2}{V_1}\right) \quad \ldots(22.3)$$

Where M is the mass of the gas in moles and R is called gas constant. The value of R is 8.314 $Jmol^{-1}K^{-1}$

During the isothermic compression process along the path C, D the amount of work W_2 is done on the gas by the piston leading to a transfer of heat Q_2 to the reservoir at temperature T2. Hence

$$Q_2 = W_2 = MRT_2 \log\left(\frac{V_3}{V_4}\right) \qquad ...(22.4)$$

Dividing Eqn. 22.3 By Eqn. 22.4 we get

$$\frac{Q_1}{Q_2} = \frac{T_1 \log(V_2/V_1)}{T_2 \log(V_3/V_4)} \qquad ...(22.5)$$

For the isothermal process along the paths A B and C D we have the following relations to hold good for an ideal gas

$$P_1T_1 = P_2V_2 \qquad ...(22.6)$$

$$P_3V_3 = P_4 V_4 \qquad ...(22.7)$$

For the adiabatic processes along the Paths B C and D A we have

$$P_2V_2^{\gamma} = P_3V_3^{\gamma} \qquad ...(22.8)$$

and

$$P_4V_4^{\gamma} = P_1V_1^{\gamma} \qquad ...(22.9)$$

Multiplying Eqs. 22.6,22.7, 22.8, and 22.9 we get

$$P_1V_1P_3V_3P_2V_2^{\gamma}P_4V_4^{\gamma} = P_2V_2P_4V_4P_3V_3^{\gamma}P_1V_1^{\gamma} \qquad ...(22.10)$$

$$\text{or } V_1V_3V_2^{\gamma}V_4^{\gamma} = V_2V_4V_1^{\gamma}V_3^{\gamma} \qquad ...(22.11)$$

$$\text{or } (V_2V_4)^{\gamma-1} = (V_1V_3)^{\gamma-1} \qquad ...(22.12)$$

$$\text{or } V_2V_4 = V_1V_3 \qquad ...(22.13)$$

Substituting Eqn. 22.14 in Eqn. 22.5 we get

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \frac{\log(V_3/V_4)}{\log(V_3/V_4)} = \frac{T_1}{T_2} \qquad ...(22.15)$$

As per the definition of the efficiency of heat engine we have

$$\nu = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

T_1 and T_2 are in 0^0K.

Example 1:

Calculate the theoretic efficiency of a steam engine operating at 10 atmosphere at which pressure water boils at 180^0C. The temperature of the condenser is 30^0C.

The efficiency of heat engine can be expressed as

$$\nu = \frac{T_1 - T_2}{T} = \frac{(273+180) - (273+30)}{(273+180)}$$

$$\nu = \frac{453 - 303}{453} = 0.33$$

The efficiency of the heat engine is 33%

Example 2:

A Carnot Cycle uses 1 mole of an ideal gas whose C = 25 J $mole^{-1}K^{-1}$ as the working substance. It operates from the most compressed stage of 10 atm. Pressures and 500 K. It expands isothermally to a pressure of 1 atm. and then adiabatically reaches a most expanded stage at a temperature of 300 K. Determine the numerical values of heat and work done in each stroke. Determine the efficiency of the system

The data given in the problem is presented in Fig 22.4

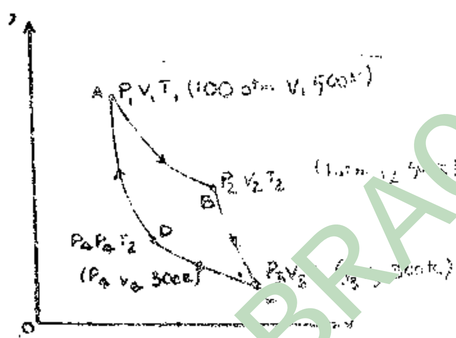


Fig 22.4

The work done by the gas when the system goes from A to B in the isothermal compression process.

$$W_1 = \int PdV = MRT_1 \int_{V_1}^{V_2} \frac{dV}{V} = MRT_1 \log \frac{V_2}{V_1}$$

Since the gas contained in the cylinder is one mole M = 1. For the isothermal process we have $P_1V_1 = P_2V_2$. Hence $\frac{V_2}{V_1} = \frac{P_1}{P_2}$

$$\text{Therefore } W_1 = RT_1 \log_e \frac{P_1}{P_2} = 2.303RT_1 \log_{10} \frac{P_1}{P_2}$$

As per the data given in the problem

$T_1 = 500\ K^0 P_1 = 1$ atm. $P_2 = 1$ atm.

Since $R = 8.314$ J mole $^{-1}(K^0)^{-1}$ we get

$$Q_1 = W = (8.314\ J\ mole^{-1} K^{-1})(1 mol)(500K)(2.303)\ Log_{10} \frac{10}{1}$$

$$Q_1 = W_1 = (8.314)(1)(500)\ 2.303(1)\ J = 9547\ J$$

Since the working substance is an ideal gas and the expression being adiabatic no heat enters or leaves the system. The work done by the gas in pushing the piston leading to the path BC shows in Fig 22.4 comes as a result of conversion of heat energy when the system's temperature changes from T_1 to T_2. Since C_V is the specific heat we have.

$$W_2 = \mu CV\ (T_1 - T_2) = 25\ J\ mole^{-1}\ (K^0)^{-1}(1 mole)\ (500\text{-}300)K^0$$

$$W_2 = 5000J$$

In the process isothermal compression shown by the path C D the quantity of heat given out by the gas to the sink is given by

$$Q_2 = W_3 = \int_{V_3}^{V_4} PdV = \mu RT_2 \int \frac{dV}{V} = \mu RT_2 \log \frac{V_4}{V_3}$$

Since the working substance is an ideal gas of 1 mol mass we have

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} = \frac{P_1}{P_2}$$

$$\therefore Q_2 = W_3 = 2.303\ RT_2 \log_{10} \frac{P_2}{P_1}$$

$$= 2.303\ (8.314)300\ \log 10(1/10)$$

$$= -2.303(8.314)300\ \log_{10} 10$$

$$Q_2 = W_3 = -5744\ J$$

In the adiabatic compression on the path of the process is D.A When work is done on the gas the system's temperature changes from T_2 to T_1 Hence

$$W_4 = \mu C_V\ (T_2 - T_1) = (1)\ (25)\ (300 - 500) = 5000\ J$$

The efficiency of the heat engine is given by

$$v = \frac{Q1 - Q2}{Q_1} = \frac{9754 - 5744}{9754} = 0.41 = 41\ \%$$

v can also be obtained as

$$v = \frac{T_1 - T_2}{T_1} = \frac{500 - 300}{500} = 0.4 = 4\%$$

22.6 CARNOT'S THEOREM

From the study of practical heat engines and the analysis of the ideal. Carnot engines Carnot proposed a theorem regarding the efficiency of heat engines. The Carnot's theorem may be stated as that the efficiency of reversible --engines operating between the same two temperatures is the same and no irreversible engine working between the same two temperatures can have a greater efficiency than the reversible engine. Clausius and Kelvin showed that the above theorem was a necessary consequence of second law of thermodynamics. The efficiency of a reversible engine is independent of the working substance and depends only on the temperatures.

To prove the Carnot's theorem let us consider two reversible engines E and E^1 as shown in Fig 22.5

The engines E E^1 operate between the temperatures T_1 and T_2 where $T_1 > T_2$. The engines may differ in their working substance, or in their initial pressures and lengths of stroke. Let the engine E runs in the forward direction (direct carnot Cycle of operation) and let the

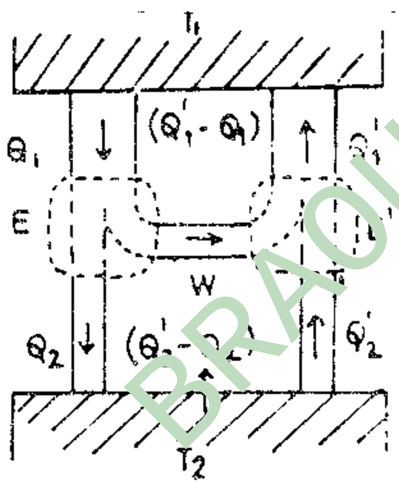


Fig 22.5

Engine E^1 runs in the backward direction (reverse Carnot cycle of operation). The engine E takes heat Q_1 at T_1 and gives out heat energy Q_2 at T_2. The backward running engine E^1 (say refrigerator) takes heat Q_2 at T_2 and give out heat Q_1, at T_1. Let the engines E and E^1 be connected mechanically as shown in Fig 22.5 and the stroke lengths be adjusted so that the work done per cycle by E is just sufficient to operate E^1

Let us assume that the efficiency of E say η be, greater than the efficiency of E^1 say η^1

$$\eta > \eta^1 \qquad \ldots(22.7)$$

Then $$\frac{Q_1 - Q_2}{Q_1} > \frac{Q^1_1 - Q^1_2}{Q^1_1} \qquad \ldots(22.18)$$

Since the work done per cycle by one engine is equal to the work done per cycle by the other engine. We have

$$W = W^1 \qquad \ldots(22.19)$$

or

$$Q_1 - Q_2 = Q^1_1 - Q^1_2 \qquad \ldots(22.20)$$

Since $Q_1 - Q_2 > Q$ and from Eqs. 22.20 and 22.18 we have

$$\frac{1}{Q_2} > \frac{1}{Q^1_2} \qquad \ldots(22.21)$$

or

$$Q_1 < Q^1_1 \qquad \ldots(22.22)$$

Since $W = W^1$ we also have

$$Q_2 < Q^1_2 \qquad \ldots(22.23)$$

Equations 22.22 and 22.23 indicate that the hot source gains heat $Q^1_1 - Q_1$, and the cool source loses heat $Q^1_2 - Q_2$. But not work is done in the process by the combined system, $E + E^1$. That is heat has been transferred from a body at T_2 (lower temperature) to a body at T_1 (higher temperature) without performing work. This behaviour is contrary to the second law of thermodynamics. Hence, we can conclude that η can not be greater than η^1 If we consider E^1 to work in the forward direction and E to work in the backward direction we can also prove on the same lines discussed above that η^1 can not be grater than η. The net result, is

$$\eta = \eta^1 \qquad \ldots(22.24)$$

there by providing the first part of the Carnot's theorem.

To prove the second part of the Carnot's theorem let us consider E to be an irreversible engine and E^1. By following the same analysis mentioned earlier we can prove that η^1, can not be greater than η_{ir}. Since E now is irreversible we cannot reverse the cycle and hence we can not prove that η^1 cannot be greater than η_{ir}. Hence η_{ir} is either equal to or less than η^1. Since $\eta = \eta^1 = \eta$ reversible we get

$$\eta_{\text{irreversible}} \leq \eta_{\text{reversible}} \qquad \ldots(22.25)$$

Thereby proving the second part of Carton's theorem

Example 3:

A steam engine takes steam from the boiler at 200^0C at 10 atmospheric pressure and exhausts directly into air at 1 atmospheric pressure at 100^0C. Calculate the maximum efficiency of the steam engine.

The maximum efficiency of the steam engine according to Carnot's theorem depends on the temperatures of hot and cold bodies and is given by

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{[(200+273)-(100+273)]}{(200+273)} = 100/473 = 0.211 \text{ or } \eta = 21.1\%$$

The actual efficiency of the heat engine is less than the maximum efficiency, which is possible only under ideal conditions. Energy is lost by friction, turbulence and heat conduction. In steam engines the usual efficiency attainable is above 15%. Lower exhaust temperatures or more complicated steam engines may raise the maximum efficiency attainable to 35% and actual efficiency realisable to 20%. The actual efficiency of ordinary automobile engine attainable is about 22%. In the case of large diesel oil engine actual efficiency realisable is about 40%.

22.7 SUMMARY

Carnot's Cycle is useful in converting heat into energy. Carnot's Cycle is a reversible cycle consisting of four process namely (1) isothermal expansion (2) adiabatic compression (3) isothermal compression (4) adiabatic expansion

The work done by Carnot engine is given $W = Q_1 = Q_2$. Where Q_1 is the heat gained by the system from high temperature heat source and Q_2 is the heat given out by the system to lower temperature heat link. The efficiency of heat engine is given by

$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$. The efficiency of heat engine depends on the temperature of hot and cold bodies and is independent of working substance.

Carnot's theorem may be stated as the efficiency of reversible engine operating between the same two temperatures is the same and no irreversible engine working between the same two temperatures can have a greater efficiency than the reversible engine.

22.8 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Show that in a Carnot Cycle the net work done by the system is equal to the difference of heat gained by system at temperature T_1 and the heat given out by the system at temperature T_2.

2. State and prove Carnot's theorem.

3. Define what is meant by efficiency of a heat engine. Derive an expression for the efficiency of the heat engine for the efficiency of the heat engine in terms of temperature of the source and sink.

II Solve the following problems

1. An ideal heat engine operates in a Carnot cycle between 300 and 150^0C. It absorbs 50×10^4J of heat at the higher temperature. How much work per cycle is turned out by this engine.

(Ans: 13.1 J)

2. In a Carnot cycle the isothermal expansion of the gas takes place at 500°K and the isothermal compression at 300°K. During the expansion 2500J of heat energy is transferred to the gas. Determine the heat rejected by the gas during the isothermal compression.

3. 3 . (Ans. 1500 J)

UNIT – 23: SECOND LAW OF THERMODYNAMICS AND ENTROPY

Contents

23.1 OBJECTIVES

This Unit discusses the formulation of second law of thermodynamics and its application. Also discusses the concept of entropy and its application in understanding the second law of thermodynamics.

To make you understand the concept this Unit explains

(1) Formation of second law of thermodynamics

(2) What is meant by entropy and how it changes in reversible and irreversible processes and

(3) The second law of thermodynamics interms of Entropy

After going through this unit you will be able to explain thermodynamic temperature scale, and make out that the Entropy is a measure of disorder.

23.2 INTRODUCTION

The first law of thermodynamics establishes the internal convertibility of heat and work. Experience tells us that no law achieves 100% efficiency in converting work into heat. For example: by devising a machine whose sole function is to create friction between moving part. The convert process, that is complete conversion of heat into work, has not been found possible. Investigations on the possibility to achieve complete conversion of heat into work led to the formulation of second law of thermodynamics. The formulation of second law of thermodynamics and its important application in the formulation of thermodynamic temperature scale are discussed in this Unit.

The second law of thermodynamics as stated by Kelvin and Clausius does not specify the direction of change in a chemical or physical process. By defining a new state function called entropy it is possible to express second law, of thermodynamics in a mathematical form. Let us now study what is meant by entropy, how it can be calculated for reversible and irreversible processes and the relation between entropy and disorder. The physical quantity entropy was introduced by Clausius.

23.3 FORMULATION OF SECOND LAW OF THERMODYNAMICS

The analysis of the various thermodynamic processes in the ideal carnot engine, which works on the Carnot cycle, indicates that it is impossible to convert heat completely into work. While it was the desire to design a perfect heat engine where all the heat input is converted into work output as shown in fig 23.1 (a). Practical experience showed that a sizeable portion of the absorbed heat by the system was still discharged at the lower temperature exhaust of the engine. A fraction of the absorbed heat alone was converted into work. This is illustrated in *Fig 23.1b*

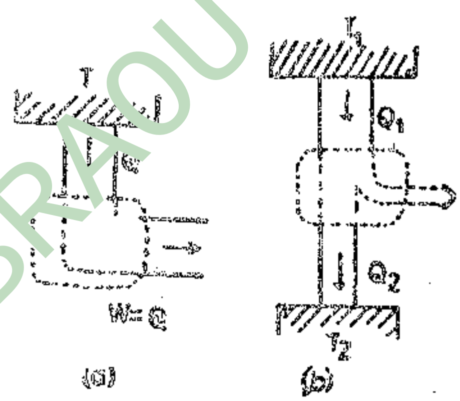


Fig 23.1

The design of heat engines working on the reverse carnot cycle, the refrigerator, to simply transfer heat from a cold body to a hot body with out the expense of outside work as illustrated in 23.2a . work W is needed to transfer heat from a low temperature reservoir to a high temperature reservoir

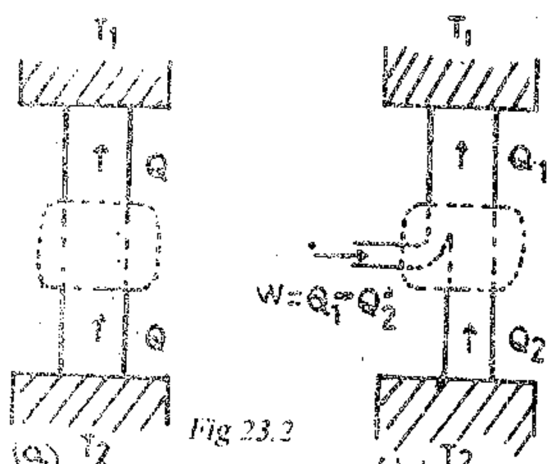


Fig 23.2

Clausius and Kelvin generalized the discovery made by carnot and stated the second law of thermodynamics in two different forms. Lord Kelvin along with Planck stated the law, which is as follows:

It is impossible to construct a device which operating on a cycle will produce no other effect than extraction of heat from a reservoir and the performance of an equivalent amount of work.

Extensive experience with refrigerating devices led to the generalisation of the second law of thermodynamics. Clausius stated the law as – it is impossible to construct a device which, operating on a cycle, will produce no other effect than the transfer of heat from a cooler to a warmer body.

There are many statements of second law, which means the same as that of the two statements given above. This law stated in either way can not be proved directly since it is in the negative form. – To show that the two statements are equivalent it is enough if we simply prove that if one statement is false the other statement also is false.

Let us suppose that the Clausius statement be false. Then we can – have a refrigerator operating without external work. By connecting this perfect refrigerator to an ordinary engine as shown in Fig 23.a heat would be returned to the hot body without expenditure of work and would become available again for use by the heat engine. Hence the combination of an ordinary engine and the perfect refrigerator would constitute a heat engine, which violates Kelvin – Planck statement.

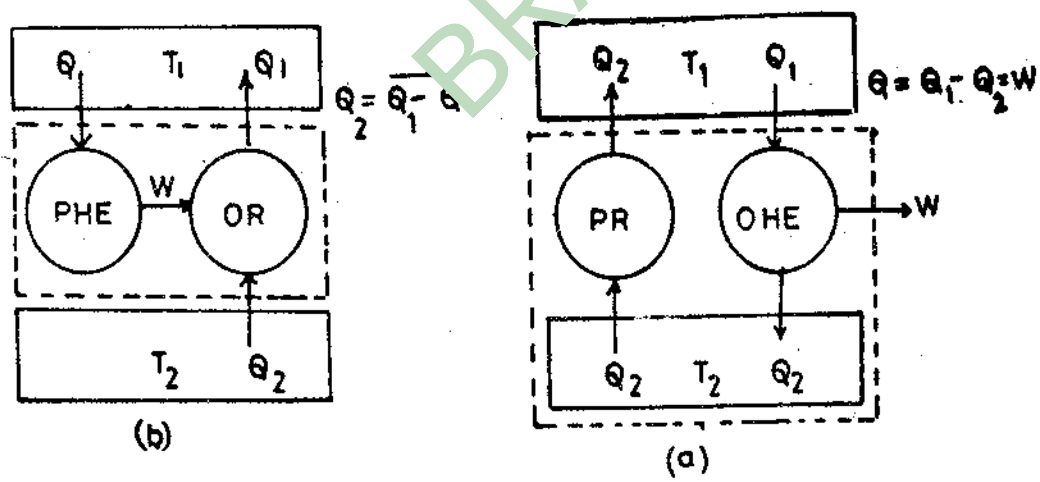


Fig 23.3

If Kelvin – Planck statement were wrong, we can have a heat engine which simply takes heat and converts it completely into work. By connecting this perfect heat engine to an ordinary refrigerator as shown in Fig 23.3(b), we can extract heat from the hot body convert it completely to work which can be used to run the refrigerator. The refrigerator extracts heat from the cold body and delivers it plus the work converted to heat by the refrigerator to the hot body. The net result is transfer of heat from cold body to hot body without expenditure of work. This process violates the Clausius' statement.

From the above argument we can conclude that if Kelvin – Planck statement is correct Clausius statement is also correct and thereby proving the second law of thermodynamics.

The second law of thermodynamics is related to equilibrium because work can be obtained from a system only if it is not already at equilibrium. If a system is at equilibrium, no process tends to occur spontaneously and there is nothing to harness to produce work. The second law also implies that spontaneous flow of heat is from a high to a low temperature and that the reverse is possible only when work is expanded.

The statements of Kelvin and Clausius provide fundamental definitions for temperature and discussed in the following section. Temperature defined based on second law of thermodynamics is identical with the temperature scale which makes the relation $PV = nRT$ which holds good for ideal gases.

23.4 THERMODYNAMIC TEMPERATURE SCALE

The change in the physical property of a substance like expansion of a liquid or gas, change in electrical resistance of platinum etc is generally employed in measuring the temperature of a substance in the Centigrade scale or Fahrenheit scale or Reanmur scale. The temperature so determined depends on the nature of the substance and hence the temperature measured is not absolute but is relative since these scales are defined with respect to the specific property of the substance namely the freezing and boiling point of water which are taken as reference standards. From the work of carnot on ideal heat engines and the second law of thermodynamics, Lord Kelvin realized the possibility to define temperature in terms of energy which is independent of the nature of any particular substance. Lord Kelvin worked out the theory of such an absolute scale starting from the result of the efficiency of ideal Carnot engine. This temperature scale is called as Kelvin (or thermodynamic) temperature scale.

The efficiency η of all reversible heat engine working between two temperatures θ_1 and θ_2 depends only on these temperatures and is independent of the working substance.

Hence $\eta = f(\theta_1, \theta_2)$ ...(23.1)

We can also represent η, as

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad ...(23.2)$$

Where Q1 is the amount of heat absorbed at higher temperature θ1, and Q2 is the amount of heat rejected at the lower temperature θ2, where θ1 and θ2 are being measured in any arbitrary scale.

From 23.1 and 23.2

$$1 - \frac{Q_2}{Q_1} = f(\theta_1, \theta_2) \quad ...(23.3)$$

Or

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f(\theta_1,\theta_2)} = F(\theta 1,\theta 2) \qquad \ldots(23.4)$$

Where F denotes some other function of θ_1, θ_2. For a reversible engine working between θ_1 and θ_3. Where $\theta_2 > \theta_3$, we have

$$\frac{Q_2}{Q_3} = F(\theta_2,\theta_3) \qquad \ldots(23.5)$$

For a reversible engine working between θ1 and θ3 where θ1 > θ3 we have

$$\frac{Q1}{Q3} = F(\theta 1,\theta 3) \qquad \ldots(23.6)$$

Multiply Eqs and 23.2 and 24.5 we get

$$\frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = \frac{Q_1}{Q_3} = F(\theta_1,\theta_3) = F(\theta_1,\theta_2) \times F(\theta_2,\theta_3) \qquad \ldots(23.7)$$

Eq. 23.7 is valid only if

$$F(\theta_1,\theta_2) = \frac{\varnothing(\theta_1)}{\varnothing(\theta_2)} \qquad \ldots(23.8)$$

Where ϕ is another function of temperature. Therefore, for any reversible engine we can write

$$\frac{Q_1}{Q_2} = \frac{\varnothing(\theta_1)}{\varnothing(\theta_2)} \qquad \ldots(23.9)$$

Since $\theta_1 > \theta_2$ and $Q_1 > Q_2$ we have $\phi(\theta_1) > \phi(\theta_2)$.

This indicates that $\phi(\theta)$ is a linear function of θ and may be used to measure the temperature. Let $\phi(\theta)$ be denoted by τ. Then Eq 23.9 becomes.

$$\frac{Q_1}{Q_2} = \frac{r_1}{r_2} \qquad \ldots(23.10)$$

Equation 23.10 can be used to define a new scale of temperature r. which is called thermodynamic scale or Kelvin scale. This scale is independent of the properties of any particular substance and Eq. 23.10 is universally true. The ratio of any two temperatures on this scale is equal to the ratio of heats taken in and rejected out by an engine working reversibly between these two temperatures.

The zero of Kelvin's temperature scale i.e., $\tau = 0$ is that temperature at which $Q_2 = 0$ Hence W = Q1. Thus all the heat taken by the engine has been converted into work and the efficiency of the engine is 100%^ i.e $\tau = 1$. τ Cannot be less than 0, that is negative since Q_2 becomes negative implying that the engine would draw heat both from the source and the sink. This is against the second law of thermodynamics. Hence, $\tau = 0$ is the lowest temperature conceivable. This is called the absolute zero of temperature.

To determine the size of each degree in Kelvin's temperature scale let us suppose that the reversible engine by working between the boiling point and freezing points of water normal pressure. Let ABCD represent indicator diagram as shown in fig 23.4 for the carnot cycle for its operation. The work done by the engine is given by the area ABCD. Let the area ABCD be divided into 100 equal parts by drawing isothermals parallel to AB and CD . The area of each point will correspond to one degree on the Kelvin's absolute scale. This degree Kelvin may be defined as the difference in the temperatures between which the reversible engine should be worked to get energy equal to one hundredth of the area of the indicator diagram representing carnot's cycle between the temperatures of boiling water and melting ice

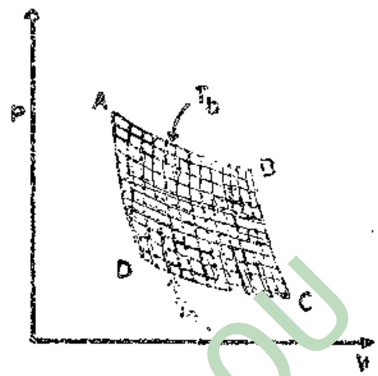


Fig 23.4

By keeping the temperature of the source constant say at the boiling point of water, the work done by the engine will be made to increase as the temperature of the sink is reduced. If we let the adiabatic expansion BC to continue so that the working substance cools continuously. A stage will be reached when the substance no longer expands. The isothermal corresponding to this stage is called the zero isothermal. The temperature of the isothermal, as shown as Fig 23.5 represents the zero degree of the absolute scale.

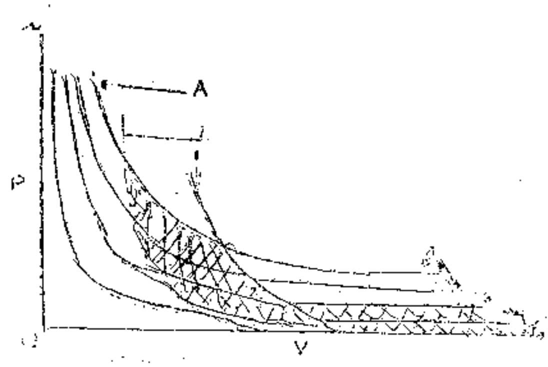


Fig 23.5

The above theoretically developed Kelvin's temperature scale can be realized in practice since it agrees completely with the perfect gas scale as proved below.

For a reversible engine-containing perfect gas as the working substance the efficiency η can be given by

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \ldots(23.11)$$

Where T_1 and T_2 represent the temperature of the source and sink measured on the perfect gas scale.As per Eq 23.11 we get

$$\frac{T_1}{T_2} = \frac{T1}{T2} \qquad \ldots\ldots(23.12)$$

Eq.23.12 indicates that the ratio of any two temperatures on the perfect gas scale and the thermodynamic gas scale are equal. If $\tau = 0$. and hence the zero of the thermodynamic scale coincides with the zero of the perfect gas scale. If T1, T2 represents the temperatures of boiling point of water and melting point of ice measured on the perfect gas scale we have

$$T_1 - T_2 = 100 \qquad \ldots(23.13)$$

On the Kelvins temperature scale we have for the two fixed points

$$T_1 - T_2 = 100 \qquad \ldots\ldots(23.14)$$

We can write

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{100}{T_1} \qquad \ldots(23.15)$$

And also

$$\eta = \frac{T1 - T2}{T_1} = \frac{100}{T1}$$

Eqs. 23.15 and 24.16 indicate that the temperatures of the boiling point of water and the melting point of ice are identical on the two scales. Hence, any temperatures has the same value on the two scales and hence the two scales as identical. The melting point of ice given in Clausius scale as O° C is equal to 273.15 K in that Kelvin's temperature scale.

It is worthwhile to note here that the fundamental feature of all cooling processes in that, the lower the temperature, the more difficult it is to go still lower. This practical experience has led to the formulation of the third law of thermodynamics. It can be states as that it is impossible by any procedure, however idealized it may be, to reduce any system to the absolute zero of temperature in a finite number of operations. Since we cannot have a sink at absolute zero, to realize a heat engine with 100^0 efficiency is a practical impossibility.

23.5 DEFINITION OF ENTROPY

In the carnot cycle, the wrong substance undergoes a thermodynamic change in the process. If Q_1 represents the heat given to the working substance at temperature T_1 and Q_2 represents the heat taken away from the working substance at temperature T_2 we have from 23.15 (See unit 22)

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \qquad \ldots(23.17)$$

In the above equation Q's are taken as positive quantities. If Q is taken as positive when it enters the system and as negative when it leaves the system. We have

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \qquad \ldots(23.18)$$

The above equation indicates that the sum of the algebraic quantities Q/T is zero for a carnot cycle.

A reversible Carnot cycle can be taken to be equivalent to a large number of Carnot cycles as illustrated in Fig 23.1. Fig 23.1a represents an arbitrary reversible cycle super imposed on a family of isotherms. By connecting these isotherms by suitable adiabatics as shown in fig 23.1b The actual cycle can be split up into as assembly of Carnot cycles. This is because the outer parts of the set of Carnot cycles trace out a curve that approximates with that of the original cycle. If one performs all the Carnot cycles in the set, the net result would be almost the same as performing the original cycle. This is because of the fact that all the Carnot cycle steps that are inside the boundary are cancelled out since each is traced in both a forward and reverse direction. By dividing the original Carnot cycle into infinitely large . Carnot cycles we can get the exact reproduction of the process. For the isothermal-adiabatic sequence of lines as shown in Fig 23.6 b we get

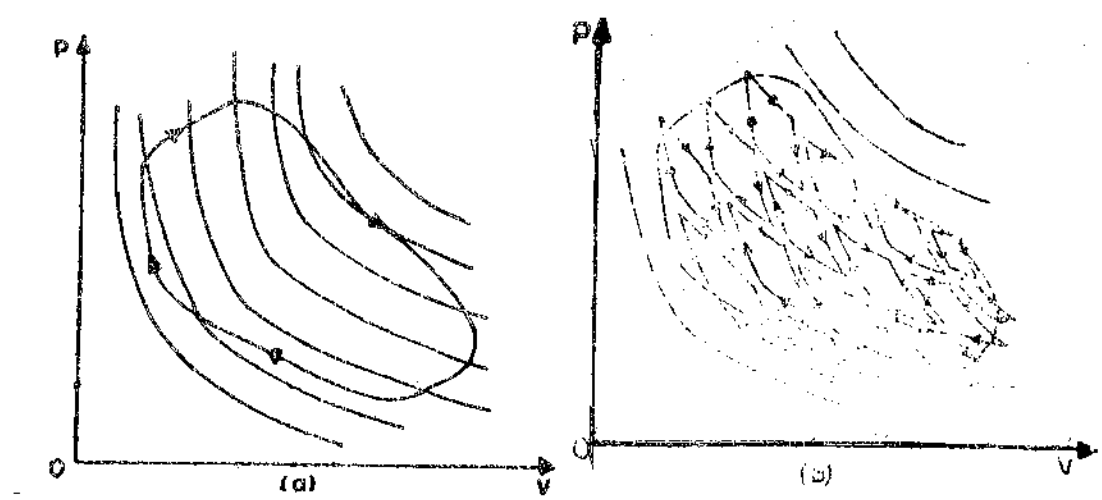


Fig 23.6

$\Sigma\ Q/T = 0$ ...(23.18)

In the limit of infinite small temperature difference between the isotherms we get

$$\oint \frac{dQ}{T} \quad ...(23.19)$$

The sign ∮ signified integration around a complete cycle. A zero value for a cyclic integral implies that the function being integrated is independent of the path over which the integration is made. Such a function is called a state function a thermodynamic property. This thermodynamic property is called entropy and hence we have

$$ds = \frac{dQ}{T} \quad ...(23.20)$$

And

$$\oint ds = 0 \quad ..(23.21)$$

The units of entropy is $J\ K^{-1}$

Entropy may be defined as that thermal property of a substance which remains constant when the substance undergoes adiabatic changes since heat is neither communicated to the system nor taken away from it . It is a physical quantity which can not be felt like temperature and pressure. It is a definite single valued function of the thermodynamic co-ordinates which define the state of the substance namely temperature, pressure volume and internal energy.

23.6 CHANGE IN ENTROPY IN A REVERSIBLE PROCESS

Consider any two equilibrium states A and B of a system and the paths connecting them are reversible as shown in Fig 23.7 . For this type of system where the process is a reversible process we have

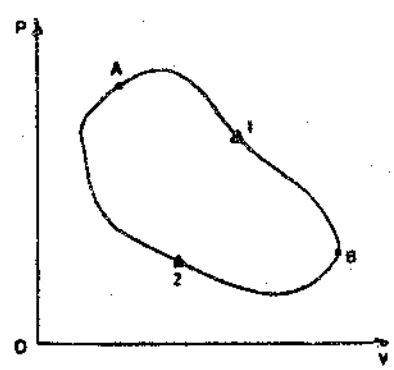


Fig 23.7

$$\int_{1A}^{B} ds + \int_{2B}^{A} ds = O \qquad \ldots (23.22)$$

where 1 and 2 describe the paths connecting the points A and B since the cycle is reversible.

$$\int_{1A}^{B} ds + \int_{2A}^{B} ds = O \qquad \ldots (23.23)$$

or

$$\int_{1A}^{B} dS = \int_{2A}^{B} dS \qquad \ldots (23.24)$$

Eqn 23.9 indicate that the quantity $^{B}\int_{A} dS$ between any two equilibrium states of a system such as A and B is independent between the states A and B. The change in entropy between the states A and B is given by

$$SB - SA = \int_{A}^{B} dS = \int_{A}^{B} \frac{dQ}{T} \qquad \ldots (23.25)$$

In a reversible carnot cycle there are two adiabatic and two isothermal processes, During the adiabatic processes no heat enters or leaves the system . Hence there is no change in entropy. Out of the two isothermal process one is an expansion at temperature T_1 and another is a compression at T_2 in the isothermal expansion process an amount of heat Q_1 is absorbed at T_1 Net gain of entropy is (Q_1/T_1). In the isothermal compression process an amount of heat Q_2 is liberated at T_2. Net loss of entropy is Q_2/T_2. Hence the effective change in entropy ΔS for the whole cycle is given by

$$\Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \qquad \ldots (23.26)$$

For a reversible cycle $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ Hence .

$$\Delta S = O$$

Eqn 23.12 indicates that the entropy of a system remains constant in all reversible processes

If a process is carried out reversibly between two states A and B then as per Eqn . 23.10

$$\int_{A}^{B} dS = -\int_{B}^{A} dS \qquad \ldots (23.27)$$

Or

$$S_B - S_A = \Delta S_{AB} = - \Delta S_{AB} \qquad ...(23.28)$$

Eqn. 23.14 indicates that the heat which is absorbed or lost by the system during the process A B must be transferred reversibly from or to the surrounding. Hence in any process carried out reversibly, the entropy gained or lost by the system must be lost or gained by the surroundings. Hence the sum of entropy changes of the system and the surroundings must be zero. Hence

$$\Delta S_{AB} \text{(System)} = - \Delta S_{AB} \text{(Surroundings)} \qquad ...(23.29)$$

$$\Delta S_{Total} = \Delta S_{BAsys} + \Delta S_{AB}\text{(Surroundings)} \qquad ...(23.30)$$

Example – 1

Determine the change in entropy in the conversion of 1 mole of liquid water at 100^0 c
To vapour 100^0 C at 1 atm pressure
The system consists of 1 mole of water and the surroundings consist of a heat reservoir at 100^0 as shown in fig 23.8.

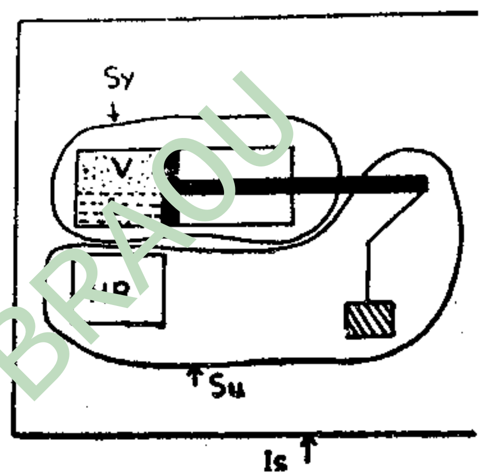


Fig 23.8

The temperature gradient between heat reservoir and the water is infinitesimally small. Hence the system absorbs the necessary heat from the reservoir reversibly and the water gets converted into vapor.

The Change in entropy of the system.

$$\Delta S = \int \frac{dQ}{T}$$

Since T is constant

$$\Delta S = \frac{1}{T} \int dQ = \frac{Q}{T}$$

The amount of heat required to change 1 g of water to vapor is the latent heat of vaporization given by 2258 j/g. Hence the heat absorbed to convert 1 mole of water to vapor Q is given by

$$Q = 2258 \text{ j/g} \times 18 = 40644 \text{ J}.$$

$$\Delta S_{system} = \frac{Q}{T} = \frac{40644J}{373} = 104JK^{-1}$$

The surroundings, that is the heat reservoir also experience a change in entropy due to loss of heat equal to 4066 J at temperature 373 K. Thus the change in entropy of the Surroundings.

$$\Delta S_{Surroundings} = -\frac{40644}{373} = -109 \text{ JK}^{-1}$$

The total change in the entropy as the isolated system consisting of the actual system and surroundings is zero.

23.7 CHANGE IN ENTROPY IN IRREVERSIBLE PROCESS

Since the entropy of a system depends only on the state of the system we can calculate the change in entropy for irreversible processes also. The only criteria required is that the irreversible process states from an equilibrium state and ends in another equilibrium state. We know that irreversible processes are spontaneous processes like transfer of heat from hot body to cold body (heat conduction) and expansion of a gas into vacuum. Let us calculate the change in entropy in the above processes.

(i) Heat conduction

As shown in fig 23.9 let heat flow from the hot body at Temp T1 to cold body at temperature T2

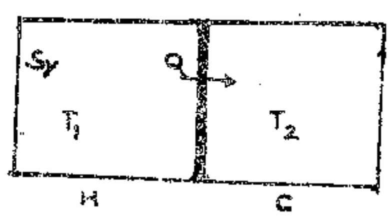


Fig 23.9

If Q represents the quantity of heat transferred from the hot body to cold body, then the change in entropy of the hot body.

$$\Delta S_{hot\ body} = -\frac{Q}{T_1} \qquad \ldots(23.31)$$

The change in entropy of the cold body

$$\Delta S_{\text{cold body}} = \frac{Q2}{T2} \qquad \ldots(23.32)$$

The entropy change of the system

$$\Delta S_{\text{system}} = \Delta S_{\text{hot body}} + \Delta S_{\text{cold body}} \qquad \ldots(23.33)$$

$$\Delta S \text{ System} = -\frac{Q}{T_1} + \frac{Q}{T_2} = Q\left[\frac{1}{T_2} - \frac{1}{T_1}\right] \qquad \ldots(23.34)$$

Since $T_1 > T_2$ ...(23.35)

ΔS System > O ... (23.36)

In the irreversible adiabatic heat conduction process the entropy of the system increases.

(II) Free expansion.

Consider an ideal gas of volume V_1 expand into vacuum and let the final volume be V_2 Since no work is done on the system in the expansion process and the gas is enclosed by non - conducting walls as shown in Fig 23.10

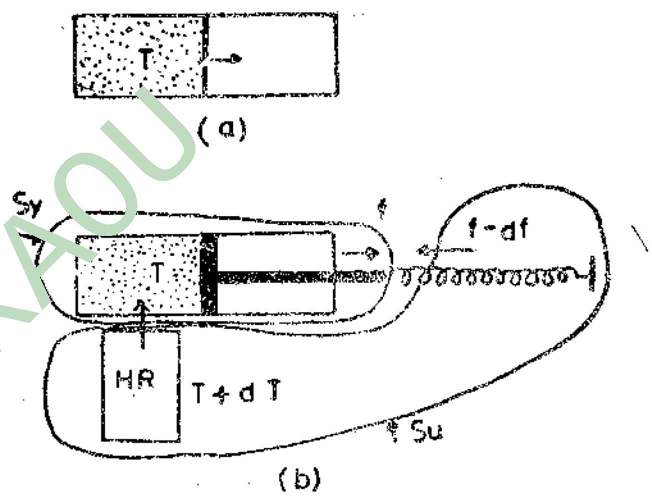


Fig. 23.10

F.Q = 0 From the first law of thermodynamics $\Delta V = 0$ that is $U_i = U_f$ where i and f represent the initial and final states. Since the gas is an ideal U depends on the temperature and not on volume or pressure. Hence the temperature of initial and final states is the same i.e. $T_i = T_f = T$.

The free expansion is a irreversible process. There occurs a change in entropy of the system when the volume changes from V_1 to V_2. We can not analyse this process directly to find the entropy difference between the expanded gas and the initial gas. It is necessary to think of a process which can be carried out reversible that takes the system from the same initial state to the same final state as in free expansion. Fig 23.5b. illustrates how this can be achieved. This is an isothermal process. The heat absorbed can be calculated based on first law of thermodynamics. Thus

$$dQ = dw = Pdv = \mu RT \frac{dv}{v} \qquad \ldots(23.37)$$

Where μ represents the number of moles of the gas.
The entropy change of the system

$$\Delta S \text{ system} = \int_{V1}^{V2} \frac{dQ}{T} = \mu R \int_{V_1}^{V2} \frac{dv}{v} \qquad \ldots(23.38)$$

$$\Delta S \text{ System} = \mu \text{ Rlong} \left(\frac{V2}{V1}\right) \qquad \ldots(23.39)$$

Since $V_2 > V_1$ ...(23.40)

$$\Delta S \text{ system} > O \qquad \ldots(23.41)$$

Hence the entropy of the system increases in the irreversible process.

23.8 SECOND LAW OF THERMODYNAMICS IN TERMS OF ENTROPY

The second law of thermodynamics can be stated in terms of entropy of the system which is as follows.

A natural process that starts in one equilibrium state and ends in another equilibrium state will go in the direction that causes the entropy of the system plus the environment to increase.

All irreversible processes which are spontaneous and hence natural are consistent with the above statement. The essence of second law is that there exists an useful thermodynamic variable called entropy. This way of viewing at the second law is similar to the statements there exists an useful thermodynamic quantity called temperature in the case of zeroth law and there exists an useful thermodynamic quantity called internal energy in the case of first law.

The second law as stated above applies only for irreversible process, which have a natural direction. An important deduction of this law can be seen in the famous view of Clausius. "The energy of the Universe is constant: the entropy of the universe tends always towards a maximum". Since all natural processes occurring in the universe are spontaneous there must occur an increase of entropy and hence the sum total of the entropy of the universe is continuously increasing. A philosophical deduction of this due to sir Arthur Eddington is that "entropy is time's arrow"

Reversible Processes can go equally well in either direction. For reversible processes the entropy of the system plus environment remains unchanged. When the process is reversible the environment and the system differ in temperature by only a differential amount dT, when the heat transfer takes place.

In adiabatic processes, which can be reversible or irreversible, there is no transfer of heat with the environment and hence entropy change occurs only in the system. As per the statement of second law in terms of entropy we have.

$$S_f = S_i \qquad \ldots(23.42)$$

for reversible adiabatic process and

$$S_f > S_i \qquad \ldots (23.42)$$

for irreversible adiabatic process. Where S_i and S_f represent the initial and final entropies of the system.

The statement of second law in terms of entropy of the system is consistent with the statement of Clausius that there is no such thing as perfect refrigerator. If there exists a perfect refrigerator then the entropy of the lower temperature reservoir should decrease by Q/T_2 and that of the upper temperature reservoir should increase by Q/T_1. The entropy of the system should remain constant since it undergoes a complete cycle. Hence the net change in the entropy of the system plus environment is a decrease. This violates the principle of second law and if the law is to be retained as applicable then there is no such thing as perfect refrigerator.

The statement of second law in terms of entropy of the system is also consistent with Kelvin-Planck statement. which implies that there is no such thing as perfect heat engine. If there is a perfect heat engine then the entropy of the reservoir at temperature T should decrease by Q/t where as the entropy of the system remains unchanged giving rise to a net decrease in entropy of the system plus environment. This violates the second law stated in terms of entropy and hence there is no such thing as a perfect heat engine.

Example – 2.

Determine the entropy change of a system consisting of 5Kg of ice at 0^0 C which melts, irreversibly to water at 0^0 C. The latent heat of melting is 333 J/g. Since ice is is made to melt irreversibily, it must be kept in contact with a heat reservoir whose temperature is 0°C by only a differential amount. When the temperature of the reservoir is lowered by a differential amount the melted ice begins to freeze. Since the process is reversible the change in entropy.

$$\Delta S = \int_0^T \frac{dQ}{T} = \frac{Q}{T}$$

now $Q = 5 \times 10^3 \text{ g} \times 333 \text{J/g} = 1665 \times 10^6$ J

$$\Delta \text{ S System} = \frac{1.665 \times 10^6 \text{ j}}{273^0 \text{ K}} = 6.1 \times 10^3 \text{ J.K}^{-1}$$

The change in entropy of the environment

$$\Delta \text{ S environment} = \frac{-1.665 \times 10^6 \text{ j}}{273^0 \text{ K}} = -6.1 \times 10^3 \text{ J.K}^{-1}$$

The net change in the entropy of the system plus reservoir is zero

In practice melting of ice is an irreversible process. Suppose ice is made to melt in a glass of water, the entropy of the system plus environment in this case increases.

23.9 ENTROPY AND DISORDER

Heat is a disordered energy. Energy can exist without disorder. A flying rifle bullet or an atom of U^{235} carries ordered energy. The moving bullet has kinetic energy. When the bullet hits a metallic plate it transfers its energy to the plate and gets stopped. The kinetic energy of the bullet is transferred into random motions of the atoms of the bullet and as well as the metal plate. This is a disordered energy which is felt in the from of heat.

The energy in U^{235} is potential energy arising out of the forces binding the nucleus inside the nucleus. When the U^{235} atom undergoes fission results in a disordered energy, which is felt in the form of heat.

The above two examples illustrate that energy, becomes heat as soon as it is disordered. Disorder can exist without energy and if it is energized it becomes heat. The quantity of disorder and energy can be expressed in terms of the physical quantities namely entropy and Joule. If disorder and entropy are related then like entropy, disorder must also increase in natural processes. Indeed this is true in the irreversible processes like free expansion and heat conduction.

In free expansion the gas molecules confined to one half of a box say left half before expansion fill the entire box. We know that in this case entropy increases. Disorder also increases. This is because before expansion we say that the molecules are in the box. That is the system is specified more generally than specifically.

In heat conduction two bodies at temperatures T_1 and T_2 come to uniform temperature T when they are brought together. The system has become more disordered in this natural process. Now we say that all molecules in the system correspond to the temperature T. Before the bodies are brought into contact we can say that all molecules of body A are at temperature T_1 and all molecules of body B are at temperature T_2. Because of heat conduction we are going from more specific to less specific state. That is disorder has increased.

The above examples indicate that there is a tendency for natural process to proceed towards a state of greater disorder.

In the universe all processes occur spontaneously. Hence entropy of the universe is increasing. That is an ordered state of the universe we are tending towards a disordered state.

Statistical mechanics enables us to connect the entropy and disorder by the following relation.

$$S = K \log_e W \qquad \ldots(23.43)$$

Where K represents the Boltzmann's constant and W represents the disorder parameter. We represents the probability that the system can exist in the state in relation to all possible states it could be in. This equation connects a thermodynamic quantity (S) with a statistical quantity (W).

In terms of disorder we can calculate the change in entropy of an ideal gas in an isothermal expansion. In this process the number of molecules (N) and the temperature (T) remain constant and volume alone changes. The probability that molecule can be found in a volume V is proportional to W. Hence the probability of finding a single molecule in V is given by

$$W_1 = cV \quad \ldots(23.44)$$

Where c is constant. The probability of finding N molecules simultaneously in the volume V is W given by

$$W = W_1^N = (CV)^N \quad \ldots(23.45)$$

Substituting Eq. 23.45 in Eq 23.43, we get

$$S = K N \log c V = KN (\log C + \log V) \quad \ldots(23.46)$$

The difference in entropy between a state of volume V_f and a state of volume Vi when T and N are constant can be given by

$$S_f - S_i = KN \ (\log C + \text{Log } V) - K N (\log C + \log V) \quad \ldots(23.47)$$

Or

$$S_f - S_i = KN \log \frac{Vf}{Vi} = \frac{RN}{No} \log \frac{Vf}{Vi} \quad \ldots(23.48)$$

$$S_f - S_i = \mu R \log \frac{Vf}{Vi} \quad \ldots(23.49)$$

Where N_o represents the Avagadro's number

Eq. 23.39 is the same as the Eq (23.39)

Due to irreversible expansion if the volume of the gas doubles then W goes from $(C_1V)^N$ to $(C_2 V)^N$ Since W represents disorder, it increases in the natural process of free expansion The second law of thermodynamics can be stated based on statistical mechanics. The direction in which natural process take place is determined by the laws of probability that is towards a more probable state. The equilibrium state is the state of maximum entropy thermodynamically and the most probable state statistically. Hence second law of thermodynamics shows us the most probable course of events and not the only possible ones.

23.10 SUMMARY

It is impossible to construct a device which operating on a cycle, will produce no other effect than the extraction of heat from a reservoir and the performance of an equivalent amount of work (According to Kelvin).

It is impossible to construct a device which operating on a cycle will produce no other effect than the extraction of heat from a cooler to a warmer body (Clausius statement) Lord Kelvin defined temperature in terms of energy, which is independent of the nature of any particular substance. The degree Kelvin is the difference in temperatures between which the reversible engine works to get energy equal to the (1/100th) of the area of the indicator diagram representing Carnot's cycle between the temperatures of boiling water and melting ice. The third law of thermodynamics can be stated as it is impossible by any procedure to reduce any system to the absolute zero of temperature in a finite number of operations.

The entropy of a system is a function of the state. The change in entropy of a system depends only on the initial and final states of the system and not on the nature of the process. The entropy of the universe is increasing. Entropy is a measure of disorder. Disorder increases with the increase in entropy.

23.11 SAMPLE EXAMINATION QUESTIONS

I. Answer the following questions in detail.

1. Discuss the formulation of thermodynamic temperature scale based on second law of thermodynamics.

2. Discuss the formulation of second law of thermodynamics from the results of the working of ideal Carnot engine. Give an account of the experimental verification of the second law.

3. Define the term entropy. Show that in all reversible processes the total entropy of the system and its surroundings remains constant.

4. Show with suitable examples the change in entropy of the system undergoing an irreversible process is always positive.

5. Show that when the entropy of a system increases its disorder also increases.

6. State and explain second law of thermodynamics in terms of entropy of the systems. Show that this statement is consistent with Clausius and Kelvin Plank statement.

UNIT: 24 - THERMODYNAMIC POTENTIALS

Contents

24.1 OBJECTIVES

This unit discusses the equation for thermodynamic potentials and their derivation by combining the I[st] Law of thermodynamics and Carnot's theorem.

After going through this Unit you should be able to find out the mathematical relation between E, H, A and G.

24.2 INTRODUCTION

Many useful thermodynamic relations can be obtained based on the laws of thermodynamics and the properties of the system. From the first law of thermodynamics we have

$$dE = dQ + dW \qquad \ldots(24.1)$$

Where dE represents the change in energy of the system dQ represents the elemental heat produced and dW represents the elemental work done. Eq. 24.1 applies to both reversible and irreversible process.

For a revrsible process we have, based on Carnot's theorem.

$$dQ = T\,ds \qquad \ldots(24.2)$$

If P represents the pressure and V represents the volume of the system then the work done on the system.

$$dW = -PdV \qquad \ldots(24.3)$$

Substituting Eqs. 24.2 and 24.3 in Eq. 24.1 we get

$$dE = Tds - pdv \qquad \ldots(24.4)$$

The above equation is called Clausius' differential relation. This equation applies to the closed system. performing PV work only. It incorporates the mathematical content of the first and second law of thermodynamics. The significance of Clausius' differential equation is that it connects the five most fundamental functions of a state.

24.3 THERMODYNAMIC POTENTIALS

Clausius' differential relation can be interpreted as the total differential of E represented as a function of S and V. Hence

$$E = E (S, V) \quad \text{...(24.5)}$$

We have

$$dE = \left(\frac{dE}{ds}\right)_V dS + \left(\frac{dE}{dV}\right)_S dV \quad \text{...(24.6)}$$

Comparing this with the Clausius's differential relation

$$dE = Tds - PdV$$

we get

$$\left(\frac{dE}{dS}\right)_V = T \quad \text{...(24.52)}$$

and

$$\left(\frac{dE}{dV}\right)_S = -P \quad \text{...(24.6)}$$

The enthalpy H of a system which is a function of S and P can be written as

$$H (S,P) = E + PV \quad \text{...(24.7)}$$

Since H = H (S,P)

$$dH = \left(\frac{dH}{ds}\right)_P ds + \left(\frac{dH}{dP}\right)_S dP \quad \text{...(24.8)}$$

Also since H = E + PV ...(24.9)

$$dH = dE + PdV + VdP \quad \text{...(24.10)}$$

Using Eq. (24.4) in Eq. (24 10)

$$dH = TdS - PdV + PdV + VdP \quad \text{...(24.11)}$$

or

$$dH = TdS + VdP \quad \text{...(24.12)}$$

Comparing Eqs. 24.8 and 24.12 we get

$$\left(\frac{dH}{dP}\right)_p = T \quad \text{...(24.13)}$$

and

$$\left(\frac{dH}{dP}\right)_s = V \quad \text{...(24.14)}$$

The Helmholtz free energy of the system A is characteristic function of T and V. It is defined as

$$A\,(T.V) = E - TS \quad \text{...(24.15)}$$

Since

$$A = A(T.V)$$

We can write

$$dA = \left(\frac{dA}{dT}\right)_V dT + \left(\frac{dA}{dv}\right)_T dV \quad \text{...(24.16)}$$

Also since

$$A = E - TS \quad \text{...(24.17)}$$

$$dA = dE - TdS - SdT \quad \text{...(24.18)}$$

Using Eq. 24.4 in Eq. 24.18 we get

$$dA = TdS - PdV - TdS - SdT$$

or

$$dA = -PdV - SdT \quad \text{...(24.20)}$$

Comparing Eqs. 24.16 and 24.20 we get

$$\left(\frac{dA}{dT}\right)_V = -S \quad \text{...(24.21)}$$

and

$$\left(\frac{dA}{dT}\right)_V = -P \quad \ldots(24.22)$$

The Gibbs free energy of the system G is a characteristic function of T and P. It is given by

$$G(T, P) = E + PV - TS \quad \ldots(24.23)$$

From Eq. 24.7

$$G(T, P) = H - TS \quad \ldots(24.24)$$

Also From Eq. 24.15

$$G(T, P) = A + PV \quad \ldots(24.28)$$

Since

$$G = G\ (T, P) \quad \ldots(24.26)$$

$$dG = \left(\frac{dG}{dT}\right)_P dT + \left(\frac{dG}{dP}\right)_T dP \quad \ldots(24.27)$$

From the relation

$$G = A + PV$$

$$dG = dA + Pdv + vdp \quad \ldots(24.28)$$

$$dG = -PdV - SdT + PdV + VdP \quad \ldots(24.29)$$

g eq.24.20

$$= -SdT + VdP \quad \ldots\ldots(24.30)$$

Comparing Eq. 24.27 with Eq. 24.30 we get

$$\left(\frac{dG}{dT}\right)_P = -S \quad \ldots(24.31)$$

$$\left(\frac{dG}{dP}\right)_T = V \quad \ldots(24.32)$$

The parameters E, H, A and G are called the thermodynamic potentials. The sequence for transformations leading to these thermodynamic potentials was first devised by Gibbs. From a knowledge of any one of these potentials as a function of its natural variables one can calculate the other thermodynamic potentials. To illustrate if E (S, V) is know we can determine T (S, V) and P (S, V) from Eqs 24.5 and 24.6. By eliminating S from the equation 24.5 and 24.6 we can get the equation of the state F (P, V, T) = 0

Since conditions of constant pressure are more prevalent than the conditions of constant volume, H and G occur more naturally in experimental thermodynamics than E and A.

Since volume can be more easily defined theoretically than pressure, E and A are more fundamental quantities in statistical thermodynamics. A and G are important in the study do thermodynamic equilibrium. The four differential expressions

$dE = TdS - PdV$

$dH = TdS + VdP$

$dA = -SdT - PdV$

$dG = -SdT + VdP$

Provide a starting point for the derivation of many useful relationships among thermodynamic variables. To remember the above differential relations a device can be used as illustrated in 24.1

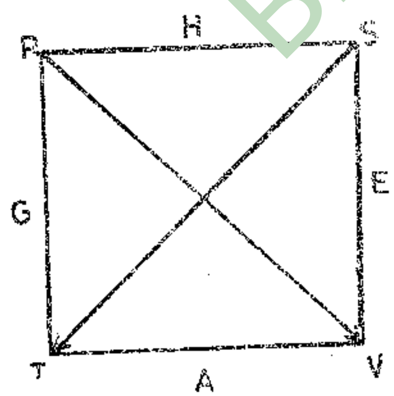


Fig 24.1.

The four independent variables P, S, T, V (in alphabetical order) are placed at the corners of a square. The potentials E, H, A, G are placed at the sides such that each lies between its natural variables. To write the differential expression for the potentials multiply the differential of each independent variable by the quantity diagonally opposite. Add if the diagonal move follows an arrow and subtract if it opposes an arrow. To illustrate let us write the differential relation for E. In the diagram dE contains dV and dS. The coefficient of dv opposes the arrow. Hence we take it as (-P). The coefficient of dS is along the arrow. So it is (+T). Hence $dE = TdS - PdV$. Similarly we can write for dH, dA and dG easily.

24.4 SUMMARY

E, A, H & G are thermodynamic potentials. The relation between them is

$$dE = TdS - PdV$$

$$dH = TdS + VdP$$

$$dA = -SdT - PdV$$

$$dG = -SdT + VdP$$

The physical quantities T, P, V, S in terms of thermodynamic potentials can be given by the following relations

$$T = \left(\frac{dE}{dS}\right)_V = (dH / dS)_P$$

$$P = -\left(\frac{dE}{dV}\right)_S = -\left(\frac{dA}{dV}\right)$$

$$V = \left(\frac{dH}{DP}\right)_S = \left(\frac{dG}{dP}\right)_T$$

$$S = -\left(\frac{dA}{dT}\right)_V = -\left(\frac{dG}{dT}\right)_P$$

23.11 SAMPLE EXAMINATION QUESTIONS

I Answer following questions in detail

1. What are thermodynamic potentials?
2. Write down the differential relations for the thermodynamic potentials

UNIT- 25: MAXWELL'S THERMO DYNAMIC EQUATIONS AND APPLICATIONS

Contents

25.1 OBJECTIVES

This unit discusses the Maxwell's thermodynamic equations and its applications.

After going through this unit you will be able to

(i) Understand the Maxwell's thermodynamic equations.

(ii) Explain the applications of these equations.

25. 2 INTRODUCTION

The main aim of this chapter is to make use of 2 laws of thermodynamics in convenient form and to a set of fundamental relations, known as thermodynamic relations. which find ready application to particular problems. Here the main aim is to deduce these relations & indicate their application to some thermal phenomena.

In general the condition of a substance is completely determined by any pair of the quantities P. V. T & S. In solving therefore, any thermodynamic problem, the pair most suitable is chosen as independent variables. Also the applications aspect especially to that of (1) clausius clapyron's latent heat equation & (2) Joule –Kelvin Effect is dealt in this chapter. though there are many other applications. The quantities dealt above as P, V, T & S are pressure. volume. temperature & entropy respectively.

25.3 MAXWELL'S EQUATIONS

Maxwell's Equations can be derived directly from thermodynamic potentials, & also using I law of Thermodynamics.

Maxwell's first relation

$du = T\,ds - P\,dv$

We write $\left[\frac{\delta u}{ds}\right]$, const = T and $\left[\frac{\delta u}{dv}\right] = P\left[\frac{Su}{dv}\right] = P$ ....(25.1)

δu being a perfect differential, the condition

$\frac{\delta^2 u}{dvds} = \frac{\delta^2 u}{dvds}$ must be satisfied

partially differentiating the expression V $\frac{\delta u}{ds} = T$ w.r.t.

and $\frac{\delta u}{ds} = -P$ w.r.t S. we have

$$\frac{\delta^2 u}{dvds} = \left[\frac{\delta T}{dv}\right]_S \text{ and } \frac{\delta^2 u}{dvds} = -\left[\frac{\delta p}{di}\right]_V$$

Substituting these values in (1) we get

$$\left[\frac{dT}{dv}\right]_S = \left[\frac{dp}{ds}\right]$$

This is Maxwell's 1st equation.

II. Relation:

Enthalpy h = U + PV

In terms of basic coordinates

dh = T ds + V dp

We have from the above equations

$$\left[\frac{dh}{ds}\right]_p = T \text{ and } \left[\frac{dh}{dp}\right]_S = V$$

differentiating partially

$$\left[\frac{d^2h}{dsdp}\right] = \left[\frac{dT}{dv}\right]_S \text{ and } \left[\frac{d^2h}{dpds}\right] = \left[\frac{dv}{ds}\right]_p$$

since dh is a perfect differential, we have

$$\left[\frac{dT}{dp}\right]_S = \left[\frac{dv}{ds}\right]_p$$

This is Maxwell's 2nd equation.

III. Relation: Helmholtinz Functions F = U – Ts

df = - Pdv – S dt (∴ du = dq – P dv From I law. & dq = T ds from II law)

Which gives $\left(\frac{df}{dq}\right)_T = -P$ and $\left(\frac{df}{dt}\right)_V = -S$

Partially differentiating w.r.v.t andwλ.v. respectively
We have

$$\frac{d^2f}{dvdt} = \left(\frac{dp}{dt}\right)_V \text{ and } \frac{d^2f}{dtdv} = \left(\frac{ds}{dv}\right)_T$$

Since df is a perfect differential we have $\left(\frac{dp}{dT}\right)_V = \left(\frac{ds}{dv}\right)_T$ This is Maxwell's 3[rd] equation.

IV. Relation: Gibb's Function G = U + Pv – Ts

In terms of basic coordinates

dG = Vdp - sdT, Which gives , $\left(\frac{dg}{dp}\right)_T = V$ and $\left(\frac{dg}{dt}\right)_P = -S$

Differentiating w.r.t.T & differentiating w.r.t.P. respectively, the above equations, we have

$$\frac{d^2G}{dpdT} = \left(\frac{dv}{dT}\right)_p \quad \& \quad \frac{d^2G}{dTdp} = \left(\frac{ds}{dp}\right)_T$$

Since dG is a perfect differential $\frac{d^2G}{dpdT} = \frac{d^2G}{dTdp}$

Hence we write

$\left(\frac{dv}{dT}\right)_p = \left(\frac{ds}{dp}\right)_T$ This is Maxwell's 4[th] equation

These above Maxwell's thermodynamic equations 1, 2, 3 & 4 must hold good for any pure substance.

25.4(a) APPLICATIONS

Clusius Clapyrons Latent Heat equation:

The clausius clapyrons latent heat equation relates the change in melting point or boiling point with change in pressure. We take the Maxwell's third equation, which contains

dp/dT term for the purpose & obtaining this equation

$$\left(\frac{dp}{dT}\right)_v = \left(\frac{ds}{dv}\right)_T \qquad \dots (25.3)$$

Multiplying both the sides by T, we have

$$T\left(\frac{dp}{dT}\right) = T\left(\frac{Tds}{dv}\right)_T = \left(\frac{Tds}{dv}\right)_T \text{ since T is a constant}$$

$$= \left(\frac{dq}{dv}\right)_T$$

$$= \frac{L}{Vf - Vi}$$ - Putting (dq/T) = L latent heat & dv = v from B.

$$\left(\frac{dp}{dT}\right)_v = \frac{L}{Vf - Vi} \qquad (25.4)$$

This equation (25.4) is known as clausius clapyrons latent heat equation. This equation pertains to first order phase transition when there is a emission or absorption of heat without any change in temperature. As there is transfer of heat in phase transitions of the orders there is a change in entropy a Volume. The pressure also remains constant. The suitable thermodynamics potential to describe such a physical process is the Gribbs potential.

Gribbs function G = U + Pv –TS (∴du = dq – Pdv from 1 & = Tds – Pdv laws).

or dG = dP – S dt

Which gives $\left(\frac{dG}{dP}\right)_T = V$ and $\left(\frac{dG}{dP}\right)_P = -S$

The first order derivative of Gribb's Functions giving Entropy and Volume, Change discontinuously.

Thus during first order phase transition such as melting ,vapourisation and sublimation. The Gibbsfunction first derivative change discontinuously at the transition point.

There are changes in Entropy & Volume because of the transference of heat.

However it should be noted that Gribbs Function in the same in both the phases at equilibrium in an Isothermal -Isobaric change.

25.4(b) JOULE – KELVIN'S EFFECT

When ever or real gas in throttled through a porous plug. there is always a change in the temperature of the gas, a cooling in general, which is reffered to as Joule kelvins or Joule-Thomson effect. This effect was discovered for the first time in 1852 by Joule-Thomson.

One essential condition of Joule-Kelvin's expansion of a real gas is that the enthalpy of the gas h = u + Pv must remain constant i.e. h = 0, although there is a pressure difference across the throttling valve or porous plug.

Joule Kelvins coefficient is defined as the ratio of the temperature of a gas to the change of pressure upon throttling at constant enthalpy & represented by μ

Thus $\mu = (dT / dP)_h$

The expn for μ can be achieved i.e. Joule – Kelvin coefficient is terms of the basic thermodynamics coordinates from enthalpy & second T.ds equations.

Enthalpy $h = u + Pv$ (25.5)

and $dh = Tds + udp$

Taking the second Tds equation

$$Tds = cPdT - T\left(\frac{dV}{dT}\right)_p dp \qquad (25.6)$$

and substituting the value in equation (25.5) above we get

$$dh = cpdt - T\left(\frac{dv}{dT}\right)_p dp + Vdp$$

$$= CpdT - \left[T\left(\frac{dv}{dT}\right)_v - v\right] dp \quad \text{or}$$

$$Cpdt = dh + \left[T\left(\frac{dv}{dT}\right)_v - v\right] dp \quad \text{or}$$

$$dT = \frac{dh}{Cp} \left[T\left(\frac{dv}{dT}\right)_v - v\right] dp$$

$dh = 0$ since enthalpy h is constant

$$\therefore dT = \frac{1}{CP}\left[\left(T\frac{dv}{dT}\right)_p - v\right] dp$$

$$\text{But } dT = \left(\frac{dT}{dP}\right)_h - dt\ dp$$

$$\text{so we write} \left(\frac{dT}{dP}\right)_h dt = \frac{1}{CP}\left[T\left(\frac{dv}{dT}\right)_p - v\right]$$

$$\mu = \left(\frac{dT}{dP}\right)_h \text{ we have}$$

Joule – Kelvin coefficient

$$\mu = \frac{1}{C_P}\left[T\left(\frac{dv}{dT}\right)_p - v\right] \qquad \ldots(25.7)$$

When i) $\mu \Rightarrow \left(\frac{dT}{dP}\right)_h$ is negative, a cooling effect is produced.

ii) $\mu \Rightarrow \left(\frac{dT}{dP}\right)_h$ is positive, a heating effect is produced.

iii) $\mu \Rightarrow \left(\frac{dT}{dP}\right)_h$

is zero there is neither cooling nor heating and the corresponding transition temperature known as Inversion Temperature which is a characteristic of the gas. Below the inversion temperature we have J.T. cooling which is of great importance in liquifaction of gases.

25.5 SUMMARY

Maxwell's relations are of great importance in cooling devices. The applications are of primary importance in liquifaction of gases. In this Joule Kelvin effect is used. Most critical thermodynamic problems can be solved by these equations. Use of these is also enunciated in Heat Engines.

25.6 SAMPLE EXAMINATION QUESTIONS

I Answer the following questions in detail

1. Maxwell's Equations - Derive them from primary considerations
2. Joule –Kelvin effect –write about this also derive the Joule –Kelvin coefficient.
3. Clasius clapyron's equation –derive it.
4. Short Notes:
   1) Inversion Temperature
   2) Enthalpy
   3) Helm Holtz Function
   4) Gibbs Function

Dr.B.R.AMBEDKAR OPEN UNIVERSITY
(under graduate programme)
SECOND YEAR SYLLABUS
PHYSICS – COURSE – 2.
ELECTROMAGNETISM AND THERMODYNAMICS

BLOCK – I : VECTORS AND ELECTROSTATICS

Unit –1 : Vectors
Unit –2 : Electric fields and Gauss Theorem
Unit –3 : Electric Potential
Unit –4 : Capacitance
Unit –5 : Parallel plate condenser with and without dielectric

BLOCK – 2 : CURRENT DENSITY, STEADY CURRENTS AND CIRCUITS

Unit –6 : Electrical Conductivity
Unit –7 : Kirchoff's Laws
Unit –8 : Networks

BLOCK – 3 : MAGNETOSTATICS

Unit –9 : Ampere's Law
Unit –10 : Biot-Savart's law
Unit –11 : Magnetic force on a circuit, Torque

BLOCK – 4 : ELECTROMAGNETIC INDUCTION

Unit –12 : Motion of charged particle
Unit –13 : Determination of isotopic masses
Unit –14 : Self inductance and mutual inductance
Unit –15 : Faraday's laws of Induction
Unit –16 : Magnetic Energy – Maxwell's Equations

BLOCK – 5 : VARYING CURRENTS

Unit –17 : LR, LC and CR circuits with A.C
Unit –18 : Transient Response
Unit –19 : Series and parallel resonance circuit

BLOCK – 6 : LAWS OF THERMODYNAMICS

Unit –20 : Zeroth and first law of thermodynamics
Unit –21 : Reversible and Irreversible processes
Unit –22 : Carnot's cycle and carnot's theorem
Unit –23 : Second law of thermodynamics and entropy
Unit –24 : Thermodynamic potentials
Unit –25 : Maxwell's thermodynamic equations and applications.

FACULTY OF SCIENCE

SECOND YEAR (3 YEAR DEGREE COURCE) EXAMINATION

MODEL QUESTION PAPER

COURSE II: ELECTROMAGNETISM & THERMODYNAMICS

Time: 3 Hours

Max Marks:70
Min Marks: 25

SECTION – A
(Marks: 3X15=45)

Instructions to the candidates:

1) **Answer <u>any three</u> of the following questions in about 30 lines each**
2) **Each question carries <u>fifteen</u> marks**

1) Define electric potential and field strength. rive the expression for potential due to a point charge.

2) Explain electric displacement. Applying Gauss theorem find the intensity of the field of uniformly charged sphere.

3) Drive an expression for energy stored in the field of a charged condenser.

4) State Kirchoff's laws. Applying these laws, calculate the potential difference and current in a multiple loop circuit.

5) Describe Thomson's method of experiment for determining e/m of an electron.

6) Drive differential relations for thermodynamic potentials.

SECTION-B
(Marks: 5x5=25)

Instructions to the candidates:

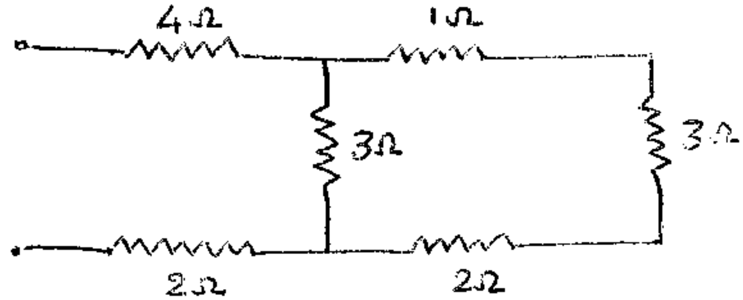
1) **Answer <u>any five</u> of the following questions in about 10 lines each.**
2) **Each question carries <u>five</u> marks.**

7) Explain zeroth law of thermodynamics.

8) Calculate the efficiency of a steam engine operating at 10 atmosphere at which pressure, water boils at 180^0 c the temperature of the condenser is 30^0 c.

9) Write short notes on parallel L C R circuit.

10) Determine the equivalent resistance in the figure.



11) Discuss the growth of current in CR circuits

12) What is equivalent capacitance when three capacitors are connected in series.

13) Drive the differential equation for enthalpy of a thermodynamic system.

14) Show that when entropy of a system increases its disorder also increases.

15) What is Hall effect?

16) Describe faraday's laws of induction